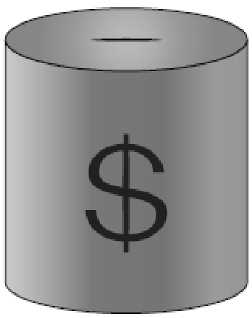
Geometry and trigonometry 5.12 *[50 marks]*

Money boxes are coin containers used by children and come in a variety of shapes. The money box shown is in the shape of a cylinder. It has a radius of 4.43 cm and a height of 12.2 cm.

diagram not to scale



1a. Find the volume of the money box. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$(V =)\ \pi(4.43)^2 \times 12.2$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into volume of a cylinder formula, ***(A1)*** for correct substitution.

752 cm^3 (752.171... cm^3) ***(A1)(C3)***

[3 marks]

1b. A second money box is in the shape of a sphere and has the same volume as the cylindrical money box. *[3 marks]*

diagram not to scale

Find the diameter of the second money box.

Markscheme

$752.171\ldots = \frac{4}{3}\pi(r)^3$ ***(M1)***

Note: Award ***(M1)*** for equating their volume to the volume of a sphere formula.

$(r =)$ 5.64169...cm ***(A1)*****(ft)**

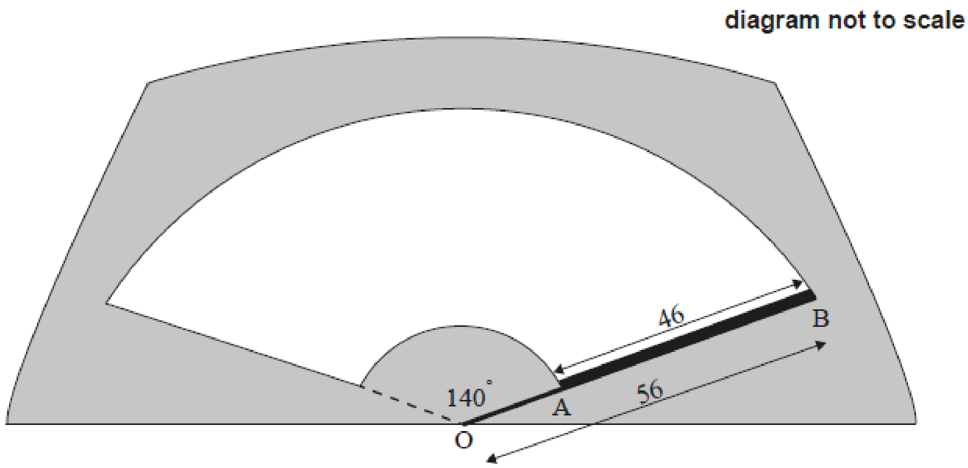
Note: Follow through from part (a).

$(d =)$ 11.3 cm (11.2833...cm) ***(A1)*****(ft)** ***(C3)***

[3 marks]

The straight metal arm of a windscreen wiper on a car rotates in a circular motion from a pivot point, O, through an angle of 140°. The windscreen is cleared by a rubber blade of length $46\ \mathrm{cm}$ that is attached to the metal arm between points A and B. The total length of the metal arm, OB, is $56\ \mathrm{cm}$.

The part of the windscreen cleared by the rubber blade is shown unshaded in the following diagram.



2a. Calculate the length of the arc made by B, the end of the rubber blade. *[2 marks]*

Markscheme

attempt to substitute into length of arc formula ***(M1)***

$\frac{140^\circ}{360^\circ} \times 2\pi \times 56$

$137\ \mathrm{cm} \quad \left(136.833\ldots,\ \frac{392\pi}{9}\ \mathrm{cm}\right)$ ***A1***

[2 marks]

2b. Determine the area of the windscreen that is cleared by the rubber blade. *[3 marks]*

Markscheme

subtracting two substituted area of sectors formulae ***(M1)***

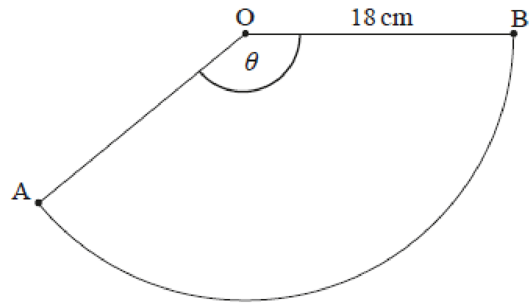
$\left(\frac{140^\circ}{360^\circ} \times \pi \times 56^2\right) - \left(\frac{140^\circ}{360^\circ} \times \pi \times 10^2\right)$ **OR** $\frac{140^\circ}{360^\circ} \times \pi \times \left(56^2 - 10^2\right)$ ***(A1)***

$3710 \text{ cm}^2 \quad \left(3709.17\ldots \text{ cm}^2\right)$ ***A1***

[3 marks]

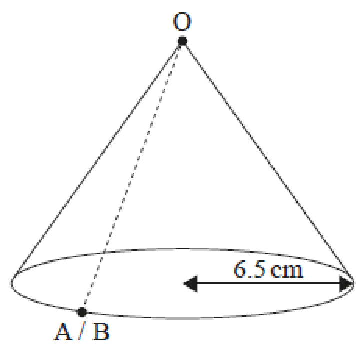
Joey is making a party hat in the form of a cone. The hat is made from a sector, AOB, of a circular piece of paper with a radius of 18 cm and $\text{A}\hat{\text{O}}\text{B} = \theta$ as shown in the diagram.

diagram not to scale



To make the hat, sides $[\text{OA}]$ and $[\text{OB}]$ are joined together. The hat has a base radius of 6.5 cm.

diagram not to scale



3a. Write down the perimeter of the base of the hat in terms of π. *[1 mark]*

Markscheme

$13\pi \text{ cm}$ ***A1***

Note: Answer must be in terms of π.

[1 mark]

3b. Find the value of θ. *[2 marks]*

Markscheme

METHOD 1

$\frac{\theta}{360} \times 2\pi(18) = 13\pi$ **OR** $\frac{\theta}{360} \times 2\pi(18) = 40.8407\ldots$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into length of an arc formula.

$(\theta =)\ 130^\circ$ ***A1***

METHOD 2

$\frac{\theta}{360} \times \pi \times 18^2 = \pi \times 6.5 \times 18$ ***(M1)***

$(\theta =)\ 130^\circ$ ***A1***

[2 marks]

3c. Find the surface area of the outside of the hat. *[2 marks]*

Markscheme

EITHER

$\frac{130}{360} \times \pi(18)^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into area of a sector formula.

OR

$\pi(6.5)(18)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into curved area of a cone formula.

THEN

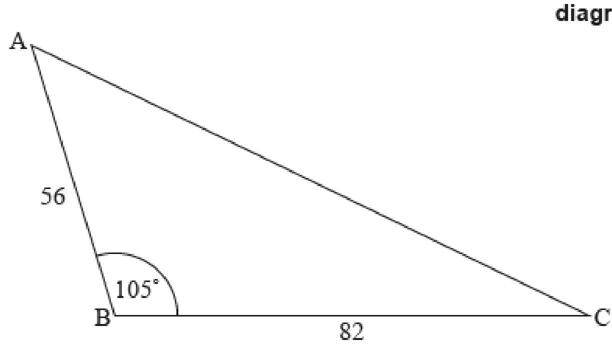
(Area=) 368 cm^2 $(367.566\ldots,\ 117\pi)$ ***A1***

Note: Allow ***FT*** from their part (a)(ii) even if their angle is not obtuse.

[2 marks]

4. A triangular field ABC is such that $\text{AB} = 56 \text{ m}$ and $\text{BC} = 82 \text{ m}$, each measured correct to the nearest metre, and the angle at B is equal to 105°, measured correct to the nearest 5°. *[5 marks]*

diagram not to scale



Calculate the maximum possible area of the field.

Markscheme

attempt to find any relevant maximum value ***(M1)***

largest sides are 56.5 and 82.5 ***(A1)***

smallest possible angle is 102.5 ***(A1)***

attempt to substitute into area of a triangle formula ***(M1)***

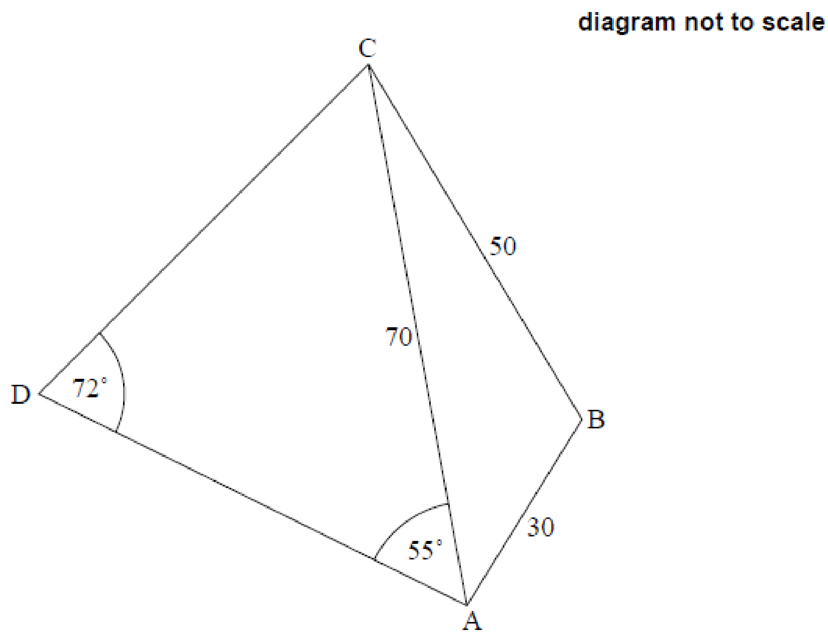
$\frac{1}{2} \times 56.5 \times 82.5 \times \sin(102.5^\circ)$

$= 2280\,(\mathrm{m}^2)\,(2275.37\ldots)$ ***A1***

[5 marks]

Haraya owns two triangular plots of land, ABC and ACD. The length of AB is $30\ \mathrm{m}$, BC is $50\ \mathrm{m}$ and AC is $70\ \mathrm{m}$. The size of $\mathrm{D\hat{A}C}$ is 55° and $\mathrm{A\hat{D}C}$ is 72°.

The following diagram shows this information.



5a. Find the length of AD. *[4 marks]*

Markscheme

$\text{A}\widehat{\text{C}}\text{D} = 53^\circ$ (or equivalent) ***(A1)***

Note: Award ***(A1)*** for 53° (or equivalent) seen.

$\frac{\text{AD}}{\sin 53^\circ} = \frac{70}{\sin 72^\circ}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into sine rule formula, ***(A1)*** for correct substitution.

OR

$\left(\text{AD}^2 =\right) 60.2915\ldots^2 + 70^2 - 2 \times 70 \times 60.2915\ldots \times \cos 53$ ***(A1)(M1)(A1)***

Note: Award ***(A1)*** for 53 or $60.2915\ldots$ seen, ***(M1)*** for substitution into cosine rule formula, ***(A1)*** for correct substitution.

$(\text{AD} =)\ 58.8\ (\text{m})\ (58.7814\ldots)$ ***(A1)(G3)***

[4 marks]

5b. Find the size of $\text{A}\widehat{\text{B}}\text{C}$. *[3 marks]*

Markscheme

$\left(\cos \text{A}\widehat{\text{B}}\text{C}\right) = \frac{30^2 + 50^2 - 70^2}{2 \times 30 \times 50}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into cosine rule formula, ***(A1)*** for correct substitution.

$\left(\text{A}\widehat{\text{B}}\text{C} =\right) 120^\circ$ ***(A1)(G2)***

[3 marks]

5c. Calculate the area of the triangular plot of land ABC. *[3 marks]*

Markscheme

Units are required in part (c)

$A = \frac{1}{2} \times 50 \times 30 \times \sin 120^{\circ}$ ***(M1)(A1)*(ft)**

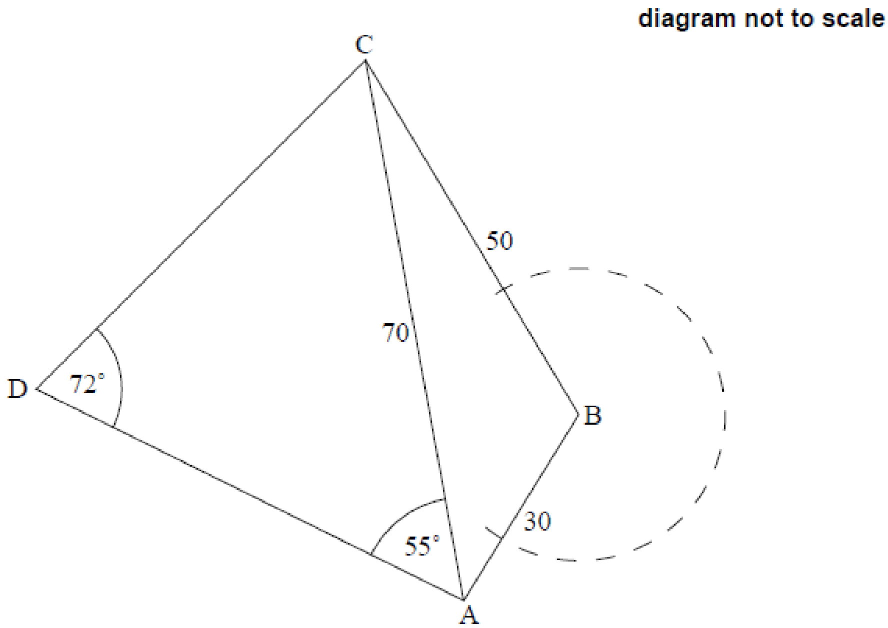
Note: Award ***(M1)*** for substitution into the area formula, ***(A1)*(ft)** for correct substitution. Award ***(M0)(A0)(A0)*** for $\frac{1}{2} \times 50 \times 30$.

$(A =)\ 650\ \mathrm{m}^2\ \left(649.519\ldots\ \mathrm{m}^2\right)$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (b).

[3 marks]

Haraya attaches a $20\ \mathrm{m}$ long rope to a vertical pole at point B.



5d. Determine whether the rope can extend into the triangular plot of land, ACD. Justify your answer. *[5 marks]*

Markscheme

METHOD 1 (equating part (c) to expression for area of triangle ABC)

$649.519\ldots = \frac{1}{2} \times 70 \times h$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for correctly substituted area of triangle formula. Award ***(A1)*(ft)** for equating the area formula to their area found in part (c).

$(h =)\ 18.6\,(\mathrm{m})\ (18.5576\ldots)$ ***(A1)*(ft)**

Note: Follow through from their part (c).

$20 > 18.5576\ldots$ ***(R1)*(ft)**

Note: Accept "the length of the rope is greater than the altitude of triangle ABC".

the rope passes inside the triangular plot of land ACD ***(A1)*(ft)**

Note: Follow through from their altitude. The final ***(A1)*** is contingent on ***(R1)*** being awarded.

METHOD 2 (finding $\text{C}\widehat{\text{A}}\text{B}$ or $\text{A}\widehat{\text{C}}\text{B}$ with sine rule and then trig ratio)

$\frac{\sin \text{C}\hat{\text{A}}\text{B}}{50} = \frac{\sin 120^\circ}{50}$ $\left(\text{C}\hat{\text{A}}\text{B} = 38.2132\ldots^\circ\right)$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into sine rule formula to find $\text{C}\hat{\text{A}}\text{B}$ or $\text{A}\hat{\text{C}}\text{B}$. Follow through from their part (b).

$(h =)\ 30 \times \sin(38.2132\ldots^\circ)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their $\text{C}\hat{\text{A}}\text{B}$ or $\text{A}\hat{\text{C}}\text{B}$ into trig formula.

$(h =)\ 18.6\,(\text{m})\ (18.5576\ldots)$ ***(A1)*(ft)**

Note: Follow through from their part (b).

$20 > 18.5576\ldots$ ***(R1)*(ft)**

Note: Accept "the length of the rope is greater than the altitude of triangle ABC".

the rope passes inside the triangular plot of land ACD ***(A1)*(ft)**

Note: Follow through from their altitude. The final ***(A1)*** is contingent on ***(R1)*** being awarded.

METHOD 3 (finding $\text{C}\widehat{\text{A}}\text{B}$ or $\text{A}\widehat{\text{C}}\text{B}$ with with cosine rule and then trig ratio)

$\cos \text{A}\hat{\text{C}}\text{B} = \frac{50^2+70^2-30^2}{2(50)(70)}$ $\left(\text{A}\hat{\text{C}}\text{B} = 21.7867\ldots^\circ\right)$ ***(M1)***

Note: Award ***(M1)*** for for their correct substitution into cosine rule formula to find $\text{C}\hat{\text{A}}\text{B}$ or $\text{A}\hat{\text{C}}\text{B}$.

$(h =)\ 50 \times \sin(21.7867\ldots^\circ)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their $\text{C}\hat{\text{A}}\text{B}$ or $\text{A}\hat{\text{C}}\text{B}$ into trig formula.

$(h =)\ 18.6\,(\text{m})\ (18.5576\ldots)$ ***(A1)*(ft)**

$20 > 18.5576\ldots$ ***(R1)*(ft)**

Note: Accept "the length of the rope is greater than the altitude of triangle ABC".

the rope passes inside the triangular plot of land ACD ***(A1)*****(ft)**

Note: Follow through from their altitude. The final ***(A1)*** is contingent on ***(R1)*** being awarded.

METHOD 4 (finding area of triangle with height 20, justifying the contradiction)

$A = \frac{1}{2}(70)(20) = 700\left(\mathrm{m}^2\right)$ ***(M1)(A1)***

Note: Award ***(M1)*** for correct substitution into area of a triangle formula for a triangle with height 20 and base 70. Award ***(A1)*** for 700. Award ***(M0)(A0)*** for unsupported 700 unless subsequent reasoning explains how the 700 was found.

$700 > 649.519\ldots$ ***(R1)***

if rope exactly touches the AC then this triangle has an area greater than ABC and as the distance AC is fixed the altitude must be less than 20 ***(R1)***

OR

$\frac{1}{2}(70)(20) > \frac{1}{2}(70)$ (height perpendicular to AC) and therefore $20 >$height perpendicular to AC ***(R1)*****(ft)**

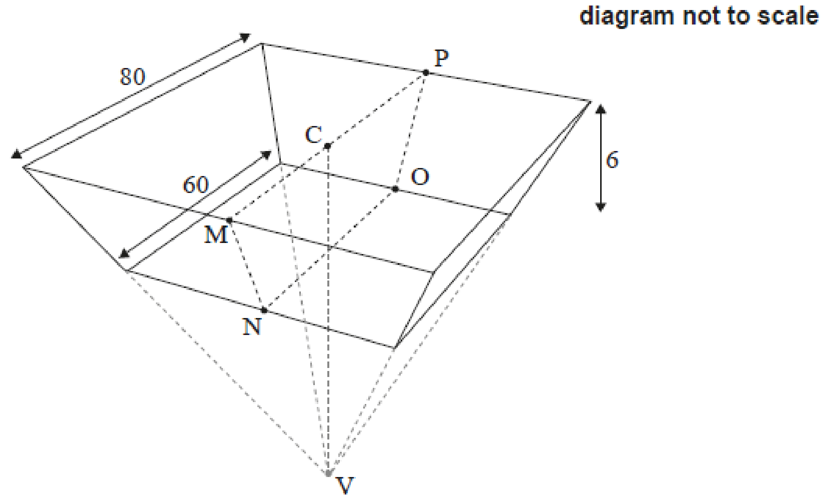
Note: Award ***(R1)*** for an explanation that recognizes the actual triangle ABC and this new triangle have the same base (70) and hence the height of triangle ABC is less than 20.

therefore, the rope passes inside the triangular plot of land ACD ***(A1)*****(ft)**

Note: Other methods, besides those listed here, may be possible. These methods can be summarized in two broad groups: the first is to find the altitude of the triangle, and compare it to 20, and the second is to create an artificial triangle with an altitude of 20 and explain why this triangle is not ABC by relating to area **and** the given lengths of the sides.

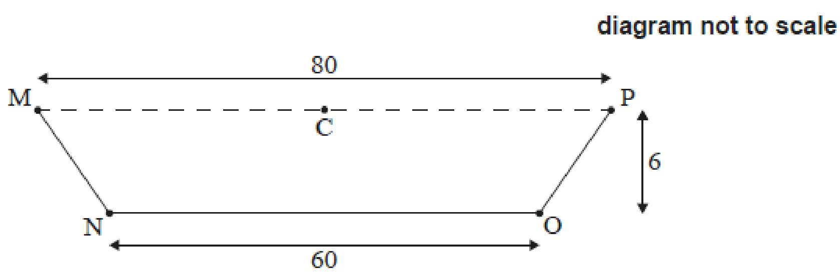
[5 marks]

A large water reservoir is built in the form of part of an upside-down right pyramid with a horizontal square base of length 80 metres. The point C is the centre of the square base and point V is the vertex of the pyramid.



The bottom of the reservoir is a square of length 60 metres that is parallel to the base of the pyramid, such that the depth of the reservoir is 6 metres as shown in the diagram.

The second diagram shows a vertical cross section, MNOPC, of the reservoir.



6a. Find the angle of depression from M to N. *[2 marks]*

Markscheme

$\tan(\theta) = \frac{6}{10}$ ***(M1)***

$(\theta =)\ 31.0^\circ\ \ (30.9637\ldots^\circ)$ **OR** $0.540\ \ (0.540419\ldots)$ ***A1***

[2 marks]

6b. Find CV. *[2 marks]*

Markscheme

$(\mathrm{CV}=)\ 40\tan(\theta)$ **OR** $(\mathrm{CV}=)\ 4\times 6$ ***(M1)***

Note: Award ***(M1)*** for an attempt at trigonometry or similar triangles (e.g. ratios).

$(\mathrm{CV}=)\ 24\ \mathrm{m}$ ***A1***

[2 marks]

6c. Hence or otherwise, show that the volume of the reservoir is $29\ 600\ \mathrm{m}^3$. *[3 marks]*

Markscheme

$(V=)\frac{1}{3}80^2\times 24-\frac{1}{3}60^2\times 18$ ***M1A1A1***

Note: Award ***M1*** for finding the difference between the volumes of two pyramids, ***A1*** for each correct volume expression. The final ***A1*** is contingent on correct working leading to the given answer.
If the correct final answer is not seen, award at most ***M1A1A0***. Award ***M0A0A0*** for any height derived from $V=29\ 600$, including 18.875 or 13.875.

$(V=)\ 29\ 600\ \mathrm{m}^3$ ***AG***

[3 marks]

Every day $80\ \mathrm{m}^3$ of water from the reservoir is used for irrigation.

Joshua states that, if no other water enters or leaves the reservoir, then when it is full there is enough irrigation water for at least one year.

6d. By finding an appropriate value, determine whether Joshua is correct. *[2 marks]*

Markscheme

METHOD 1

$\left(\frac{29600}{80}=\right) 370$ (days) ***A1***

$(370 > 366)$ Joshua is correct ***A1***

Note: Award ***A0A0*** for unsupported answer of “Joshua is correct”. Accept $1.01\ldots > 1$ for the first ***A1*** mark.

METHOD 2

$80 \times 366 = 29280 \text{ m}^3$ **OR** $80 \times 365 = 29200 \text{ m}^3$ ***A1***

$(29280 < 29600)$ Joshua is correct ***A1***

Note: The second ***A1*** can be awarded for an answer consistent with their result.

[2 marks]

6e. To avoid water leaking into the ground, the five interior sides of the reservoir have been painted with a watertight material. *[5 marks]*

Find the area that was painted.

Markscheme

height of trapezium is $\sqrt{10^2+6^2}$ $(=11.6619\ldots)$ ***(M1)***

area of trapezium is $\frac{80+60}{2}\times\sqrt{10^2+6^2}$ $(=816.333\ldots)$ ***(M1)(A1)***

$(SA=)\ 4\times\left(\frac{80+60}{2}\times\sqrt{10^2+6^2}\right)+60^2$ ***(M1)***

Note: Award ***M1*** for adding 4 times their (MNOP) trapezium area to the area of the (60×60) base.

$(SA=)\ 6870\ \mathrm{m}^2\ \ \left(6865.33\ \mathrm{m}^2\right)$ ***A1***

Note: No marks are awarded if the correct shape is not identified.

[5 marks]