Probability 19.12 *[59 marks]*

A group of 130 applicants applied for admission into either the Arts programme or the Sciences programme at a university. The outcomes of their applications are shown in the following table.

	Accepted	Rejected
Arts programme	17	24
Sciences programme	25	64

1a. Find the probability that a randomly chosen applicant from this group was accepted by the university. *[1 mark]*

Markscheme

$\left(\frac{17+25}{130} =\right) \frac{42}{130}$ $\left(\frac{21}{65}, 0.323076\ldots\right)$ ***A1***

[1 mark]

An applicant is chosen at random from this group. It is found that they were accepted into the programme of their choice.

1b. Find the probability that the applicant applied for the Arts programme. *[2 marks]*

Markscheme

$\left(\frac{17}{17+25} =\right) \frac{17}{42}$ $(0.404761\ldots)$ ***A1A1***

Note: Award ***A1*** for correct numerator and ***A1*** for correct denominator.
Award ***A1A0*** for working of $\frac{17/130}{\text{their answer to (a)}}$ if followed by an incorrect answer.

[2 marks]

1c. Two different applicants are chosen at random from the original group. *[3 marks]*

Find the probability that both applicants applied to the Arts programme.

Markscheme

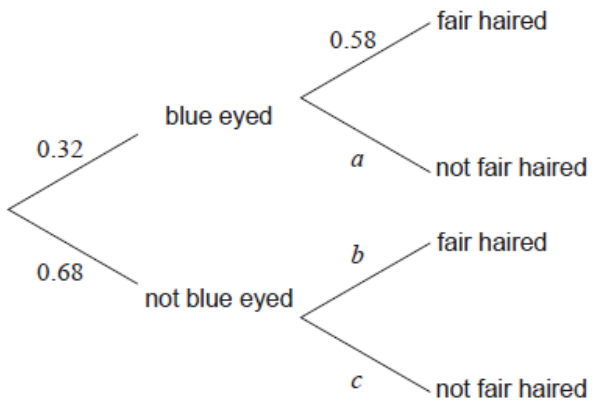
$\frac{41}{130} \times \frac{40}{129}$ ***A1M1***

Note: Award ***A1*** for two correct fractions seen, ***M1*** for multiplying their fractions.

$= \frac{1640}{16770} \approx 0.0978 \left(0.0977936\ldots, \frac{164}{1677}\right)$ ***A1***

[3 marks]

In a city, 32% of people have blue eyes. If someone has blue eyes, the probability that they also have fair hair is 58%. This information is represented in the following tree diagram.



2a. Write down the value of a. *[1 mark]*

Markscheme

$a = 0.42$ ***A1***

[1 mark]

2b. Find an expression, in terms of b, for the probability of a person not having blue eyes **and** having fair hair. *[1 mark]*

Markscheme

$(\mathrm{P}(B' \cap F) =)\ b \times 0.68$ **A1**

[1 mark]

It is known that 41% of people in this city have fair hair.

Calculate the value of

2c. b. *[2 marks]*

Markscheme

$0.32 \times 0.58 + 0.68b = 0.41$ **(M1)**

Note: Award ***(M1)*** for setting up equation for fair-haired or equivalent.

$b = 0.33$ **A1**

[2 marks]

2d. c. *[1 mark]*

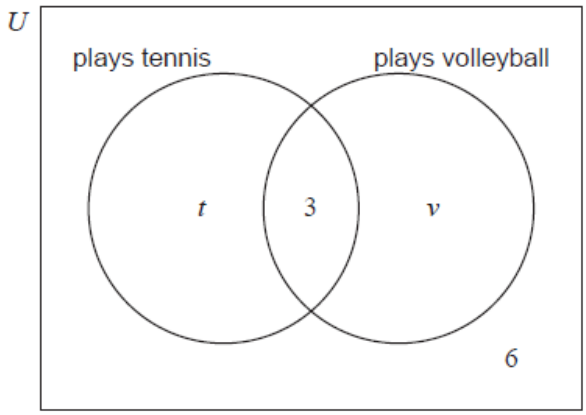
Markscheme

$c = 0.67$ **A1**

[1 mark]

In a class of 30 students, 19 play tennis, 3 play both tennis and volleyball, and 6 do not play either sport.

The following Venn diagram shows the events “plays tennis” and “plays volleyball”. The values t and v represent numbers of students.



3a. Find the value of t. *[2 marks]*

Markscheme

valid approach to find t ***(M1)***

eg $t + 3 = 19,\ 19 - 3$

$t = 16$ (may be seen on Venn diagram) ***A1 N2***

[2 marks]

3b. Find the value of v. *[2 marks]*

Markscheme

valid approach to find v ***(M1)***

eg $t + 3 + v + 6 = 30,\ 30 - 19 - 6$

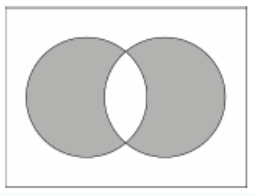
$v = 5$ (may be seen on Venn diagram) ***A1 N2***

[2 marks]

3c. Find the probability that a randomly selected student from the class plays tennis or volleyball, but not both. *[2 marks]*

Markscheme

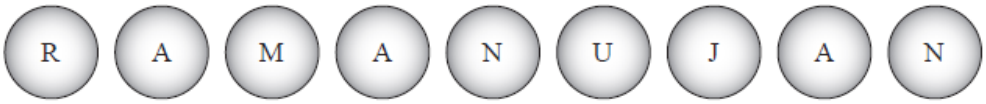
valid approach ***(M1)***

eg $16 + 5$, 21 students, $1 - \frac{3+6}{30}$, 

$\frac{21}{30}$ $\left(= \frac{7}{10}\right)$ ***A1 N2***

[2 marks]

Srinivasa places the nine labelled balls shown below into a box.



Srinivasa then chooses two balls at random, one at a time, from the box. The first ball is **not replaced** before he chooses the second.

4a. Find the probability that the first ball chosen is labelled A. *[1 mark]*

Markscheme

$\frac{3}{9}$ $\left(\frac{1}{3},\ 0.333,\ 0.333333\ldots,\ 33.3\%\right)$ ***(A1)*** ***(C1)***

[1 mark]

4b. Find the probability that the first ball chosen is labelled A or labelled N. *[1 mark]*

Markscheme

$\frac{5}{9}$ $(0.556,\ 0.555555\ldots,\ 55.6\%)$ ***(A1)*** ***(C1)***

[1 mark]

4c. Find the probability that the second ball chosen is labelled A, given that the first ball chosen was labelled N. *[2 marks]*

Markscheme

$\frac{3}{8}\ (0.375,\ 37.5\%)$ ***(A1)(A1) (C2)***

Note: Award ***(A1)*** for correct numerator, ***(A1)*** for correct denominator.

[2 marks]

4d. Find the probability that both balls chosen are labelled N. *[2 marks]*

Markscheme

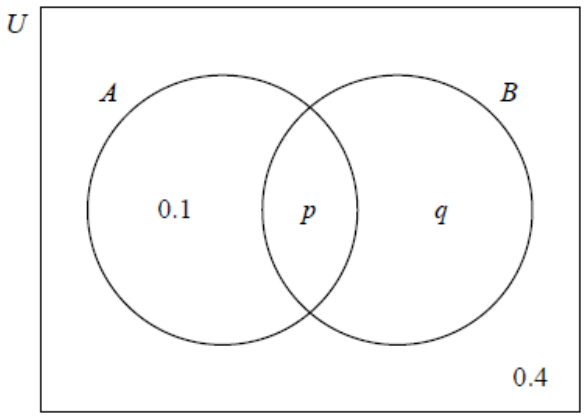
$\frac{2}{9} \times \frac{1}{8}$ ***(M1)***

Note: Award ***(M1)*** for a correct compound probability calculation seen.

$\frac{2}{72}\ \left(\frac{1}{36},\ 0.0278,\ 0.0277777\ldots,\ 2.78\%\right)$ ***(A1) (C2)***

[2 marks]

The following Venn diagram shows the events A and B, where $\mathrm{P}(A) = 0.3$. The values shown are probabilities.



5a. Find the value of p. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

valid approach ***(M1)***

eg $0.30 - 0.1, p + 0.1 = 0.3$

$p = 0.2$ ***A1 N2***

[2 marks]

5b. Find the value of q. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $1 - (0.3 + 0.4), 1 - 0.4 - 0.1 - p$

$q = 0.3$ ***A1 N2***

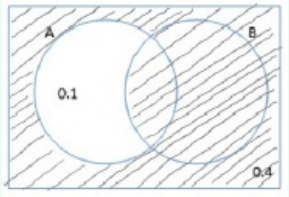
[2 marks]

5c. Find $\mathrm{P}(A' \cup B)$. *[2 marks]*

Markscheme

valid approach ***(M1)***

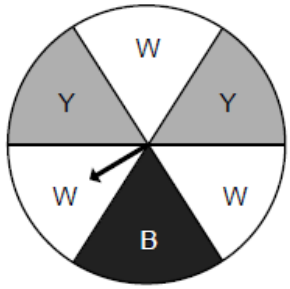
eg $0.7 + 0.5 - 0.3,\ p + q + 0.4,\ 1 - 0.1,$
$\mathrm{P}(A' \cup B) = \mathrm{P}(A') + \mathrm{P}(B) - \mathrm{P}(A' \cap B)$



$\mathrm{P}(A' \cup B) = 0.9$ ***A1 N2***

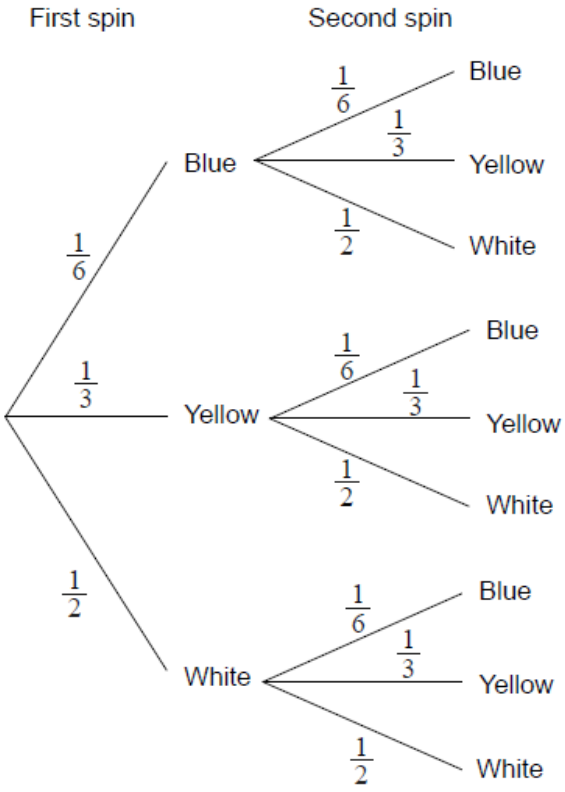
[2 marks]

The diagram shows a circular horizontal board divided into six equal sectors. The sectors are labelled white (W), yellow (Y) and blue (B).



A pointer is pinned to the centre of the board. The pointer is to be spun and when it stops the colour of the sector on which the pointer stops is recorded. The pointer is equally likely to stop on any of the six sectors.

Eva will spin the pointer twice. The following tree diagram shows all the possible outcomes.



6a. Find the probability that both spins are yellow. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\frac{1}{3} \times \frac{1}{3}$ **OR** $\left(\frac{1}{3}\right)^2$ ***(M1)***

Note: Award ***(M1)*** for multiplying correct probabilities.

$\frac{1}{9}$ (0.111, 0.111111..., 11.1%) ***(A1)*** ***(C2)***

[2 marks]

6b. Find the probability that at least one of the spins is yellow. *[3 marks]*

Markscheme

$\left(\frac{1}{2} \times \frac{1}{3}\right) + \left(\frac{1}{6} \times \frac{1}{3}\right) + \frac{1}{3}$ ***(M1)(M1)***

Note: Award ***(M1)*** for $\left(\frac{1}{2} \times \frac{1}{3}\right)$ and $\left(\frac{1}{6} \times \frac{1}{3}\right)$ or equivalent, and ***(M1)*** for $\frac{1}{3}$ **and** adding only the three correct probabilities.

OR

$1 - \left(\frac{2}{3}\right)^2$ ***(M1)(M1)***

Note: Award ***(M1)*** for $\frac{2}{3}$ seen and ***(M1)*** for subtracting $\left(\frac{2}{3}\right)^2$ from 1. This may be shown in a tree diagram with “yellow” and “not yellow” branches.

$\frac{5}{9}$ (0.556, 0.555555..., 55.6%) ***(A1)*****(ft)** ***(C3)***

Note: Follow through marks may be awarded if their answer to part (a) is used in a correct calculation.

[3 marks]

6c. Write down the probability that the second spin is yellow, given that the first spin is blue. *[1 mark]*

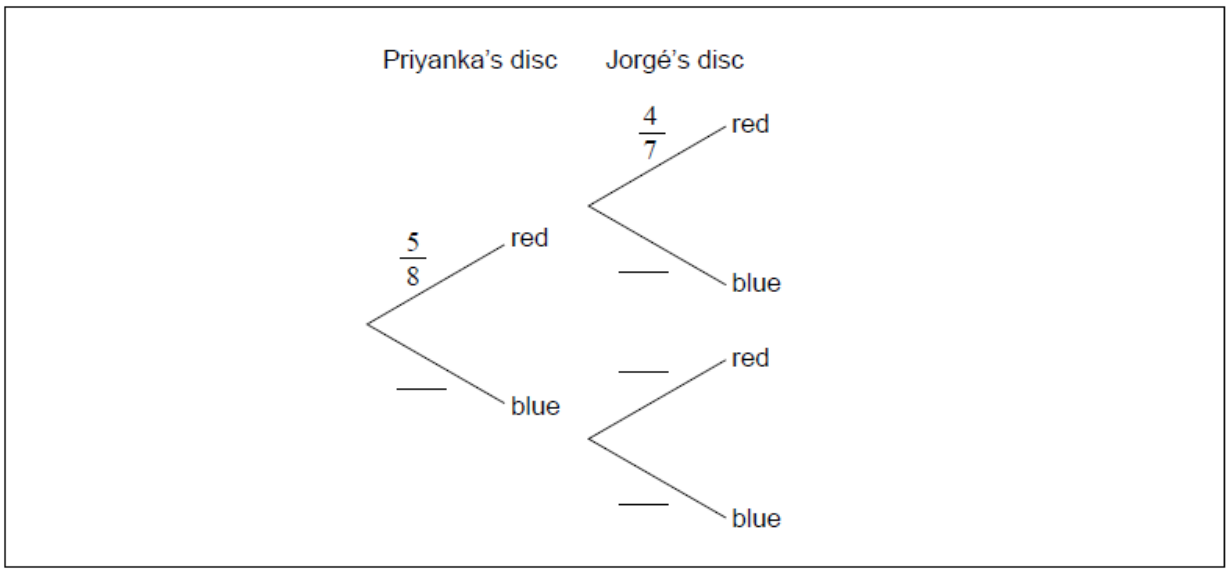
Markscheme

$\frac{1}{3}$ (0.333, 0.333333..., 33.3%) ***(A1)*** ***(C1)***

[1 mark]

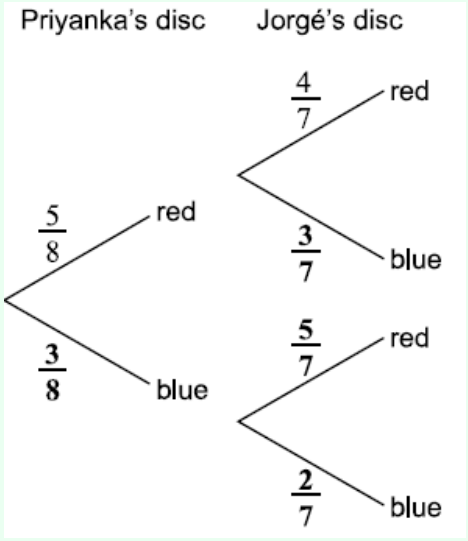
A bag contains 5 red and 3 blue discs, all identical except for the colour. First, Priyanka takes a disc at random from the bag and then Jorgé takes a disc at random from the bag.

7a. Complete the tree diagram. *[3 marks]*



Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.



(A1)(A1)(A1) (C3)

Note: Award ***(A1)*** for each correct pair of branches.

[3 marks]

7b. Find the probability that Jorgé chooses a red disc. *[3 marks]*

Markscheme

$\frac{5}{8} \times \frac{4}{7} + \frac{3}{8} \times \frac{5}{7}$ ***(A1)*****(ft)*****(M1)***

Note: Award ***(A1)*****(ft)** for **their** two correct products from their tree diagram. Follow through from part (a), award ***(M1)*** for adding their two products. Award ***(M0)*** if additional products or terms are added.

$= \frac{5}{8}$ $\left(\frac{35}{56},\ 0.625,\ 62.5\,\%\right)$ ***(A1)*****(ft)** ***(C3)***

Note: Follow through from their tree diagram, only if probabilities are [0,1].

[3 marks]

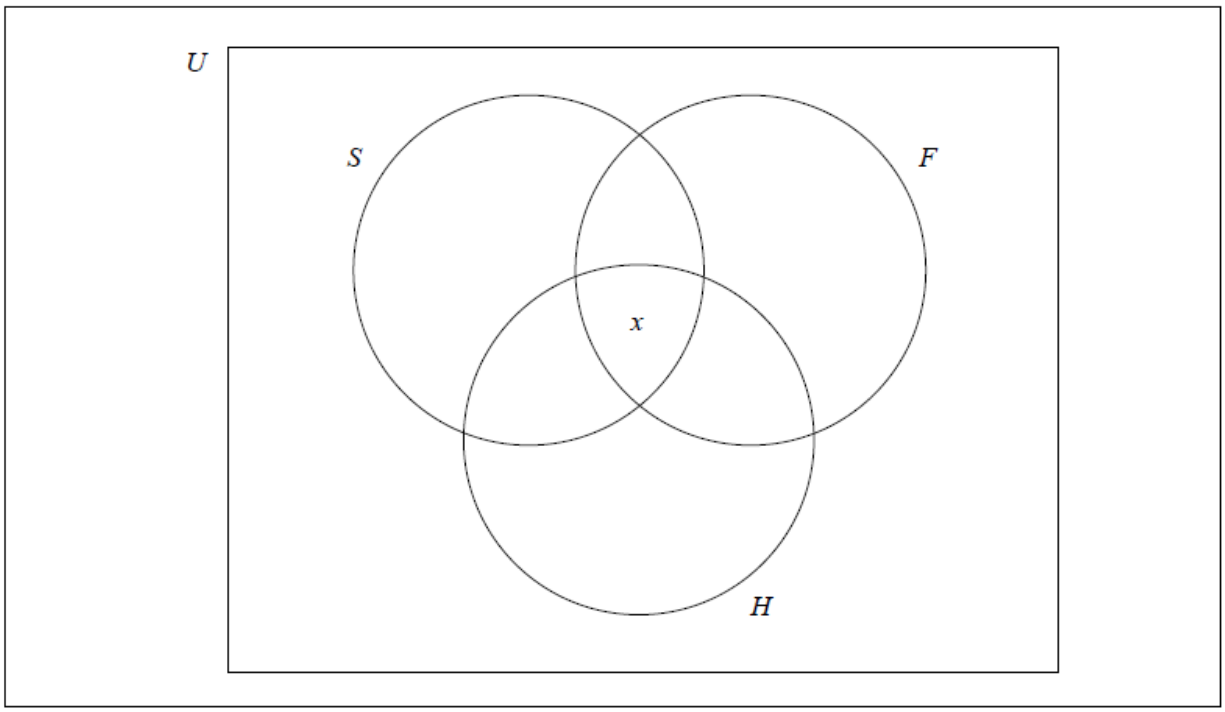
A group of 60 sports enthusiasts visited the PyeongChang 2018 Winter Olympic games to watch a variety of sporting events.

The most popular sports were snowboarding (S), figure skating (F) and ice hockey (H).

For this group of 60 people:

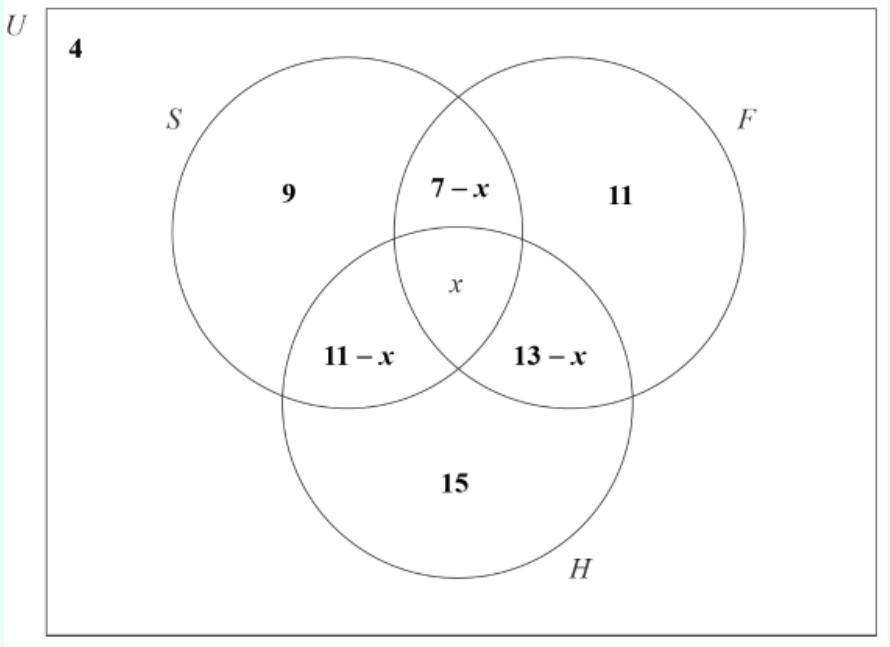
4 did not watch any of the most popular sports,
x watched all three of the most popular sports,
9 watched snowboarding only,
11 watched figure skating only,
15 watched ice hockey only,
7 watched snowboarding and figure skating,
13 watched figure skating and ice hockey,
11 watched snowboarding and ice hockey.

8a. Complete the Venn diagram using the given information. *[3 marks]*



Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.



(A1)(A1)(A1) (C3)

Note: Award ***(A1)*** for 4 in correct place.

Award ***(A1)*** for 9, 11, 15 in correct place.

Award ***(A1)*** for $7 - x$, $13 - x$, $11 - x$ in correct place.

Accept 2, 8 and 6 in place of $7 - x$, $13 - x$, $11 - x$.

[3 marks]

8b. Find the value of x. *[2 marks]*

Markscheme

$4 + 9 + 11 + 15 + x + (7 - x) + (11 - x) + (13 - x) = 60$ ***(M1)***

Note: Award ***(M1)*** for equating the sum of at least seven of the entries in their Venn diagram to 60.

$(x =)\ 5$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from part (a), but only if answer is positive.

[2 marks]

8c. Write down the value of $n\left((F \cup H) \cap S'\right)$. *[1 mark]*

Markscheme

34 ***(A1)*(ft) *(C1)***

Note: Follow through from their Venn diagram.

[1 mark]

In an international competition, participants can answer questions in **only one** of the three following languages: Portuguese, Mandarin or Hindi. 80 participants took part in the competition. The number of participants answering in Portuguese, Mandarin or Hindi is shown in the table.

		Languages			
		Portuguese	Mandarin	Hindi	**Total**
Participants	Boys	20	18	5	**43**
	Girls	18	7	12	**37**
	Total	**38**	**25**	**17**	**80**

9a. State the number of boys who answered questions in Portuguese. *[1 mark]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

20 ***(A1) (C1)***

[1 mark]

A boy is chosen at random.

9b. Find the probability that the boy answered questions in Hindi. *[2 marks]*

Markscheme

null ***(A1)(A1) (C2)***

Note: Award ***(A1)*** for correct numerator, ***(A1)*** for correct denominator.

[2 marks]

9c. Two girls are selected at random. *[3 marks]*

Calculate the probability that one girl answered questions in Mandarin and the other answered questions in Hindi.

Markscheme

$\frac{7}{37} \times \frac{12}{36} + \frac{12}{37} \times \frac{7}{36}$ ***(A1)(M1)***

Note: Award ***(A1)*** for first or second correct product seen, ***(M1)*** for adding their two products or for multiplying their product by two.

$= \frac{14}{111}$ $(0.12612\ldots,\ 12.6126\,\%)$ ***(A1) (C3)***

[3 marks]

Consider the following Venn diagrams.

Diagram 1

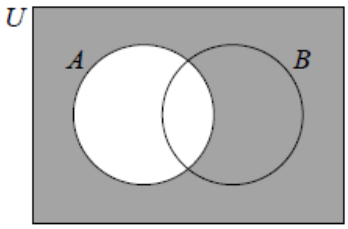


Diagram 2

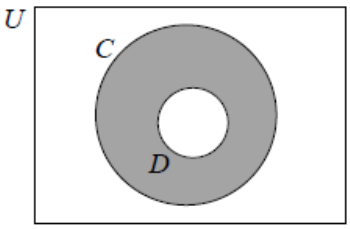
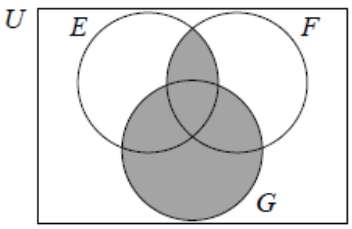


Diagram 3



10a. Write down an expression, in set notation, for the **shaded** region represented by Diagram 1. *[1 mark]*

Markscheme

A' ***(A1)***

Note: Accept alternative set notation for complement such as $U - A$.

[1 mark]

10b. Write down an expression, in set notation, for the **shaded** region represented by Diagram 2. *[1 mark]*

Markscheme

$C \cap D'$ **OR** $D' \cap C$ ***(A1)***

Note: Accept alternative set notation for complement.

[1 mark]

10c. Write down an expression, in set notation, for the shaded region represented by Diagram 3. *[2 marks]*

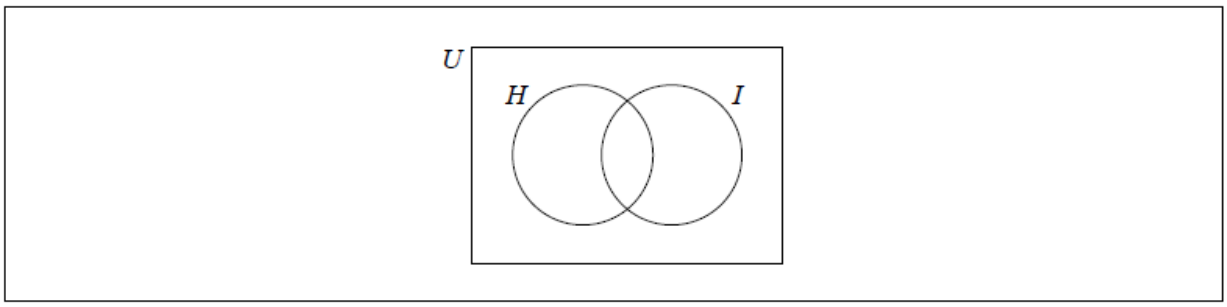
Markscheme

$(E \cap F) \cup G$ **OR** $G \cup (E \cap F)$ ***(A2) (C4)***

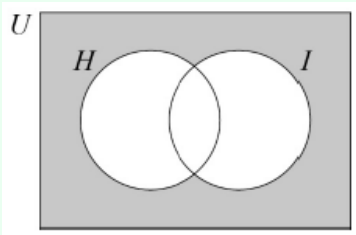
Note: Accept equivalent answers, for example $(E \cup G) \cap (F \cup G)$.

[2 marks]

10d. Shade, on the Venn diagram, the region represented by the set $(H \cup I)'$ *[1 mark]*.



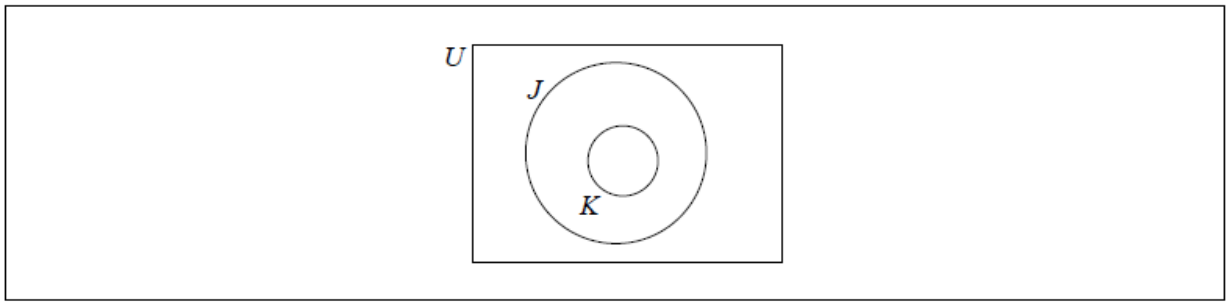
Markscheme



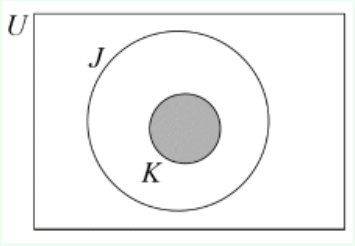
(A1)

[1 mark]

10e. Shade, on the Venn diagram, the region represented by the set $J \cap K$. *[1 mark]*



Markscheme



(A1) (C2)

[1 mark]