# Combinatorics *[46 marks]*

Mary, three female friends, and her brother, Peter, attend the theatre. In the theatre there is a row of $10$ empty seats. For the first half of the show, they decide to sit next to each other in this row.

1a. Find the number of ways these five people can be seated in this row. *[3 marks]*

For the second half of the show, they return to the same row of $10$ empty seats. The four girls decide to sit at least one seat apart from Peter. The four girls do not have to sit next to each other.

1b. Find the number of ways these five people can now be seated in this row. *[4 marks]*

Consider the set of six-digit positive integers that can be formed from the digits $0, 1, 2, 3, 4, 5, 6, 7, 8$ and $9$.

Find the total number of six-digit positive integers that can be formed such that

2a. the digits are distinct. *[2 marks]*

2b. the digits are distinct and are in increasing order. *[2 marks]*

A farmer has six sheep pens, arranged in a grid with three rows and two columns as shown in the following diagram.
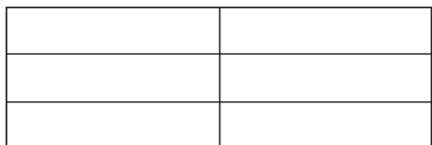


Five sheep called Amber, Brownie, Curly, Daisy and Eden are to be placed in the pens. Each pen is large enough to hold all of the sheep. Amber and Brownie are known to fight.

Find the number of ways of placing the sheep in the pens in each of the following cases:

3a. Each pen is large enough to contain five sheep. Amber and Brownie must not be placed in the same pen. *[4 marks]*

3b. Each pen may only contain one sheep. Amber and Brownie must not be   *[4 marks]*
    placed in pens which share a boundary.

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Eight runners compete in a race where there are no tied finishes. Andrea and Jack are two of the eight competitors in this race.

Find the total number of possible ways in which the eight runners can finish if Jack finishes

4a. in the position immediately after Andrea.   *[2 marks]*

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4b. in any position after Andrea.   *[3 marks]*

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A team of four is to be chosen from a group of four boys and four girls.

5a. Find the number of different possible teams that could be chosen.   *[3 marks]*

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5b. Find the number of different possible teams that could be chosen, given *[2 marks]*
    that the team must include at least one girl and at least one boy.

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6a.
   Express the binomial coefficient $\begin{pmatrix} 3n+1 \\ 3n-2 \end{pmatrix}$ as a polynomial in $n$.   *[3 marks]*

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6b.
   Hence find the least value of $n$ for which $\begin{pmatrix} 3n+1 \\ 3n-2 \end{pmatrix} > 10^6$.   *[3 marks]*

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7.  Three girls and four boys are seated randomly on a straight bench. Find   *[5 marks]*
    the probability that the girls sit together and the boys sit together.

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8.  In a trial examination session a candidate at a school has to take 18   *[6 marks]*
    examination papers including the physics paper, the chemistry paper
    and the biology paper. No two of these three papers may be taken consecutively.
    There is no restriction on the order in which the other examination papers may be
    taken.

    Find the number of different orders in which these 18 examination papers may be
    taken.

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