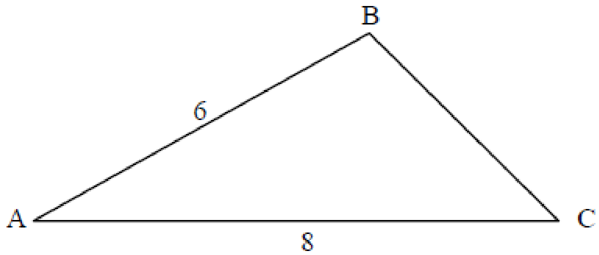
Trig review *[164 marks]*

The following diagram shows triangle ABC, with $AB = 6$ and $AC = 8$.

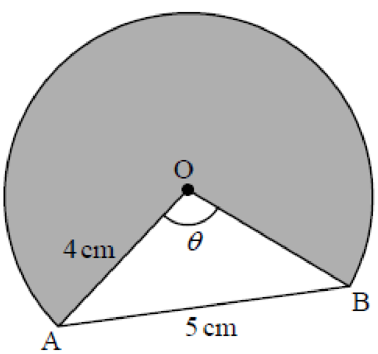
diagram not to scale



1a. Given that $\cos \hat{A} = \frac{5}{6}$ find the value of $\sin \hat{A}$. *[3 marks]*

1b. Find the area of triangle ABC. *[2 marks]*

The following diagram shows part of a circle with centre O and radius 4 cm.



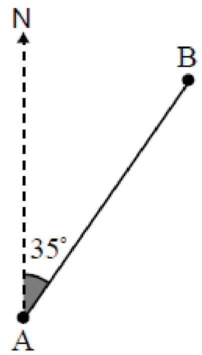
Chord AB has a length of 5 cm and $A\hat{O}B = \theta$.

2a. Find the value of θ, giving your answer in radians. *[3 marks]*

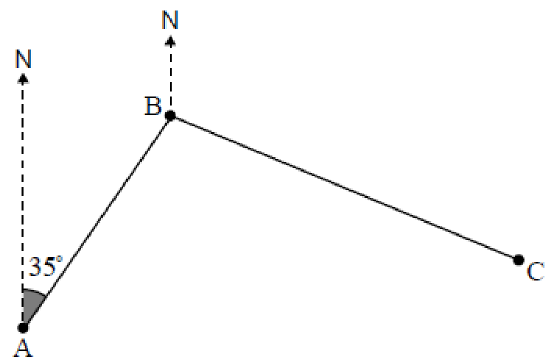
2b. Find the area of the shaded region. *[3 marks]*

Adam sets out for a hike from his camp at point A. He hikes at an average speed of 4.2 km/h for 45 minutes, on a bearing of 035° from the camp, until he stops for a break at point B.

3a. Find the distance from point A to point B. *[2 marks]*



Adam leaves point B on a bearing of 114° and continues to hike for a distance of 4.6 km until he reaches point C.



3b. Show that $\mathrm{A}\hat{\mathrm{B}}\mathrm{C}$ is 101°. *[2 marks]*

3c. Find the distance from the camp to point C. *[3 marks]*

3d. Find $\mathrm{B}\hat{\mathrm{C}}\mathrm{A}$. *[3 marks]*

Adam's friend Jacob wants to hike directly from the camp to meet Adam at point C .

3e. Find the bearing that Jacob must take to point C. *[3 marks]*

3f. Jacob hikes at an average speed of 3.9 km/h. *[3 marks]*

Find, to the nearest minute, the time it takes for Jacob to reach point C.

Consider a function f, such that $f(x) = 5.8\sin\left(\frac{\pi}{6}(x+1)\right) + b$, 0 ≤ x ≤ 10, $b \in \mathbb{R}$.

4a. Find the period of f. *[2 marks]*

The function f has a local maximum at the point (2, 21.8) , and a local minimum at (8, 10.2).

4b. Find the value of b. *[2 marks]*

4c. Hence, find the value of f(6). *[2 marks]*

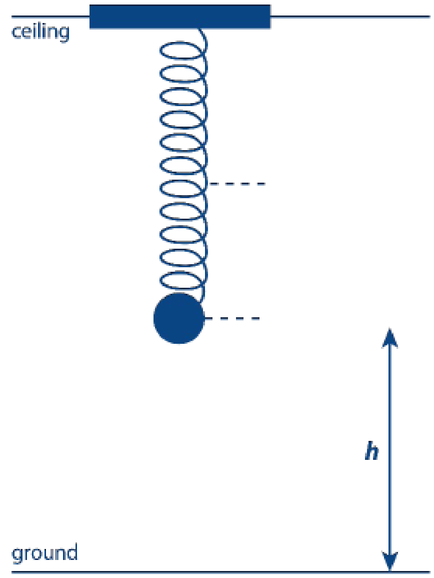
A second function g is given by $g(x) = p\sin\left(\frac{2\pi}{9}(x-3.75)\right) + q$, 0 ≤ x ≤ 10; p, $q \in \mathbb{R}$.

The function g passes through the points (3, 2.5) and (6, 15.1).

4d. Find the value of p and the value of q. *[5 marks]*

4e. Find the value of x for which the functions have the greatest difference. *[2 marks]*

The following diagram shows a ball attached to the end of a spring, which is suspended from a ceiling.



The height, h metres, of the ball above the ground at time t seconds after being released can be modelled by the function $h(t) = 0.4\cos(\pi t) + 1.8$ where $t \geq 0$.

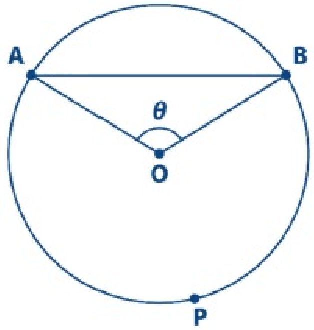
5a. Find the height of the ball above the ground when it is released. *[2 marks]*

5b. Find the minimum height of the ball above the ground. *[2 marks]*

5c. Show that the ball takes 2 seconds to return to its initial height above the ground for the first time. *[2 marks]*

5d. For the first 2 seconds of its motion, determine the amount of time that the ball is less than $1.8 + 0.2\sqrt{2}$ metres above the ground. *[5 marks]*

The following diagram shows a circle with centre O and radius 3.



Points A, P and B lie on the circumference of the circle.

Chord $[\mathrm{AB}]$ has length L and $\mathrm{A\hat{O}B} = \theta$ radians.

6a. Show that arc APB has length $6\pi - 3\theta$. *[2 marks]*

6b. Show that $L = \sqrt{18 - 18\cos\theta}$. *[2 marks]*

6c. Arc APB is twice the length of chord $[\mathrm{AB}]$. *[3 marks]*

Find the value of θ.

Consider $f(x) = 4\sin x + 2.5$ and $g(x) = 4\sin\left(x - \frac{3\pi}{2}\right) + 2.5 + q$, where $x \in \mathbb{R}$ and $q > 0$.

The graph of g is obtained by two transformations of the graph of f.

7a. Describe these two transformations. *[2 marks]*

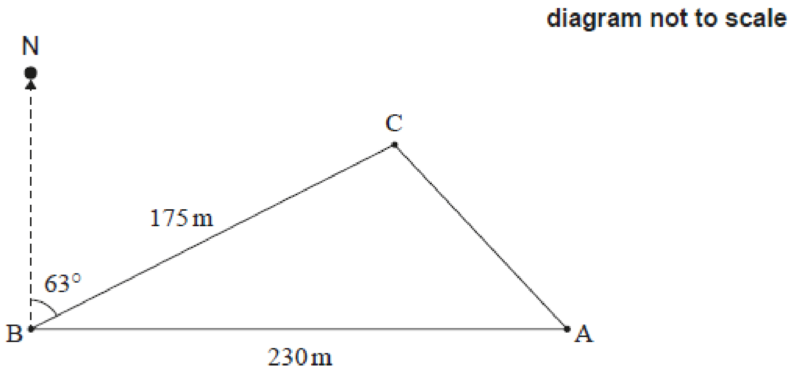
7b. The y-intercept of the graph of g is at $(0,\ r)$. *[5 marks]*

Given that $g(x) \geq 7$, find the smallest value of r.

A farmer is placing posts at points A, B, and C in the ground to mark the boundaries of a triangular piece of land on his property.

From point A, he walks due west 230 metres to point B.
From point B, he walks 175 metres on a bearing of 063° to reach point C.

This is shown in the following diagram.

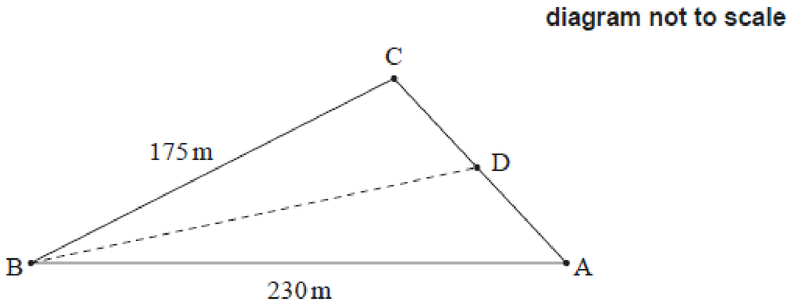


8a. Find the distance from point A to point C. *[4 marks]*

8b. Find the area of this piece of land. *[2 marks]*

8c. Find $\mathrm{C\hat{A}B}$. *[3 marks]*

The farmer wants to divide the piece of land into two sections. He will put a post at point D, which is between A and C. He wants the boundary BD to divide the piece of land such that the sections have equal area. This is shown in the following diagram.



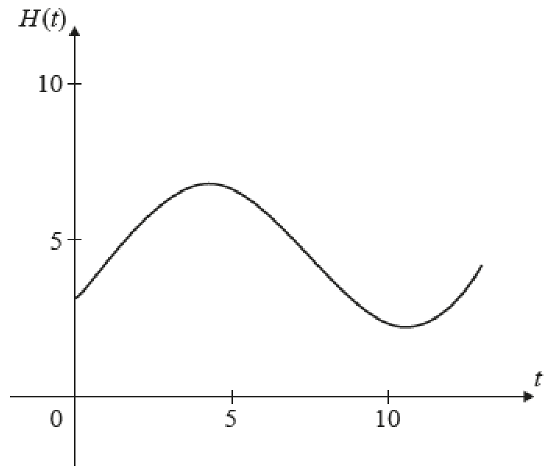
8d. Find the distance from point B to point D. *[5 marks]*

9. Consider a triangle ABC, where $\mathrm{AC} = 12,\ \mathrm{CB} = 7$ and $\mathrm{B\widehat{A}C} = 25^\circ$. *[5 marks]*

Find the smallest possible perimeter of triangle ABC.

The height of water, in metres, in Dungeness harbour is modelled by the function $H(t) = a\sin(b(t - c)) + d$, where t is the number of hours after midnight, and $a,\ b,\ c$ and d are constants, where $a > 0,\ b > 0$ and $c > 0$.

The following graph shows the height of the water for 13 hours, starting at midnight.



The first high tide occurs at $04:30$ and the next high tide occurs 12 hours later. Throughout the day, the height of the water fluctuates between $2.2\ \mathrm{m}$ and $6.8\ \mathrm{m}$.

All heights are given correct to one decimal place.

10a. Show that $b = \frac{\pi}{6}$. *[1 mark]*

10b. Find the value of a. *[2 marks]*

10c. Find the value of d. *[2 marks]*

10d. Find the smallest possible value of c. *[3 marks]*

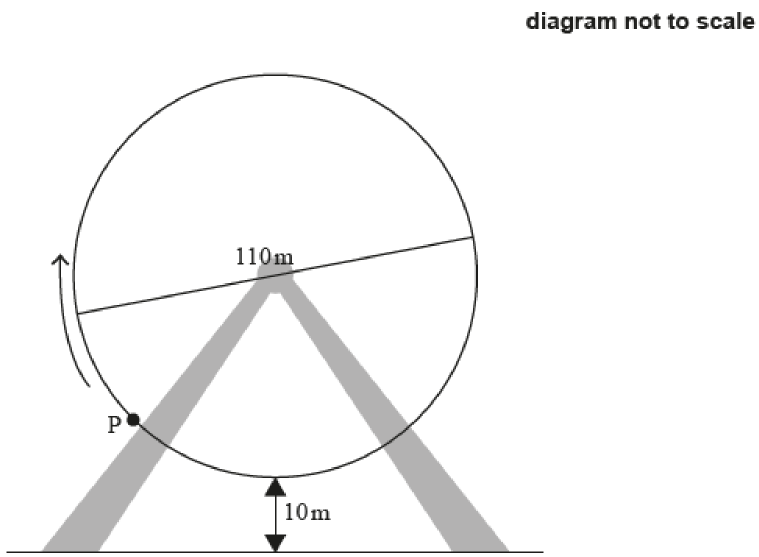
10e. Find the height of the water at $12:00$. *[2 marks]*

10f. Determine the number of hours, over a 24-hour period, for which the tide is higher than 5 metres. *[3 marks]*

10g. A fisherman notes that the water height at nearby Folkestone harbour follows the same sinusoidal pattern as that of Dungeness harbour, with the exception that high tides (and low tides) occur 50 minutes earlier than at Dungeness. *[2 marks]*

Find a suitable equation that may be used to model the tidal height of water at Folkestone harbour.

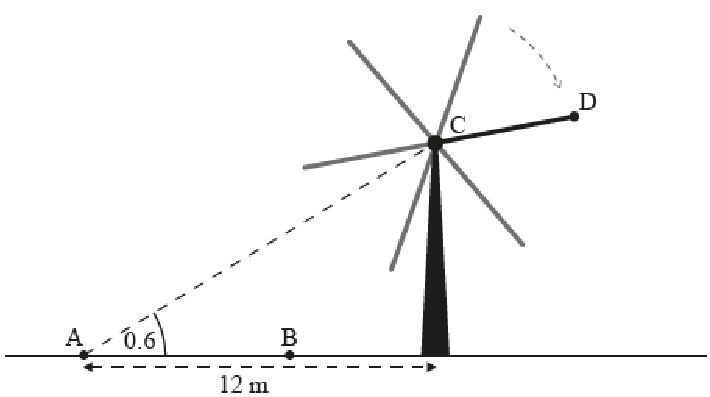
11. A Ferris wheel with diameter 110 metres rotates at a constant speed. The lowest point on the wheel is 10 metres above the ground, as shown on the following diagram. P is a point on the wheel. The wheel starts moving with P at the lowest point and completes one revolution in 20 minutes. *[5 marks]*



The height, h metres, of P above the ground after t minutes is given by $h(t) = a\cos(bt) + c$, where $a, b, c \in \mathbb{R}$.

Find the values of a, b and c.

The six blades of a windmill rotate around a centre point C. Points A and B and the base of the windmill are on level ground, as shown in the following diagram.



From point A the angle of elevation of point C is 0.6 radians.

12a. Given that point A is 12 metres from the base of the windmill, find the height of point C above the ground. *[2 marks]*

An observer walks 7 metres from point A to point B.

12b. Find the angle of elevation of point C from point B. *[2 marks]*

The observer keeps walking until he is standing directly under point C. The observer has a height of 1.8 metres, and as the blades of the windmill rotate, the end of each blade passes 2.5 metres over his head.

12c. Find the length of each blade of the windmill. *[2 marks]*

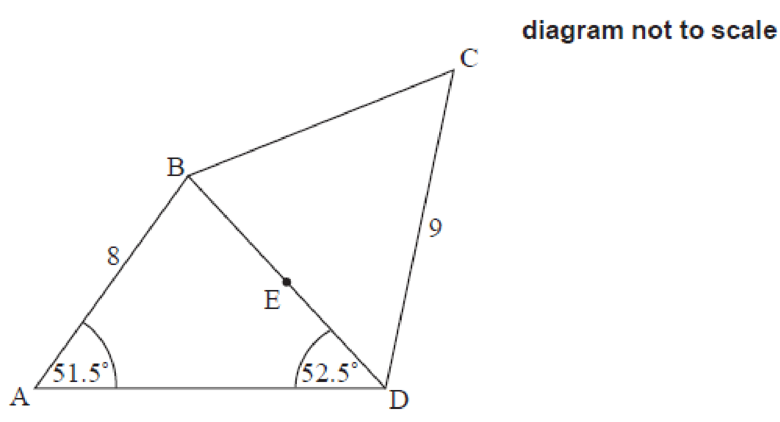
One of the blades is painted a different colour than the others. The end of this blade is labelled point D. The height h, in metres, of point D above the ground can be modelled by the function $h(t) = p\cos\left(\frac{3\pi}{10}t\right) + q$, where t is in seconds and $p,\ q \in \mathbb{R}$. When $t = 0$, point D is at its maximum height.

12d. Find the value of p and the value of q. *[4 marks]*

12e. If the observer stands directly under point C for one minute, point D will pass over his head n times. *[3 marks]*

Find the value of n.

Using geometry software, Pedro draws a quadrilateral ABCD. $\mathrm{AB} = 8\ \mathrm{cm}$ and $\mathrm{CD} = 9\ \mathrm{cm}$. Angle $\mathrm{BAD} = 51.5^\circ$ and angle $\mathrm{ADB} = 52.5^\circ$. This information is shown in the diagram.



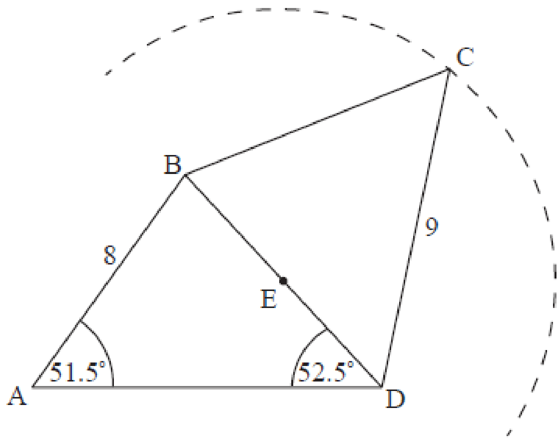
13a. Calculate the length of BD. *[3 marks]*

$\mathrm{CE} = 7\ \mathrm{cm}$, where point E is the midpoint of BD.

13b. Show that angle $\mathrm{EDC} = 48.0^\circ$, correct to three significant figures. *[4 marks]*

13c. Calculate the area of triangle BDC. *[3 marks]*

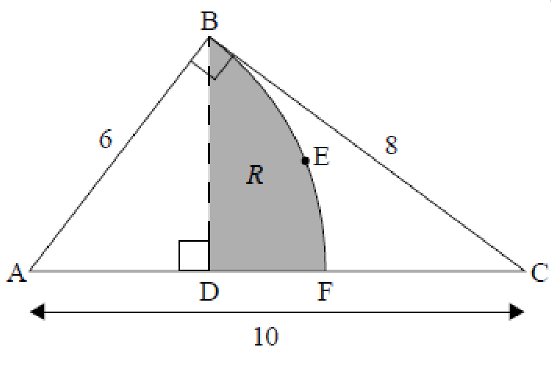
13d. Pedro draws a circle, with centre at point E, passing through point C. Part of the circle is shown in the diagram. *[5 marks]*



Show that point A lies outside this circle. Justify your reasoning.

The following diagram shows a right-angled triangle, ABC, with $\mathrm{AC} = 10\,\mathrm{cm}$, $\mathrm{AB} = 6\,\mathrm{cm}$ and $\mathrm{BC} = 8\,\mathrm{cm}$.

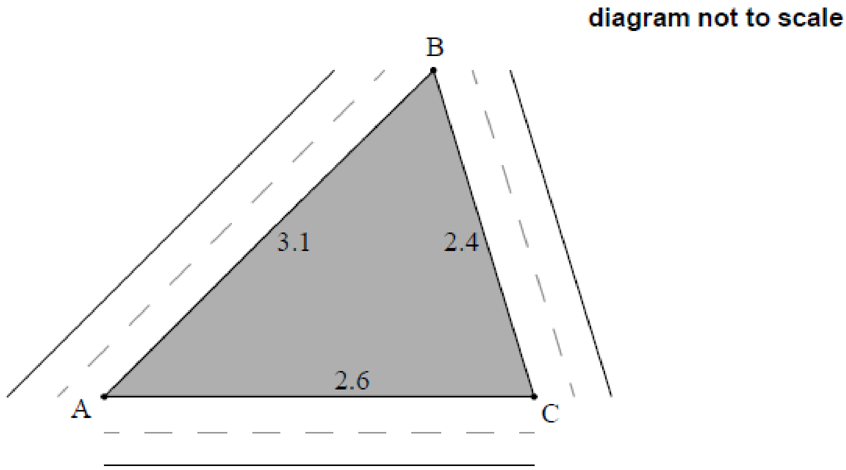
The points D and F lie on $[\mathrm{AC}]$.
$[\mathrm{BD}]$ is perpendicular to $[\mathrm{AC}]$.
BEF is the arc of a circle, centred at A.
The region R is bounded by $[\mathrm{BD}]$, $[\mathrm{DF}]$ and arc BEF.



14a. Find $\mathrm{B\hat{A}C}$. *[2 marks]*

14b. Find the area of R. *[5 marks]*

Three airport runways intersect to form a triangle, ABC. The length of AB is 3.1 km, AC is 2.6 km, and BC is 2.4 km.

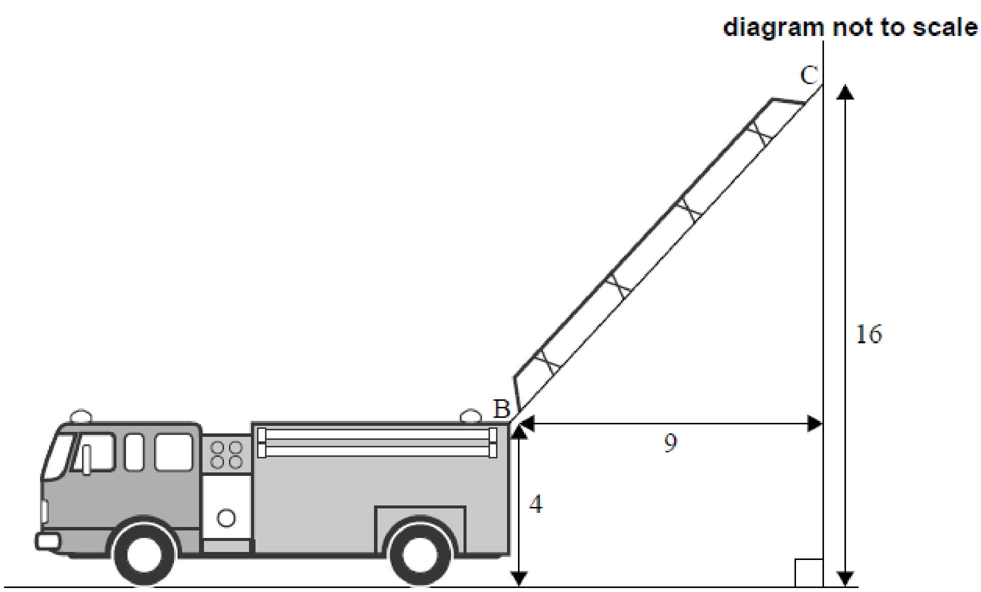


A company is hired to cut the grass that grows in triangle ABC, but they need to know the area.

15a. Find the size, in degrees, of angle BÂC. *[3 marks]*

15b. Find the area, in km^2, of triangle ABC. *[3 marks]*

A ladder on a fire truck has its base at point B which is 4 metres above the ground. The ladder is extended and its other end rests on a vertical wall at point C, 16 metres above the ground. The horizontal distance between B and C is 9 metres.



16a. Find the angle of elevation from B to C. *[3 marks]*

16b. A second truck arrives whose ladder, when fully extended, is 30 metres long. The base of this ladder is also 4 metres above the ground. For safety reasons, the maximum angle of elevation that the ladder can make is 70º. *[3 marks]*

Find the maximum height on the wall that can be reached by the ladder on the second truck.

Let $f\left(x\right)=2\sin\left(3x\right)+4$ for $x\in\mathbb{R}$.

17a. The range of f is $k \le f\left(x\right) \le m$. Find k and m. *[3 marks]*

Let $g\left(x\right)=5f\left(2x\right)$.

17b. Find the range of g. *[2 marks]*

The function g can be written in the form $g\left(x\right)=10\sin\left(bx\right)+c$.

17c. Find the value of b and of c. *[3 marks]*

17d. Find the period of g. *[2 marks]*

17e. The equation $g\left(x\right)=12$ has two solutions where $\pi \le x \le \frac{4\pi}{3}$. Find both solutions. *[3 marks]*