Maclaurin *[53 marks]*

The function f is defined by $f(x) = \arcsin(2x)$, where $-\frac{1}{2} \leqslant x \leqslant \frac{1}{2}$.

1a. By finding a suitable number of derivatives of f, find the first two non-zero terms in the Maclaurin series for f. *[8 marks]*

1b. Hence or otherwise, find $\lim_{x \to 0} \frac{\arcsin(2x) - 2x}{(2x)^3}$. *[3 marks]*

The function f is defined by $f(x) = e^{\sin x}$.

2a. Find the first two derivatives of $f(x)$ and hence find the Maclaurin series for $f(x)$ up to and including the x^2 term. *[8 marks]*

2b. Show that the coefficient of x^3 in the Maclaurin series for $f(x)$ is zero. *[4 marks]*

2c. Using the Maclaurin series for $\arctan x$ and $e^{3x} - 1$, find the Maclaurin series for $\arctan\left(e^{3x} - 1\right)$ up to and including the x^3 term. *[6 marks]*

2d. Hence, or otherwise, find $\lim_{x \to 0} \frac{f(x) - 1}{\arctan(e^{3x} - 1)}$. *[3 marks]*

The function f is defined by $f(x) = e^x \sin x$, where $x \in \mathbb{R}$.

3a. Find the Maclaurin series for $f(x)$ up to and including the x^3 term. *[4 marks]*

3b. Hence, find an approximate value for $\int_0^1 e^{x^2} \sin\left(x^2\right) \mathrm{d}\,x$. *[4 marks]*

The function g is defined by $g(x) = e^x \cos x$, where $x \in \mathbb{R}$.

3c. Show that $g(x)$ satisfies the equation $g''(x) = 2(g'(x) - g(x))$. *[4 marks]*

3d. Hence, deduce that $g^{(4)}(x) = 2(g'''(x) - g''(x))$. *[1 mark]*

3e. Using the result from part (c), find the Maclaurin series for $g(x)$ up to and including the x^4 term. *[5 marks]*

3f. Hence, or otherwise, determine the value of $\lim_{x \to 0} \frac{e^x \cos x - 1 - x}{x^3}$. *[3 marks]*