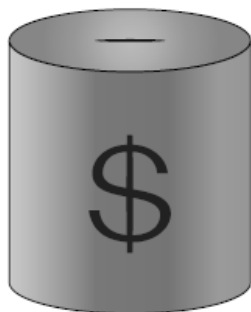# Geometry and trigonometry
5.12 *[50 marks]*

Money boxes are coin containers used by children and come in a variety of shapes. The money box shown is in the shape of a cylinder. It has a radius of 4.43 cm and a height of 12.2 cm.

diagram not to scale

1a. Find the volume of the money box.   *[3 marks]*

1b. A second money box is in the shape of a sphere and has the same   *[3 marks]*
   volume as the cylindrical money box.

diagram not to scale

Find the diameter of the second money box.

The straight metal arm of a windscreen wiper on a car rotates in a circular motion from a pivot point, O, through an angle of $140°$. The windscreen is cleared by a rubber blade of length 46 cm that is attached to the metal arm between points A and B. The total length of the metal arm, OB, is 56 cm.

The part of the windscreen cleared by the rubber blade is shown unshaded in the following diagram.

diagram not to scale
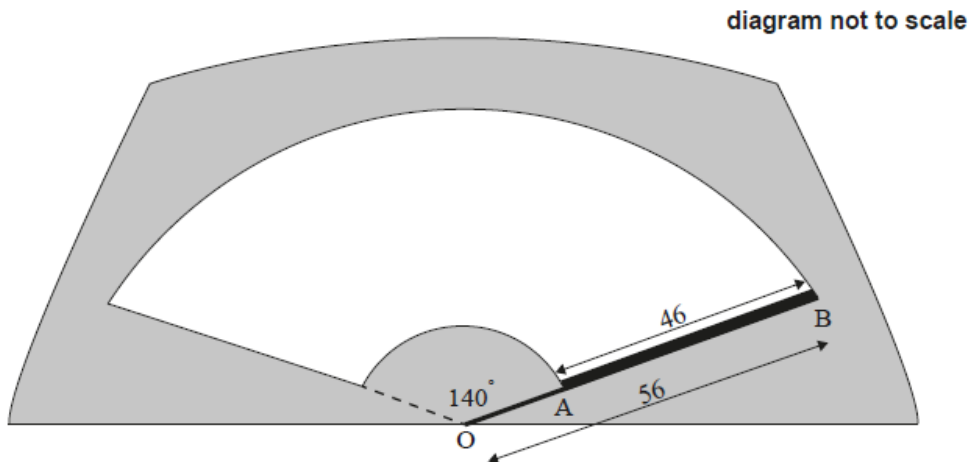
2a. Calculate the length of the arc made by B, the end of the rubber blade.   *[2 marks]*

2b. Determine the area of the windscreen that is cleared by the rubber blade.   *[3 marks]*

Joey is making a party hat in the form of a cone. The hat is made from a sector, AOB, of a circular piece of paper with a radius of 18 cm and $\text{A}\hat{\text{O}}\text{B} = \theta$ as shown in the diagram.

diagram not to scale
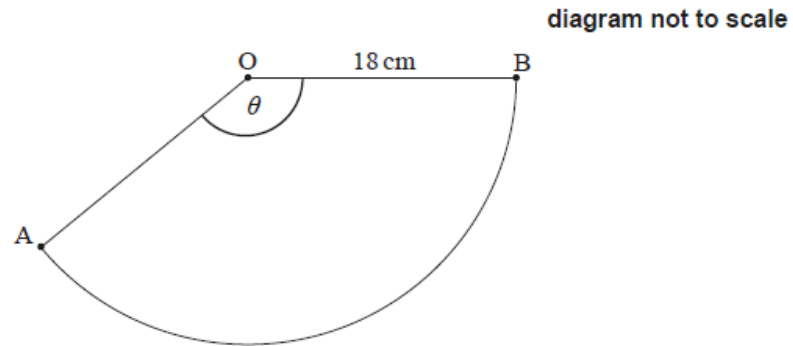
To make the hat, sides $[\text{OA}]$ and $[\text{OB}]$ are joined together. The hat has a base radius of 6.5 cm.
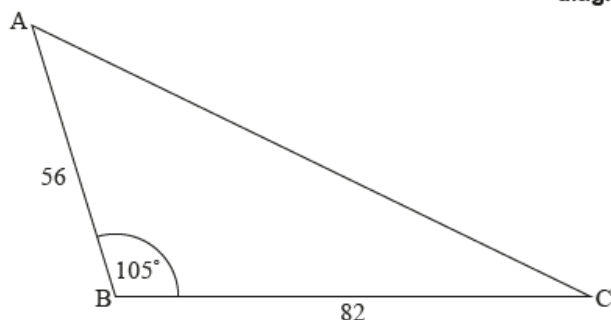
diagram not to scale
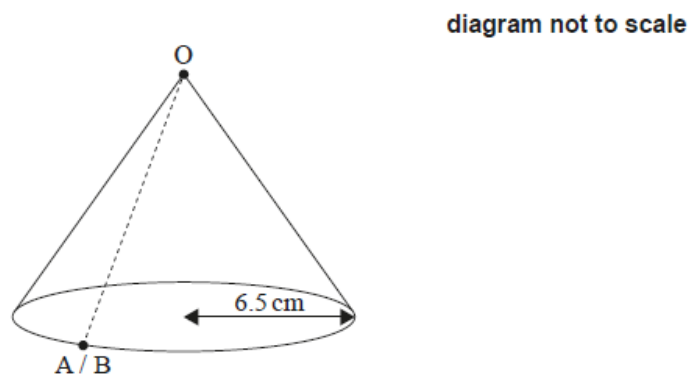
3a. Write down the perimeter of the base of the hat in terms of π.　　　　　*[1 mark]*

3b. Find the value of θ.　　　　　*[2 marks]*

3c. Find the surface area of the outside of the hat.　　　　　*[2 marks]*

4. A triangular field ABC is such that $AB = 56$ m and $BC = 82$ m, each **[5 marks]** measured correct to the nearest metre, and the angle at B is equal to $105°$, measured correct to the nearest $5°$.
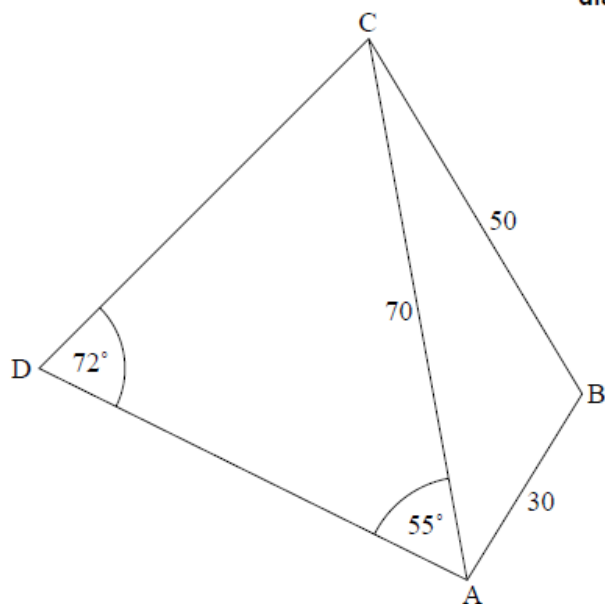
diagram not to scale

Calculate the maximum possible area of the field.

Haraya owns two triangular plots of land, ABC and ACD. The length of AB is 30 m, BC is 50 m and AC is 70 m. The size of $D\hat{A}C$ is $55°$ and $A\hat{D}C$ is $72°$.

The following diagram shows this information.
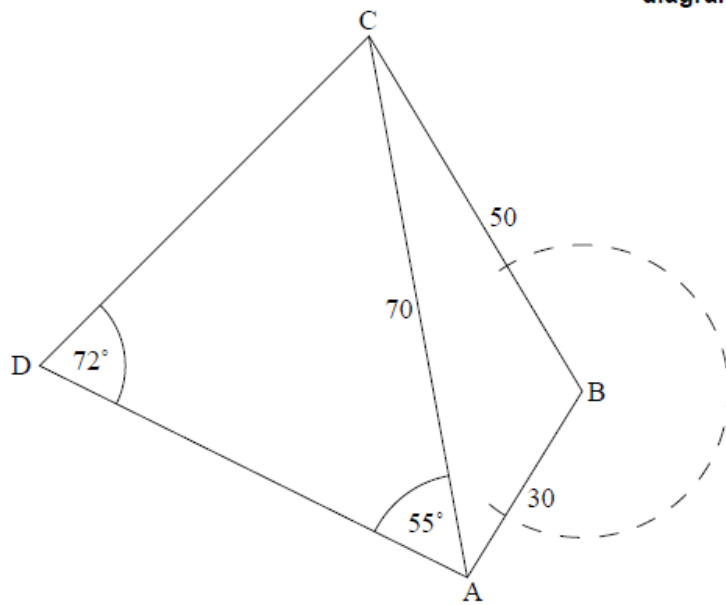
diagram not to scale

5a. Find the length of AD. **[4 marks]**

5b. Find the size of $A\hat{B}C$. **[3 marks]**

5c. Calculate the area of the triangular plot of land ABC. **[3 marks]**

Haraya attaches a 20 m long rope to a vertical pole at point B.

diagram not to scale

5d. Determine whether the rope can extend into the triangular plot of land, *[5 marks]* ACD. Justify your answer.

A large water reservoir is built in the form of part of an upside-down right pyramid with a horizontal square base of length 80 metres. The point C is the centre of the square base and point V is the vertex of the pyramid.

diagram not to scale
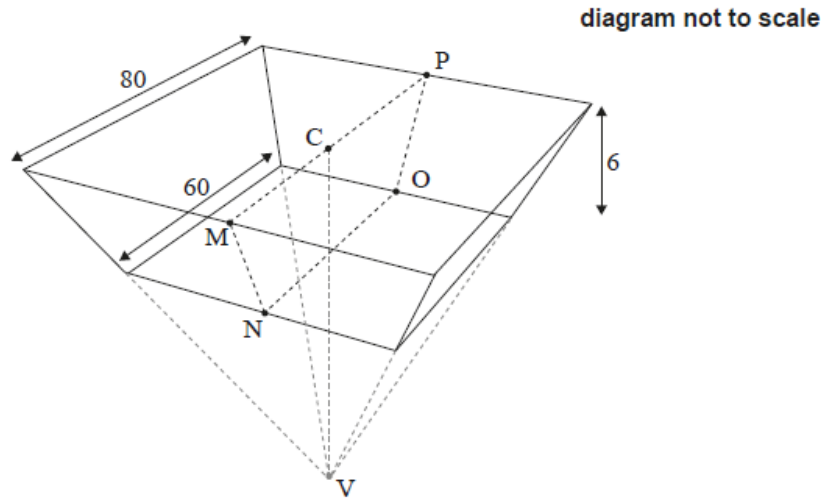
The bottom of the reservoir is a square of length 60 metres that is parallel to the base of the pyramid, such that the depth of the reservoir is 6 metres as shown in the diagram.

The second diagram shows a vertical cross section, $MNOPC$, of the reservoir.

diagram not to scale
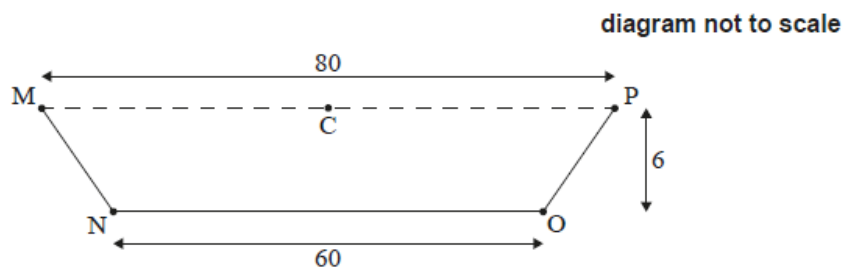
6a. Find the angle of depression from M to N.   *[2 marks]*

6b. Find CV.   *[2 marks]*

6c. Hence or otherwise, show that the volume of the reservoir is $29\ 600 \text{ m}^3$.   *[3 marks]*

Every day 80 m^3 of water from the reservoir is used for irrigation.

Joshua states that, if no other water enters or leaves the reservoir, then when it is full there is enough irrigation water for at least one year.

6d. By finding an appropriate value, determine whether Joshua is correct.   *[2 marks]*

6e. To avoid water leaking into the ground, the five interior sides of the reservoir have been painted with a watertight material.

[5 marks]

Find the area that was painted.