Monday 12.12 *[55 marks]*

Let $f(x) = \frac{4x-5}{x^2-3x+2}$ $x \neq 1, x \neq 2$.

1a. Express $f(x)$ in partial fractions. *[6 marks]*

1b. Use part (a) to show that $f(x)$ is always decreasing. *[3 marks]*

1c. *[4 marks]*

Use part (a) to find the exact value of $\int_{-1}^{0} f(x)dx$, giving the answer in the form $\ln q, \quad q \in \mathbb{Q}$.

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Consider the series $\ln x + p \ln x + \frac{1}{3}\ln x + \ldots$, where $x \in \mathbb{R},\ x > 1$ and $p \in \mathbb{R},\ p \neq 0$.

Consider the case where the series is geometric.

2a. Show that $p = \pm\frac{1}{\sqrt{3}}$. *[2 marks]*

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2b. Given that $p > 0$ and $S_\infty = 3 + \sqrt{3}$, find the value of x. *[3 marks]*

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Now consider the case where the series is arithmetic with common difference d.

2c. Show that $p = \frac{2}{3}$. *[3 marks]*

2d. Write down d in the form $k \ln x$, where $k \in \mathbb{Q}$. *[1 mark]*

2e. The sum of the first n terms of the series is $-3 \ln x$. *[6 marks]*

Find the value of n.

3. Consider the graphs of $y = \frac{x^2}{x-3}$ and $y = m(x+3)$, $m \in \mathbb{R}$. *[5 marks]*

Find the set of values for m such that the two graphs have no intersection points.

The function f is defined by $f(x) = \frac{4x+1}{x+4}$, where $x \in \mathbb{R},\ x \neq -4$.

For the graph of f

4a. write down the equation of the vertical asymptote. *[1 mark]*

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4b. find the equation of the horizontal asymptote. *[2 marks]*

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4c. Find $f^{-1}(x)$. *[4 marks]*

4d. Using an algebraic approach, show that the graph of f^{-1} is obtained by a reflection of the graph of f in the y-axis followed by a reflection in the x-axis. *[4 marks]*

The graphs of f and f^{-1} intersect at $x = p$ and $x = q$, where $p < q$.

4e. Find the value of p and the value of q. *[2 marks]*

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4f. Hence, find the area enclosed by the graph of f and the graph of f^{-1}. *[3 marks]*

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A company produces bags of sugar whose masses, in grams, can be modelled by a normal distribution with mean 1000 and standard deviation 3.5. A bag of sugar is rejected for sale if its mass is less than 995 grams.

5a. Find the probability that a bag selected at random is rejected. *[2 marks]*

5b. Estimate the number of bags which will be rejected from a random sample of 100 bags. *[1 mark]*

5c. Given that a bag is not rejected, find the probability that it has a mass greater than 1005 grams. *[3 marks]*