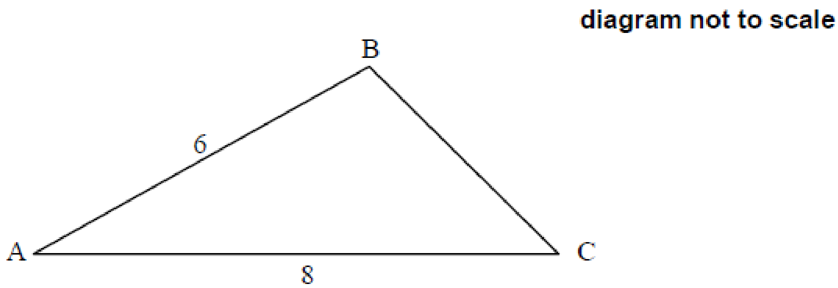
Trig review *[168 marks]*

The following diagram shows triangle ABC, with $AB = 6$ and $AC = 8$.



1a. Given that $\cos \hat{A} = \frac{5}{6}$ find the value of $\sin \hat{A}$. *[3 marks]*

Markscheme

valid approach using Pythagorean identity ***(M1)***

$\sin^2 A + \left(\frac{5}{6}\right)^2 = 1$ (or equivalent) ***(A1)***

$\sin A = \frac{\sqrt{11}}{6}$ ***A1***

[3 marks]

1b. Find the area of triangle ABC. *[2 marks]*

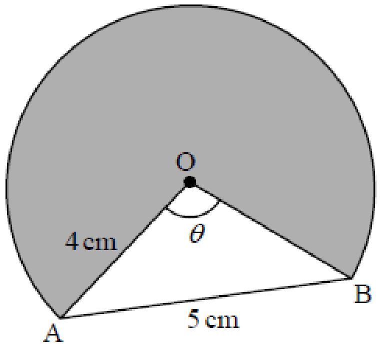
Markscheme

$\frac{1}{2} \times 8 \times 6 \times \frac{\sqrt{11}}{6}$ (or equivalent) ***(A1)***

area $= 4\sqrt{11}$ ***A1***

[2 marks]

The following diagram shows part of a circle with centre O and radius 4 cm.



Chord AB has a length of 5 cm and AÔB = θ.

2a. Find the value of θ, giving your answer in radians. *[3 marks]*

Markscheme

METHOD 1

attempt to use the cosine rule ***(M1)***

$\cos\theta = \frac{4^2+4^2-5^2}{2\times4\times4}$ (or equivalent) ***A1***

$\theta = 1.35$ ***A1***

METHOD 2

attempt to split triangle AOB into two congruent right triangles ***(M1)***

$\sin\left(\frac{\theta}{2}\right) = \frac{2.5}{4}$ ***A1***

$\theta = 1.35$ ***A1***

[3 marks]

2b. Find the area of the shaded region. *[3 marks]*

Markscheme

attempt to find the area of the shaded region ***(M1)***

$\frac{1}{2}\times4\times4\times(2\pi-1.35\ldots)$ ***A1***

$= 39.5\,(\text{cm}^2)$ ***A1***

[3 marks]

Adam sets out for a hike from his camp at point A. He hikes at an average speed of 4.2 km/h for 45 minutes, on a bearing of 035° from the camp, until he stops for a break at point B.

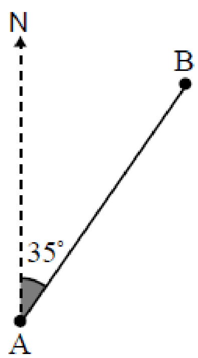
3a. Find the distance from point A to point B. *[2 marks]*

Markscheme

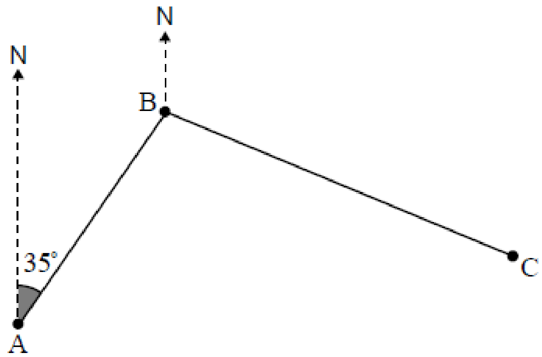
$\frac{4.2}{60} \times 45$ ***A1***

AB = 3.15 (km) ***A1***

[2 marks]



Adam leaves point B on a bearing of 114° and continues to hike for a distance of 4.6 km until he reaches point C.



3b. Show that $\text{A}\hat{\text{B}}\text{C}$ is 101°. *[2 marks]*

Markscheme

66° or (180 − 114) ***A1***

35 + 66 ***A1***

$A\hat{B}C = 101°$ ***AG***

[2 marks]

3c. Find the distance from the camp to point C. *[3 marks]*

Markscheme

attempt to use cosine rule ***(M1)***

$AC^2 = 3.15^2 + 4.6^2 - 2 \times 3.15 \times 4.6 \cos 101°$ (or equivalent) ***A1***

AC = 6.05 (km) ***A1***

[3 marks]

3d. Find $B\hat{C}A$. *[3 marks]*

Markscheme

valid approach to find angle BCA ***(M1)***

eg sine rule

correct substitution into sine rule ***A1***

eg $\frac{\sin\left(B\hat{C}A\right)}{3.15} = \frac{\sin 101}{6.0507\ldots}$

$B\hat{C}A = 30.7°$ ***A1***

[3 marks]

Adam's friend Jacob wants to hike directly from the camp to meet Adam at point C
.

3e. Find the bearing that Jacob must take to point C. *[3 marks]*

Markscheme

$\mathrm{B}\hat{\mathrm{A}}\mathrm{C}$ = 48.267 (seen anywhere) ***A1***

valid approach to find correct bearing ***(M1)***

eg 48.267 + 35

bearing = 83.3° (accept 083°) ***A1***

[3 marks]

3f. Jacob hikes at an average speed of 3.9 km/h. *[3 marks]*

Find, to the nearest minute, the time it takes for Jacob to reach point C.

Markscheme

attempt to use $\text{time} = \frac{\text{distance}}{\text{speed}}$ ***M1***

$\frac{6.0507}{3.9}$ or 0.065768 km/min ***(A1)***

t = 93 (minutes) ***A1***

[3 marks]

Consider a function f, such that $f\left(x\right) = 5.8\sin\left(\frac{\pi}{6}(x+1)\right) + b$, 0 ≤ x ≤ 10,
$b \in \mathbb{R}$.

4a. Find the period of f. *[2 marks]*

Markscheme

correct approach ***A1***

eg $\frac{\pi}{6} = \frac{2\pi}{period}$ (or equivalent)

period = 12 ***A1***

[2 marks]

The function f has a local maximum at the point (2, 21.8) , and a local minimum at (8, 10.2).

4b. Find the value of b. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $\frac{\max + \min}{2}$ $b = \max - \text{amplitude}$

$\frac{21.8+10.2}{2}$, or equivalent

b = 16 ***A1***

[2 marks]

4c. Hence, find the value of $f(6)$. *[2 marks]*

Markscheme

attempt to substitute into **their** function ***(M1)***

$5.8\sin\left(\frac{\pi}{6}(6+1)\right)+16$

f(6) = 13.1 ***A1***

[2 marks]

A second function g is given by $g(x)=p\sin\left(\frac{2\pi}{9}(x-3.75)\right)+q$, 0 ≤ x ≤ 10; p, $q\in\mathbb{R}$.

The function g passes through the points (3, 2.5) and (6, 15.1).

4d. Find the value of p and the value of q. *[5 marks]*

Markscheme

valid attempt to set up a system of equations ***(M1)***

two correct equations ***A1***

$p\sin\left(\frac{2\pi}{9}(3-3.75)\right)+q=2.5,\ p\sin\left(\frac{2\pi}{9}(6-3.75)\right)+q=15.1$

valid attempt to solve system ***(M1)***

p = 8.4; q = 6.7 ***A1A1***

[5 marks]

4e. Find the value of x for which the functions have the greatest difference. *[2 marks]*

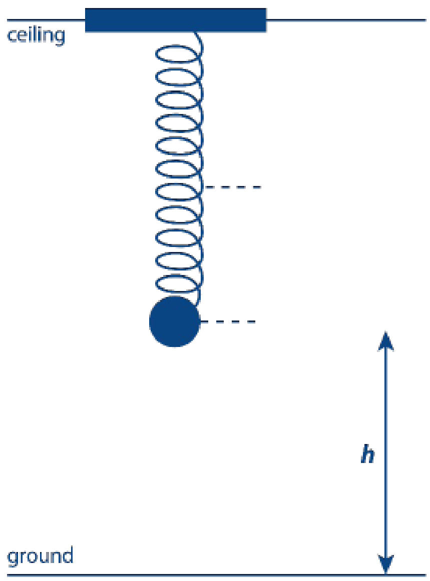
Markscheme

attempt to use $|f(x) - g(x)|$ to find maximum difference ***(M1)***

x = 1.64 ***A1***

[2 marks]

The following diagram shows a ball attached to the end of a spring, which is suspended from a ceiling.



The height, h metres, of the ball above the ground at time t seconds after being released can be modelled by the function $h(t) = 0.4\cos(\pi t) + 1.8$ where $t \geq 0$.

5a. Find the height of the ball above the ground when it is released. *[2 marks]*

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

attempts to find $h(0)$ **(M1)**

$h(0) = 0.4\cos(0) + 1.8 (= 2.2)$

2.2 (m) (above the ground) **A1**

[2 marks]

5b. Find the minimum height of the ball above the ground. *[2 marks]*

Markscheme

EITHER

uses the minimum value of $\cos(\pi t)$ which is -1 **M1**

$0.4(-1) + 1.8$ (m)

OR

the amplitude of motion is 0.4 (m) and the mean position is 1.8 (m) **M1**

OR

finds $h'(t) = -0.4\pi\sin(\pi t)$, attempts to solve $h'(t) = 0$ for t and determines that the minimum height above the ground occurs at $t = 1,\ 3,\ \ldots$ **M1**

$0.4(-1) + 1.8$ (m)

THEN

1.4 (m) (above the ground) **A1**

[2 marks]

5c. Show that the ball takes 2 seconds to return to its initial height above the ground for the first time. *[2 marks]*

Markscheme

EITHER

the ball is released from its maximum height and returns there a period later **R1**

the period is $\frac{2\pi}{\pi}(=2)(\text{s})$ **A1**

OR

attempts to solve $h(t)=2.2$ for t **M1**

$\cos(\pi t)=1$

$t=0,\ 2,\ \ldots$ **A1**

THEN

so it takes 2 seconds for the ball to return to its initial position for the first time **AG**

[2 marks]

5d. For the first 2 seconds of its motion, determine the amount of time that the ball is less than $1.8+0.2\sqrt{2}$ metres above the ground. *[5 marks]*

Markscheme

$0.4\cos(\pi t)+1.8=1.8+0.2\sqrt{2}$ **(M1)**

$0.4\cos(\pi t)=0.2\sqrt{2}$

$\cos(\pi t)=\frac{\sqrt{2}}{2}$ **A1**

$\pi t=\frac{\pi}{4},\ \frac{7\pi}{4}$ **(A1)**

Note: Accept extra correct positive solutions for πt.

$t=\frac{1}{4},\ \frac{7}{4}(0\leq t\leq 2)$ **A1**

Note: Do not award **A1** if solutions outside $0\leq t\leq 2$ are also stated.

the ball is less than $1.8+0.2\sqrt{2}$ metres above the ground for $\frac{7}{4}-\frac{1}{4}$ (s)

1.5 (s) **A1**

[5 marks]

5e. Find the rate of change of the ball’s height above the ground when $t = \frac{1}{3}$. Give your answer in the form $p\pi\sqrt{q}\,\mathrm{ms}^{-1}$ where $p \in \mathbb{Q}$ and $q \in \mathbb{Z}^{+}$. *[4 marks]*

Markscheme

EITHER

attempts to find $h'(t)$ **(M1)**

OR

recognizes that $h'(t)$ is required **(M1)**

THEN

$h'(t) = -0.4\pi \sin(\pi t)$ **A1**

attempts to evaluate their $h'\left(\frac{1}{3}\right)$ **(M1)**

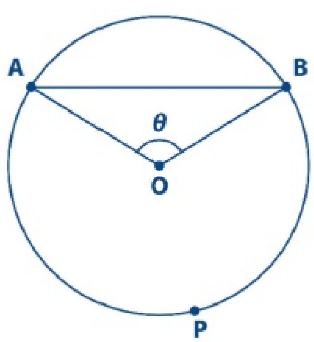
$h'\left(\frac{1}{3}\right) = -0.4\pi \sin \frac{\pi}{3}$

$= 0.2\pi\sqrt{3}\left(\mathrm{ms}^{-1}\right)$ **A1**

Note: Accept equivalent correct answer forms where $p \in \mathbb{Q}$. For example, $-\frac{1}{5}\pi\sqrt{3}$.

[4 marks]

The following diagram shows a circle with centre O and radius 3.



Points A, P and B lie on the circumference of the circle.

Chord $[\mathrm{AB}]$ has length L and $\mathrm{A\hat{O}B} = \theta$ radians.

6a. Show that arc APB has length $6\pi - 3\theta$. *[2 marks]*

Markscheme

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EITHER

uses the arc length formula **(M1)**

arc length is $3(2\pi - \theta)$ **A1**

OR

length of arc AB is 3θ **A1**

the sum of the lengths of arc AB and arc APB is 6π **A1**

THEN

so arc APB has length $6\pi - 3\theta$ **AG**

[2 marks]

6b. Show that $L = \sqrt{18 - 18\cos\theta}$. *[2 marks]*

Markscheme

uses the cosine rule **(M1)**

$L^2 = 3^2 + 3^2 - 2(3)(3)\cos\theta$ **A1**

so $L = \sqrt{18 - 18\cos\theta}$ **AG**

[2 marks]

6c. Arc APB is twice the length of chord $[\mathrm{AB}]$. *[3 marks]*

Find the value of θ.

Markscheme

$6\pi - 3\theta = 2\sqrt{18 - 18\cos\theta}$ **A1**

attempts to solve for θ **(M1)**

$\theta = 2.49$ **A1**

[3 marks]

Consider $f(x) = 4\sin x + 2.5$ and $g(x) = 4\sin\left(x - \frac{3\pi}{2}\right) + 2.5 + q$, where $x \in \mathbb{R}$ and $q > 0$.

The graph of g is obtained by two transformations of the graph of f.

7a. Describe these two transformations. *[2 marks]*

Markscheme

translation (shift) by $\frac{3\pi}{2}$ to the right/positive horizontal direction ***A1***

translation (shift) by q upwards/positive vertical direction ***A1***

Note: accept translation by $\begin{pmatrix} \frac{3\pi}{2} \\ q \end{pmatrix}$

Do not accept 'move' for translation/shift.

[2 marks]

7b. The y-intercept of the graph of g is at $(0,\ r)$. *[5 marks]*

Given that $g(x) \geq 7$, find the smallest value of r.

Markscheme

METHOD 1

minimum of $4\sin\left(x-\frac{3\pi}{2}\right)$ is -4 (may be seen in sketch) ***(M1)***

$-4+2.5+q\geq 7$

$q\geq 8.5$ (accept $q=8.5$) ***A1***

substituting $x=0$ and their $q\ (=8.5)$ to find r ***(M1)***

$(r=)\ 4\sin\left(\frac{-3\pi}{2}\right)+2.5+8.5$

$4+2.5+8.5$ ***(A1)***

smallest value of r is 15 ***A1***

METHOD 2

substituting $x=0$ to find an expression (for r) in terms of q ***(M1)***

$(g(0)=r=)\ 4\sin\left(\frac{-3\pi}{2}\right)+2.5+q$

$(r=)\ 6.5+q$ ***A1***

minimum of $4\sin\left(x-\frac{3\pi}{2}\right)$ is -4 ***(M1)***

$-4+2.5+q\geq 7$

$-4+2.5+(r-6.5)\geq 7$ (accept $=$) ***(A1)***

smallest value of r is 15 ***A1***

METHOD 3

$4\sin\left(x-\frac{3\pi}{2}\right)+2.5+q=4\cos x+2.5+q$ ***A1***

y-intercept of $4\cos x+2.5+q$ is a maximum ***(M1)***

amplitude of $g(x)$ is 4 ***(A1)***

attempt to find least maximum ***(M1)***

$r=2\times 4+7$

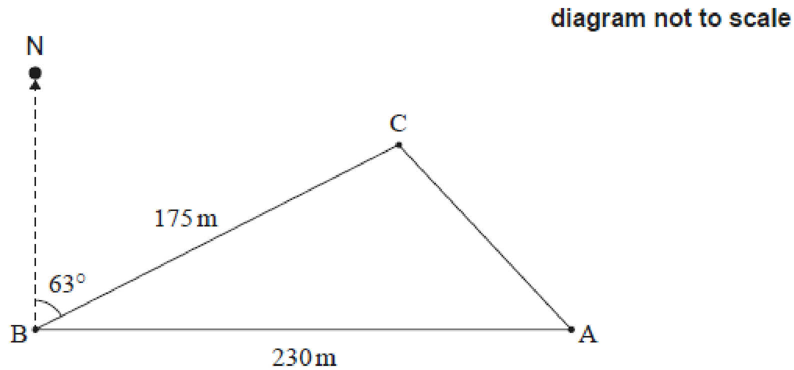
smallest value of r is 15 ***A1***

[5 marks]

A farmer is placing posts at points A, B, and C in the ground to mark the boundaries of a triangular piece of land on his property.

From point A, he walks due west 230 metres to point B.
From point B, he walks 175 metres on a bearing of 063° to reach point C.

This is shown in the following diagram.



8a. Find the distance from point A to point C. *[4 marks]*

Markscheme

$\mathrm{A\hat{B}C} = 27^\circ$ ***(A1)***

attempt to substitute into cosine rule ***(M1)***

$175^2 + 230^2 - 2(175)(230)\cos 27^\circ$ ***(A1)***

$108.62308\ldots$

$\mathrm{AC} = 109\,(\mathrm{m})$ ***A1***

[4 marks]

8b. Find the area of this piece of land. *[2 marks]*

Markscheme

correct substitution into area formula ***(A1)***

$\frac{1}{2} \times 175 \times 230 \times \sin 27^\circ$

$9136.55\ldots$

area $= 9140\left(\mathrm{m}^2\right)$ ***A1***

[2 marks]

8c. Find $\mathrm{C\hat{A}B}$. *[3 marks]*

Markscheme

attempt to substitute into sine rule or cosine rule ***(M1)***

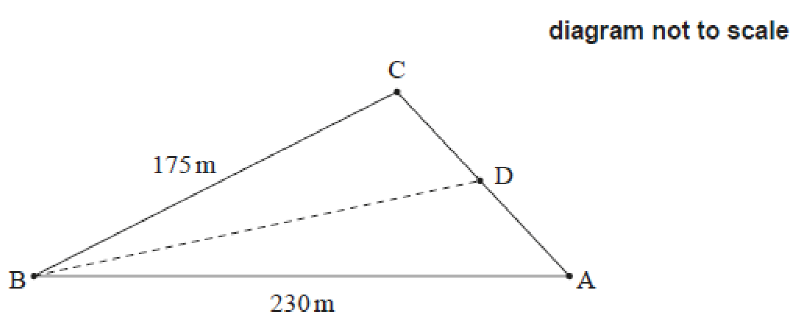
$\frac{\sin 27^\circ}{108.623\ldots} = \frac{\sin \hat{\mathrm{A}}}{175}$ OR $\cos \mathrm{A} = \frac{(108.623\ldots)^2 + 230^2 - 175^2}{2 \times 108.623\ldots \times 230}$ ***(A1)***

$47.0049\ldots$

$\mathrm{C\hat{A}B} = 47.0^\circ$ ***A1***

[3 marks]

The farmer wants to divide the piece of land into two sections. He will put a post at point D, which is between A and C. He wants the boundary BD to divide the piece of land such that the sections have equal area. This is shown in the following diagram.



8d. Find the distance from point B to point D. *[5 marks]*

Markscheme

METHOD 1

recognizing that for areas to be equal, $\mathrm{AD} = \mathrm{DC}$ ***(M1)***

$\mathrm{AD} = \frac{1}{2}\mathrm{AC} = 54.3115\ldots$ ***A1***

attempt to substitute into cosine rule to find BD ***(M1)***

correct substitution into cosine rule ***(A1)***

$\mathrm{BD}^2 = 230^2 + 54.3115^2 - 2(230)(54.3115)\cos 47.0049^\circ$

$\mathrm{BD} = 197.009\ldots$

$\mathrm{BD} = 197\,(\mathrm{m})$ ***A1***

METHOD 2

correct expressions for areas of triangle BDA and triangle BCD using BD ***A1***

$\frac{1}{2} \times \mathrm{BD} \times 230 \times \sin x^\circ$ and $\frac{1}{2} \times \mathrm{BD} \times 175 \times \sin(27 - x)^\circ$ OR

$\frac{1}{2} \times \mathrm{BD} \times 230 \times \sin(27 - x)^\circ$ and $\frac{1}{2} \times \mathrm{BD} \times 175 \times \sin x^\circ$

correct equation in terms of x ***(A1)***

$175 \sin(27 - x) = 230 \sin x$ or $175 \sin x = 230 \sin(27 - x)$

$x = 11.6326\ldots$ or $x = 15.3673\ldots$ ***(A1)***

substituting their value of x into equation to solve for BD ***(M1)***

$\frac{1}{2} \times \mathrm{BD} \times 230 \times \sin 11.6326\ldots = \frac{1}{2} \times \mathrm{BD} \times 175 \times \sin 15.3673\ldots$ or

$\frac{1}{2} \times \mathrm{BD} \times 230 \times \sin 11.6326\ldots = \frac{1}{2} \times 9136.55\ldots$

$\mathrm{BD} = 197\,(\mathrm{m})$ ***A1***

[5 marks]

9. Consider a triangle ABC, where $\mathrm{AC} = 12,\ \mathrm{CB} = 7$ and $\mathrm{B\hat{A}C} = 25^\circ$. *[5 marks]*

Find the smallest possible perimeter of triangle ABC.

Markscheme

EITHER

attempt to use cosine rule ***(M1)***

$12^2 + \mathrm{AB}^2 - 2 \times 12 \times \cos 25^\circ \times \mathrm{AB} = 7^2$ OR
$\mathrm{AB}^2 - 21.7513\ldots\mathrm{AB} + 95 = 0$ ***(A1)***

at least one correct value for AB ***(A1)***

$\mathrm{AB} = 6.05068\ldots$ OR $\mathrm{AB} = 15.7007\ldots$

using their smaller value for AB to find minimum perimeter ***(M1)***

$12 + 7 + 6.05068\ldots$

OR

attempt to use sine rule ***(M1)***

$\frac{\sin \mathrm{B}}{12} = \frac{\sin 25^\circ}{7}$ OR $\sin \mathrm{B} = 0.724488\ldots$ OR $\widehat{\mathrm{B}} = 133.573\ldots^\circ$ OR
$\widehat{\mathrm{B}} = 46.4263\ldots^\circ$ ***(A1)***

at least one correct value for C ***(A1)***

$\widehat{C} = 21.4263\ldots^\circ$ OR $\widehat{C} = 108.573\ldots^\circ$

using their acute value for $\widehat{C}$ to find minimum perimeter ***(M1)***

$12 + 7 + \sqrt{12^2 + 7^2 - 2 \times 12 \times 7 \cos 21.4263\ldots^\circ}$ OR
$12 + 7 + \frac{7 \sin 21.4263\ldots^\circ}{\sin 25^\circ}$

THEN

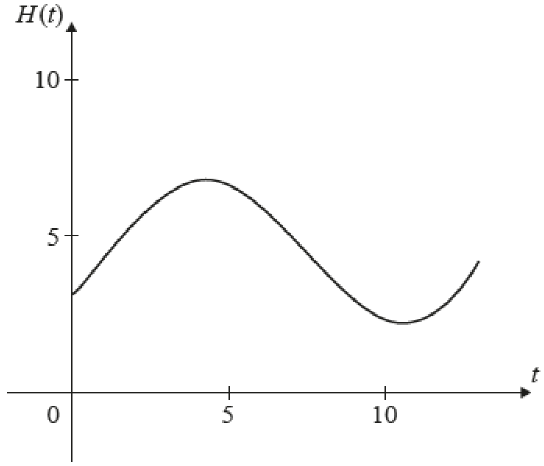
$25.0506\ldots$

minimum perimeter $= 25.1$. ***A1***

[5 marks]

The height of water, in metres, in Dungeness harbour is modelled by the function $H(t) = a\sin(b(t-c)) + d$, where t is the number of hours after midnight, and $a,\ b,\ c$ and d are constants, where $a > 0,\ b > 0$ and $c > 0$.

The following graph shows the height of the water for 13 hours, starting at midnight.



The first high tide occurs at $04:30$ and the next high tide occurs 12 hours later. Throughout the day, the height of the water fluctuates between $2.2\ \text{m}$ and $6.8\ \text{m}$.

All heights are given correct to one decimal place.

10a. Show that $b = \frac{\pi}{6}$. *[1 mark]*

Markscheme

$12 = \frac{2\pi}{b}$ OR $b = \frac{2\pi}{12}$ ***A1***

$b = \frac{\pi}{6}$ ***AG***

[1 mark]

10b. Find the value of a. *[2 marks]*

Markscheme

$a = \frac{6.8-2.2}{2}$ OR $a = \frac{\max-\min}{2}$ ***(M1)***

$= 2.3\,(\text{m})$ ***A1***

[2 marks]

10c. Find the value of d. *[2 marks]*

Markscheme

$d = \frac{6.8+2.2}{2}$ OR $d = \frac{\max+\min}{2}$ ***(M1)***

$= 4.5\,(\text{m})$ ***A1***

[2 marks]

10d. Find the smallest possible value of c. *[3 marks]*

Markscheme

METHOD 1

substituting $t = 4.5$ and $H = 6.8$ for example into their equation for H ***(A1)***

$6.8 = 2.3\sin\left(\frac{\pi}{6}(4.5 - c)\right) + 4.5$

attempt to solve their equation ***(M1)***

$c = 1.5$ ***A1***

METHOD 2

using horizontal translation of $\frac{12}{4}$ ***(M1)***

$4.5 - c = 3$ ***(A1)***

$c = 1.5$ ***A1***

METHOD 3

$H'(t) = (2.3)\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}(t - c)\right)$ ***(A1)***

attempts to solve their $H'(4.5) = 0$ for c ***(M1)***

$(2.3)\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}(4.5 - c)\right) = 0$

$c = 1.5$ ***A1***

[3 marks]

10e. Find the height of the water at $12:00$. *[2 marks]*

Markscheme

attempt to find H when $t=12$ or $t=0$, graphically or algebraically ***(M1)***

$H=2.87365\ldots$

$H=2.87\,(\mathrm{m})$ ***A1***

[2 marks]

10f. Determine the number of hours, over a 24-hour period, for which the tide is higher than 5 metres. *[3 marks]*

Markscheme

attempt to solve $5=2.3\sin\left(\frac{\pi}{6}(t-1.5)\right)+4.5$ ***(M1)***

times are $t=1.91852\ldots$ and $t=7.08147\ldots$, $(t=13.9185\ldots,\ t=19.0814\ldots)$ ***(A1)***

total time is $2\times(7.081\ldots-1.919\ldots)$

$10.3258\ldots$

$=10.3$ (hours) ***A1***

Note: Accept 10.

[3 marks]

10g. A fisherman notes that the water height at nearby Folkestone harbour follows the same sinusoidal pattern as that of Dungeness harbour, with the exception that high tides (and low tides) occur 50 minutes earlier than at Dungeness. *[2 marks]*

Find a suitable equation that may be used to model the tidal height of water at Folkestone harbour.

Markscheme

METHOD 1

substitutes $t=\frac{11}{3}$ and $H=6.8$ into their equation for H and attempts to solve for c ***(M1)***

$6.8=2.3\sin\left(\frac{\pi}{6}\left(\frac{11}{3}-c\right)\right)+4.5 \Rightarrow c=\frac{2}{3}$

$H(t)=2.3\sin\left(\frac{\pi}{6}\left(t-\frac{2}{3}\right)\right)+4.5$ ***A1***

METHOD 2

uses their horizontal translation $\left(\frac{12}{4}=3\right)$ ***(M1)***

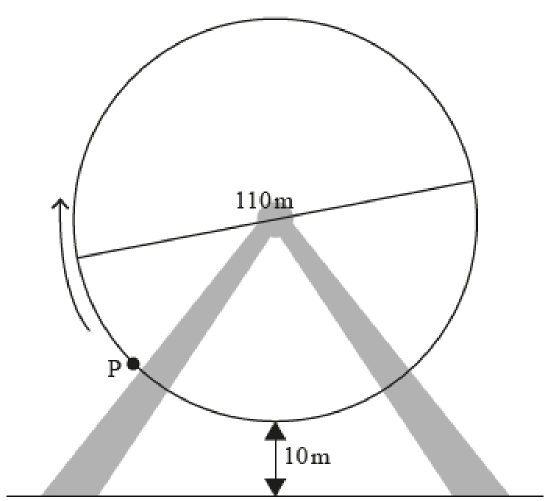
$\frac{11}{3}-c=3 \Rightarrow c=\frac{2}{3}$

$H(t)=2.3\sin\left(\frac{\pi}{6}\left(t-\frac{2}{3}\right)\right)+4.5$ ***A1***

[2 marks]

11. A Ferris wheel with diameter 110 metres rotates at a constant speed. The lowest point on the wheel is 10 metres above the ground, as shown on the following diagram. P is a point on the wheel. The wheel starts moving with P at the lowest point and completes one revolution in 20 minutes. *[5 marks]*

diagram not to scale



The height, h metres, of P above the ground after t minutes is given by $h(t)=a\cos(bt)+c$, where $a, b, c \in \mathbb{R}$.

Find the values of a, b and c.

Markscheme

amplitude is $\frac{110}{2} = 55$ ***(A1)***

$a = -55$ ***A1***

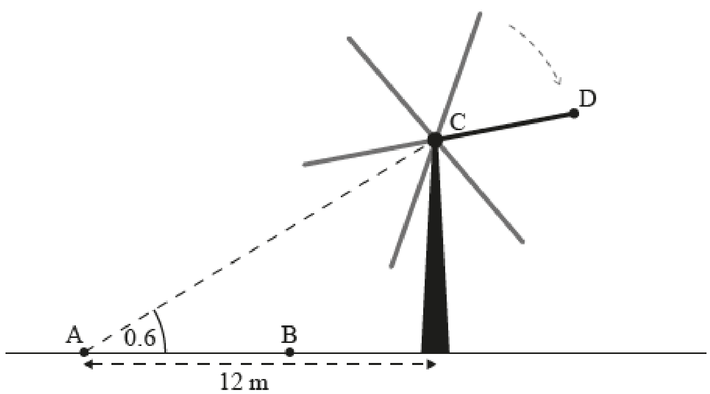
$c = 65$ ***A1***

$\frac{2\pi}{b} = 20$ OR $-55\cos(20b) + 65 = 10$ ***(M1)***

$b = \frac{\pi}{10} (= 0.314)$ ***A1***

[5 marks]

The six blades of a windmill rotate around a centre point C. Points A and B and the base of the windmill are on level ground, as shown in the following diagram.



From point A the angle of elevation of point C is 0.6 radians.

12a. Given that point A is 12 metres from the base of the windmill, find the height of point C above the ground. *[2 marks]*

Markscheme

$\tan 0.6 = \frac{h}{12}$ ***(M1)***

$8.20964\ldots$

$8.21\,(\mathrm{m})$ ***A1***

[2 marks]

An observer walks 7 metres from point A to point B.

12b. Find the angle of elevation of point C from point B. *[2 marks]*

Markscheme

$\tan B = \frac{8.2096...}{5}$ OR $\tan^{-1} 1.6419...$ ***(A1)***

$1.02375...$

1.02 (radians) (accept 58.7°) ***A1***

[2 marks]

The observer keeps walking until he is standing directly under point C. The observer has a height of 1.8 metres, and as the blades of the windmill rotate, the end of each blade passes 2.5 metres over his head.

12c. Find the length of each blade of the windmill. *[2 marks]*

Markscheme

$x + 1.8 + 2.5 = 8.20964...$ (or equivalent) ***(A1)***

$3.90964...$

3.91 (m) ***A1***

[2 marks]

One of the blades is painted a different colour than the others. The end of this blade is labelled point D. The height h, in metres, of point D above the ground can be modelled by the function $h(t) = p\cos\left(\frac{3\pi}{10}t\right) + q$, where t is in seconds and $p,\ q \in \mathbb{R}$. When $t = 0$, point D is at its maximum height.

12d. Find the value of p and the value of q. *[4 marks]*

Markscheme

METHOD 1

recognition that blade length = amplitude, $p = \frac{\max - \min}{2}$ ***(M1)***

$p = 3.91$ ***A1***

centre of windmill = vertical shift, $q = \frac{\max + \min}{2}$ ***(M1)***

$q = 8.21$ ***A1***

METHOD 2

attempting to form two equations in terms of p and q ***(M1)(M1)***

$12.1192\ldots = p\cos\left(\frac{3\pi}{10}\cdot 0\right) + q, 4.3000\ldots = p\cos\left(\frac{3\pi}{10}\cdot\frac{10}{3}\right) + q$

$p = 3.91$ ***A1***

$q = 8.21$ ***A1***

[4 marks]

12e. If the observer stands directly under point C for one minute, point D will pass over his head n times. *[3 marks]*

Find the value of n.

Markscheme

appropriate working towards finding the period ***(M1)***

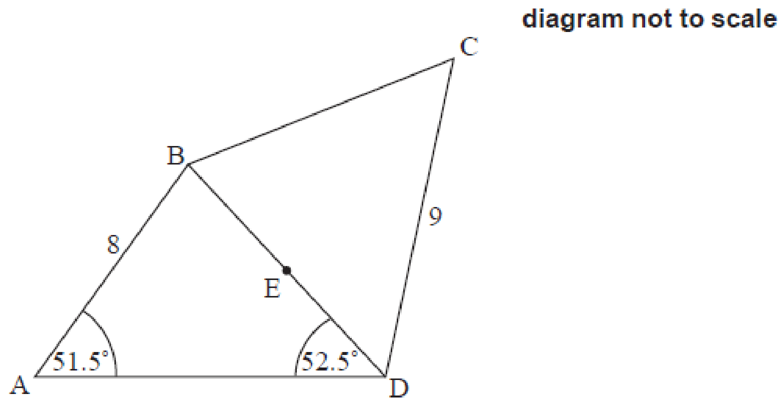
$\text{period} = \frac{2\pi}{\frac{3\pi}{10}} (= 6.6666\ldots)$

rotations per minute $= \frac{60}{\text{their period}}$ ***(M1)***

$n = 9$ (must be an integer) (accept $n = 10$, $n = 18$, $n = 19$) ***A1***

[3 marks]

Using geometry software, Pedro draws a quadrilateral ABCD. $\mathrm{AB} = 8\ \mathrm{cm}$ and $\mathrm{CD} = 9\ \mathrm{cm}$. Angle $\mathrm{BAD} = 51.5^\circ$ and angle $\mathrm{ADB} = 52.5^\circ$. This information is shown in the diagram.



13a. Calculate the length of BD. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\frac{\mathrm{BD}}{\sin 51.5^\circ} = \frac{8}{\sin 52.5^\circ}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted sine rule, ***(A1)*** for correct substitution.

$(\mathrm{BD} =)\ 7.89\ (\mathrm{cm})\ \ (7.89164\ldots)$ ***(A1)(G2)***

Note: If radians are used the answer is $9.58723\ldots$ award at most ***(M1)(A1)(A0)***.

[3 marks]

$\mathrm{CE} = 7\ \mathrm{cm}$, where point E is the midpoint of BD.

13b. Show that angle $\mathrm{EDC} = 48.0^\circ$, correct to three significant figures. *[4 marks]*

Markscheme

$\cos \mathrm{EDC} = \frac{9^2 + 3.94582\ldots^2 - 7^2}{2 \times 9 \times 3.94582\ldots}$ ***(A1)*(ft)*(M1)(A1)*(ft)**

Note: Award ***(A1)*** for $3.94582\ldots$ or $\frac{7.89164\ldots}{2}$ seen, ***(M1)*** for substituted cosine rule, ***(A1)*(ft)** for correct substitutions.

$(\mathrm{EDC} =)\ 47.9515\ldots^\circ$ ***(A1)***

48.0° (3 sig figures) ***(AG)***

Note: Both an unrounded answer that rounds to the given answer and the rounded value must be seen for the final ***(M1)*** to be awarded.
Award at most ***(A1)*(ft)*(M1)(A1)*(ft)*(A0)*** if the known angle 48.0° is used to validate the result. Follow through from their BD in part (a).

[4 marks]

13c. Calculate the area of triangle BDC. *[3 marks]*

Markscheme

Units are required in this question.

$(\text{area} =)\ \frac{1}{2} \times 7.89164\ldots \times 9 \times \sin 48.0^\circ$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substituted area formula. Award ***(A1)*** for correct substitution.

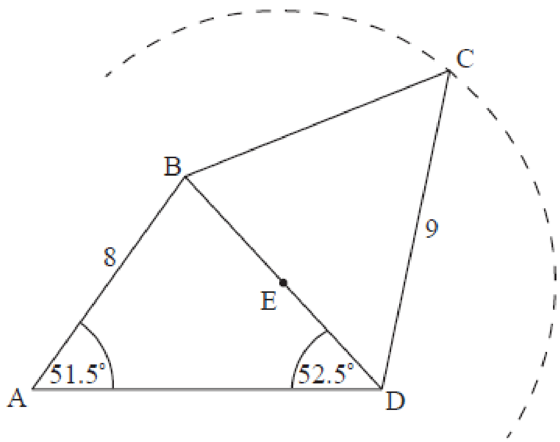
$(\text{area} =)\ 26.4\ \mathrm{cm}^2\ \ (26.3908\ldots)$ ***(A1)(ft)(G3)***

Note: Follow through from part (a).

[3 marks]

13d. Pedro draws a circle, with centre at point E, passing through point C. *[5 marks]*
Part of the circle is shown in the diagram.

diagram not to scale



Show that point A lies outside this circle. Justify your reasoning.

Markscheme

$\mathrm{AE}^2 = 8^2 + (3.94582\ldots)^2 - 2 \times 8 \times 3.94582\ldots \cos(76^\circ)$ ***(A1)(M1)(A1)*****(ft)**

Note: Award ***(A1)*** for 76° seen. Award ***(M1)*** for substituted cosine rule to find AE, ***(A1)*****(ft)** for correct substitutions.

$(\mathrm{AE} =)\ 8.02\,(\mathrm{cm})\ \ (8.01849\ldots)$ ***(A1)(ft)(G3)***

Note: Follow through from part (a).

OR

$\mathrm{AE}^2 = 9.78424\ldots^2 + (3.94582\ldots)^2 - 2 \times 9.78424\ldots \times 3.94582\ldots \cos(52.5^\circ$ ***(A1)(M1)(A1)*****(ft)**

Note: Award ***(A1)*** for $\mathrm{AD}\ (9.78424\ldots)$ or 76° seen. Award ***(M1)*** for substituted cosine rule to find AE (do not award ***(M1)*** for cosine or sine rule to find AD), ***(A1)*****(ft)** for correct substitutions.

$(\mathrm{AE} =)\ 8.02\,(\mathrm{cm})\ \ (8.01849\ldots)$ ***(A1)(ft)(G3)***

Note: Follow through from part (a).

$8.02 > 7.$ ***(A1)(ft)***

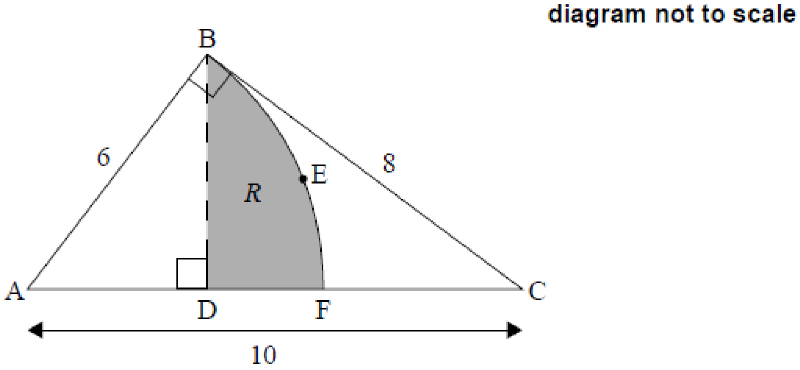
point A is outside the circle. ***(AG)***

Note: Award ***(A1)*** for a numerical comparison of AE and CE. Follow through for the final ***(A1)(ft)*** within the part for their 8.02. The final ***(A1)(ft)*** is contingent on a valid method to find the value of AE.
Do not award the final ***(A1)(ft)*** if the ***(AG)*** line is not stated.
Do not award the final ***(A1)(ft)*** if their point A is inside the circle.

[5 marks]

The following diagram shows a right-angled triangle, ABC, with $\mathrm{AC} = 10\,\mathrm{cm}$, $\mathrm{AB} = 6\,\mathrm{cm}$ and $\mathrm{BC} = 8\,\mathrm{cm}$.

The points D and F lie on $[\mathrm{AC}]$.
$[\mathrm{BD}]$ is perpendicular to $[\mathrm{AC}]$.
BEF is the arc of a circle, centred at A.
The region R is bounded by $[\mathrm{BD}]$, $[\mathrm{DF}]$ and arc BEF.



14a. Find $\mathrm{B\hat{A}C}$. *[2 marks]*

Markscheme

correct working *(A1)*

eg $\sin\alpha = \frac{8}{10}$, $\cos\theta = \frac{6}{10}$, $\cos\mathrm{B\hat{A}C} = \frac{6^2+10^2-8^2}{2\times6\times10}$

0.927295

$\mathrm{B\hat{A}C} = 0.927\ \ (= 53.1°)$ ***(A1) N2***

[2 marks]

14b. Find the area of R. *[5 marks]*

Markscheme

Note: There may be slight differences in the final answer, depending on the approach the candidate uses in part (b). Accept a final answer that is consistent with their working.

correct area of sector ABF (seen anywhere) ***(A1)***

eg $\frac{1}{2} \times 6^2 \times 0.927,\ \frac{53.1301^\circ}{360^\circ} \times \pi \times 6^2,\ 16.6913$

correct expression (or value) for either $[\mathrm{AD}]$ or $[\mathrm{BD}]$ (seen anywhere) ***(A1)***

eg $\mathrm{AD} = 6\cos\left(\mathrm{B\hat{A}C}\right)\ \ (= 3.6)$

$\mathrm{BD} = 6\sin(53.1^\circ)\ \ (= 4.8)$

correct area of triangle ABD (seen anywhere) ***(A1)***

eg $\frac{1}{2} \times 6\cos\mathrm{B\hat{A}D} \times 6\sin\mathrm{B\hat{A}D},\ 9\sin\left(2\,\mathrm{B\hat{A}C}\right),\ 8.64$ (exact)

appropriate approach (seen anywhere) ***(M1)***

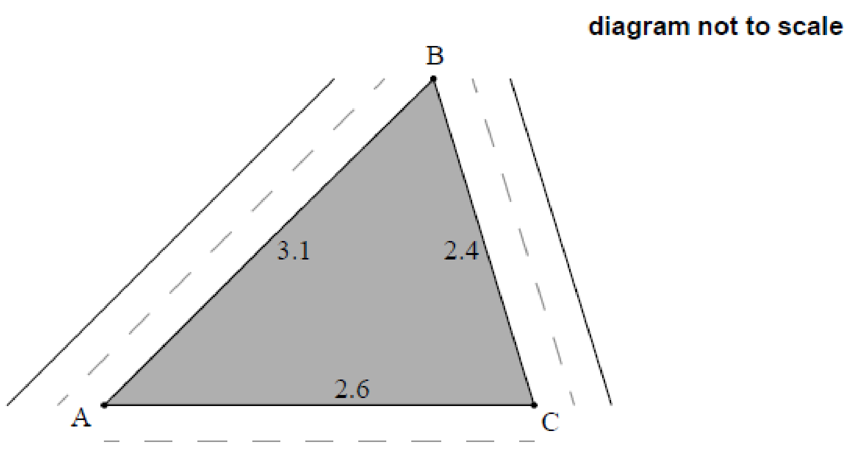
eg $\mathrm{A}_{\text{triangle ABD}} - \mathrm{A}_{\text{sector}}$, their sector − their triangle ABD

8.05131

area of shaded region $= 8.05$ (cm^2) ***A1 N2***

[5 marks]

Three airport runways intersect to form a triangle, ABC. The length of AB is 3.1 km, AC is 2.6 km, and BC is 2.4 km.



A company is hired to cut the grass that grows in triangle ABC, but they need to know the area.

15a. Find the size, in degrees, of angle BÂC. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$(\cos\ A =)\ \frac{2.6^2+3.1^2-2.4^2}{2(2.6)(3.1)}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted cosine rule formula, ***(A1)*** for correct substitutions.

48.8° (48.8381…°) ***(A1) (C3)***

[3 marks]

15b. Find the area, in km^2, of triangle ABC. *[3 marks]*

Markscheme

$\frac{1}{2} \times 2.6 \times 3.1 \times \sin\left(48.8381\ldots^{\circ}\right)$ ***(M1)(A1)*****(ft)**

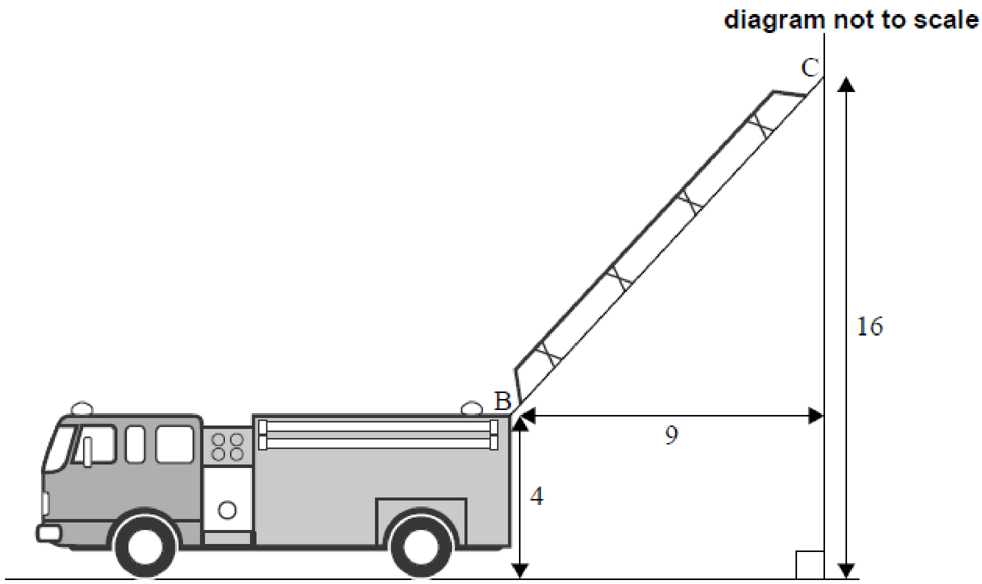
Note: Award ***(M1)*** for substituted area of a triangle formula, ***(A1)*** for correct substitution.

3.03 (km^2) (3.033997…(km^2)) ***(A1)*****(ft)** ***(C3)***

Note: Follow through from part (a).

[3 marks]

A ladder on a fire truck has its base at point B which is 4 metres above the ground. The ladder is extended and its other end rests on a vertical wall at point C, 16 metres above the ground. The horizontal distance between B and C is 9 metres.



16a. Find the angle of elevation from B to C. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\tan B = \frac{12}{9}$ ***(A1)(M1)***

Note: Award ***(A1)*** for 12 seen, ***(M1)*** for correct substitution into tan (or equivalent). Accept equivalent methods, such as Pythagoras, to find BC and correct substitution into other trig ratios. If $\tan^{-1}\left(\frac{16}{9}\right)$ seen award ***(A0)(M1)(A0)***.

53.1° (53.1301...°) ***(A1) (C3)***

Note: If radians are used the answer is 0.927295...; award at most ***(A1)(M1)(A0)***.

[3 marks]

16b. A second truck arrives whose ladder, when fully extended, is 30 metres long. The base of this ladder is also 4 metres above the ground. For safety reasons, the maximum angle of elevation that the ladder can make is 70º. *[3 marks]*

Find the maximum height on the wall that can be reached by the ladder on the second truck.

Markscheme

$30 \sin 70° + 4$ ***(M1)(M1)***

Note: Award ***(M1)*** for $\sin 70° = \frac{x}{30}$ (or equivalent) and ***(M1)*** for adding 4.

32.2 (32.1907...) (m) ***(A1) (C3)***

Note: If radians are used the answer is 27.2167...; award at most ***(M1)(M1) (A0)***.

[3 marks]

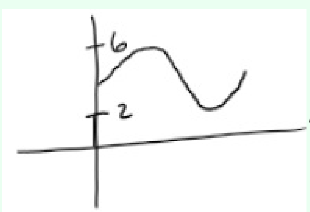
Let $f(x) = 2 \sin(3x) + 4$ for $x \in \mathbb{R}$.

17a. The range of f is $k \le f(x) \le m$. Find k and m. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

valid attempt to find range ***(M1)***

eg 

, max = 6 min = 2,

$2 \sin\left(3 \times \frac{\pi}{6}\right) + 4$ and $2 \sin\left(3 \times \frac{\pi}{2}\right) + 4$, $2(1) + 4$ and $2(-1) + 4$,

$k = 2,\ m = 6$ ***A1A1 N3***

[3 marks]

Let $g(x) = 5f(2x)$.

17b. Find the range of g. *[2 marks]*

Markscheme

$10 \leq y \leq 30$ ***A2 N2***

[2 marks]

The function g can be written in the form $g(x) = 10\sin(bx) + c$.

17c. Find the value of b and of c. *[3 marks]*

Markscheme

evidence of substitution (may be seen in part (b)) ***(M1)***

eg $5(2\sin(3(2x)) + 4)$, $3(2x)$

$b = 6$, $c = 20$ (accept $10\sin(6x) + 20$) ***A1A1 N3***

Note: If no working shown, award ***N2*** for one correct value.

[3 marks]

17d. Find the period of g. *[2 marks]*

Markscheme

correct working ***(A1)***

eg $\frac{2\pi}{b}$

1.04719

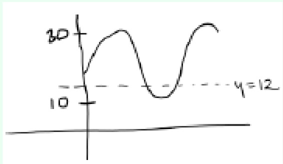
$\frac{2\pi}{6}$ $\left(= \frac{\pi}{3}\right)$, 1.05 ***A1 N2***

[2 marks]

17e. The equation $g(x) = 12$ has two solutions where $\pi \leq x \leq \frac{4\pi}{3}$. Find both solutions. *[3 marks]*

Markscheme

valid approach ***(M1)***

eg 

,

$\sin^{-1}\left(-\frac{8}{10}\right)$, $6x = -0.927,\ -0.154549,\ x = 0.678147$

Note: Award ***M1*** for any correct value for x or $6x$ which lies outside the domain of f.

3.81974, 4.03424

$x = 3.82,\ x = 4.03$ (do not accept answers in degrees) ***A1A1 N3***

[3 marks]