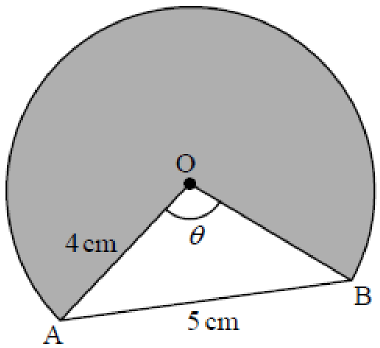
Revision Trigonometry *[213 marks]*

The following diagram shows part of a circle with centre O and radius 4 cm.



Chord AB has a length of 5 cm and AÔB = θ.

1a. Find the value of θ, giving your answer in radians. *[3 marks]*

1b. Find the area of the shaded region. *[3 marks]*

Consider a function f, such that $f\left(x\right) = 5.8\sin\left(\frac{\pi}{6}(x+1)\right) + b$, $0 \le x \le 10$, $b \in \mathbb{R}$.

2a. Find the period of f. *[2 marks]*

The function f has a local maximum at the point (2, 21.8) , and a local minimum at (8, 10.2).

2b. Find the value of b. *[2 marks]*

2c. Hence, find the value of $f(6)$. *[2 marks]*

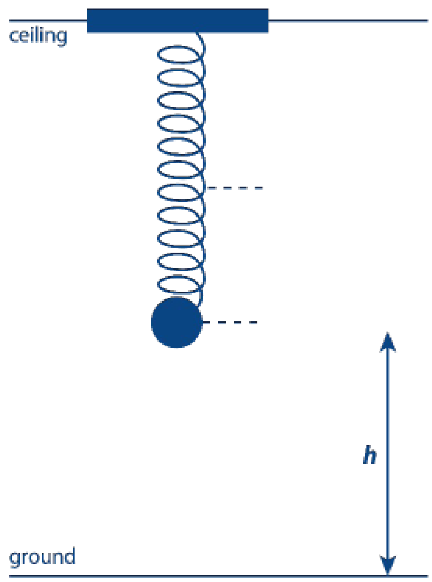
A second function g is given by $g(x) = p\sin\left(\frac{2\pi}{9}(x - 3.75)\right) + q$, $0 \le x \le 10$; p, $q \in \mathbb{R}$.

The function g passes through the points (3, 2.5) and (6, 15.1).

2d. Find the value of p and the value of q. *[5 marks]*

2e. Find the value of x for which the functions have the greatest difference. *[2 marks]*

The following diagram shows a ball attached to the end of a spring, which is suspended from a ceiling.



The height, h metres, of the ball above the ground at time t seconds after being released can be modelled by the function $h(t) = 0.4\cos(\pi t) + 1.8$ where $t \ge 0$.

3a. Find the height of the ball above the ground when it is released. *[2 marks]*

3b. Find the minimum height of the ball above the ground. *[2 marks]*

3c. Show that the ball takes 2 seconds to return to its initial height above the ground for the first time. *[2 marks]*

3d. For the first 2 seconds of its motion, determine the amount of time that the ball is less than $1.8 + 0.2\sqrt{2}$ metres above the ground. *[5 marks]*

Consider $f(x) = 4\sin x + 2.5$ and $g(x) = 4\sin\left(x - \frac{3\pi}{2}\right) + 2.5 + q$, where $x \in \mathbb{R}$ and $q > 0$.

The graph of g is obtained by two transformations of the graph of f.

4a. Describe these two transformations. *[2 marks]*

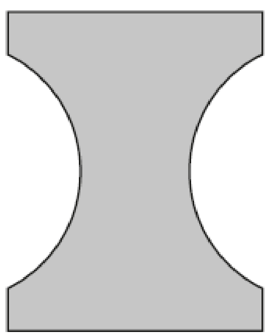
4b. The y-intercept of the graph of g is at $(0,\ r)$. *[5 marks]*

Given that $g(x) \geq 7$, find the smallest value of r.

5. Find the least positive value of x for which $\cos\left(\frac{x}{2} + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$. *[5 marks]*

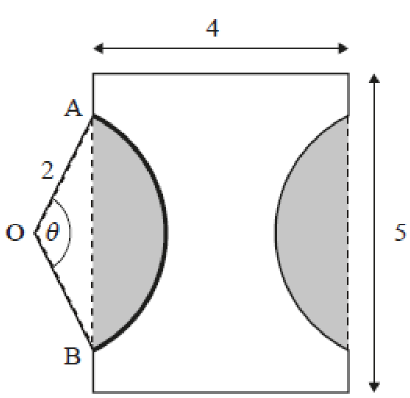
A company is designing a new logo. The logo is created by removing two equal segments from a rectangle, as shown in the following diagram.

diagram not to scale



The rectangle measures $5\ \text{cm}$ by $4\ \text{cm}$. The points A and B lie on a circle, with centre O and radius $2\ \text{cm}$, such that $\text{A}\hat{\text{O}}\text{B} = \theta$, where $0 < \theta < \pi$. This information is shown in the following diagram.

diagram not to scale



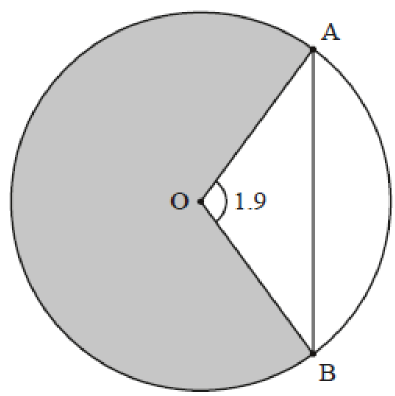
6a. Find the area of one of the shaded segments in terms of θ. *[3 marks]*

6b. Given that the area of the logo is $13.4\ \text{cm}^2$, find the value of θ. *[3 marks]*

The following diagram shows a circle with centre O and radius 5 metres.

Points A and B lie on the circle and $\text{A}\hat{\text{O}}\text{B} = 1.9$ radians.

diagram not to scale



7a. Find the length of the chord $[\text{AB}]$. *[3 marks]*

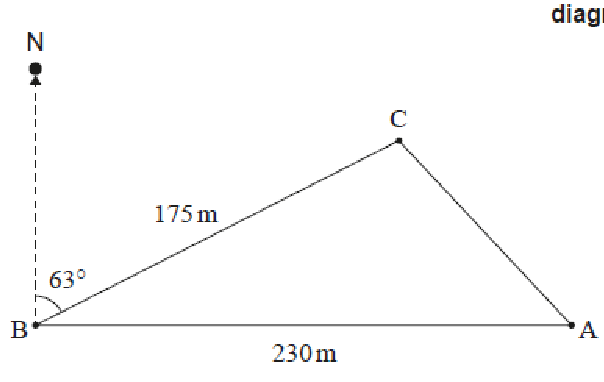
7b. Find the area of the shaded sector. *[3 marks]*

A farmer is placing posts at points A, B, and C in the ground to mark the boundaries of a triangular piece of land on his property.

From point A, he walks due west 230 metres to point B.
From point B, he walks 175 metres on a bearing of 063° to reach point C.

This is shown in the following diagram.

diagram not to scale

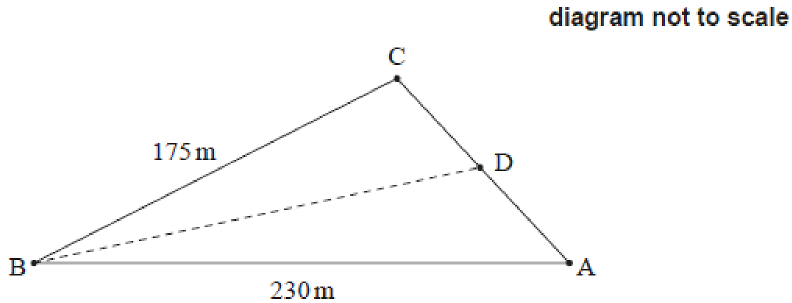


8a. Find the distance from point A to point C. *[4 marks]*

8b. Find the area of this piece of land. *[2 marks]*

8c. Find $\mathrm{C\hat{A}B}$. *[3 marks]*

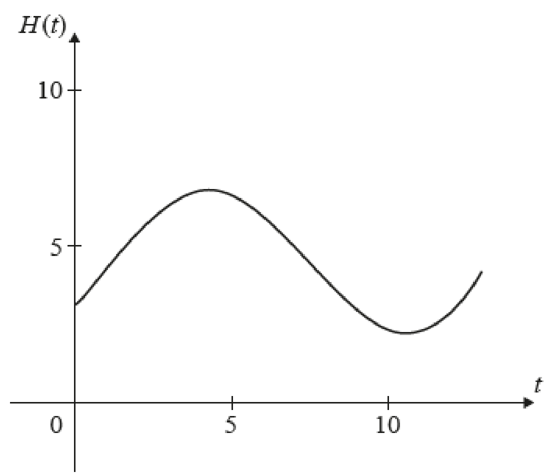
The farmer wants to divide the piece of land into two sections. He will put a post at point D, which is between A and C. He wants the boundary BD to divide the piece of land such that the sections have equal area. This is shown in the following diagram.



8d. Find the distance from point B to point D. *[5 marks]*

The height of water, in metres, in Dungeness harbour is modelled by the function $H(t) = a\sin(b(t-c)) + d$, where t is the number of hours after midnight, and $a,\ b,\ c$ and d are constants, where $a > 0,\ b > 0$ and $c > 0$.

The following graph shows the height of the water for 13 hours, starting at midnight.



The first high tide occurs at $04:30$ and the next high tide occurs 12 hours later. Throughout the day, the height of the water fluctuates between $2.2\ \mathrm{m}$ and $6.8\ \mathrm{m}$.

All heights are given correct to one decimal place.

9a. Show that $b = \frac{\pi}{6}$. *[1 mark]*

9b. Find the value of a. *[2 marks]*

9c. Find the value of d. *[2 marks]*

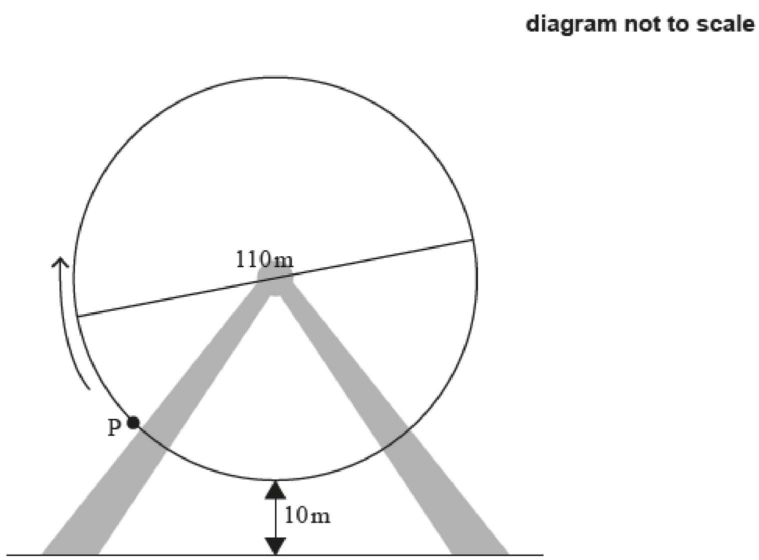
9d. Find the smallest possible value of c. *[3 marks]*

9e. Find the height of the water at $12:00$. *[2 marks]*

9f. Determine the number of hours, over a 24-hour period, for which the tide is higher than 5 metres. *[3 marks]*

10. Solve the equation $2\cos^2 x + 5\sin x = 4, 0 \le x \le 2\pi$. *[7 marks]*

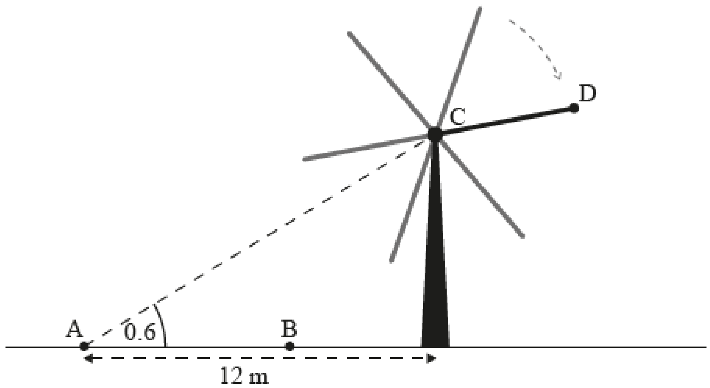
11. A Ferris wheel with diameter 110 metres rotates at a constant speed. The lowest point on the wheel is 10 metres above the ground, as shown on the following diagram. P is a point on the wheel. The wheel starts moving with P at the lowest point and completes one revolution in 20 minutes. *[5 marks]*



The height, h metres, of P above the ground after t minutes is given by $h(t) = a\cos(bt) + c$, where $a, b, c \in \mathbb{R}$.

Find the values of a, b and c.

The six blades of a windmill rotate around a centre point C. Points A and B and the base of the windmill are on level ground, as shown in the following diagram.



From point A the angle of elevation of point C is 0.6 radians.

12a. Given that point A is 12 metres from the base of the windmill, find the height of point C above the ground. *[2 marks]*

An observer walks 7 metres from point A to point B.

12b. Find the angle of elevation of point C from point B. *[2 marks]*

The observer keeps walking until he is standing directly under point C. The observer has a height of 1.8 metres, and as the blades of the windmill rotate, the end of each blade passes 2.5 metres over his head.

12c. Find the length of each blade of the windmill. *[2 marks]*

One of the blades is painted a different colour than the others. The end of this blade is labelled point D. The height h, in metres, of point D above the ground can be modelled by the function $h(t) = p\cos\left(\frac{3\pi}{10}t\right) + q$, where t is in seconds and $p,\ q \in \mathbb{R}$. When $t = 0$, point D is at its maximum height.

12d. Find the value of p and the value of q. *[4 marks]*

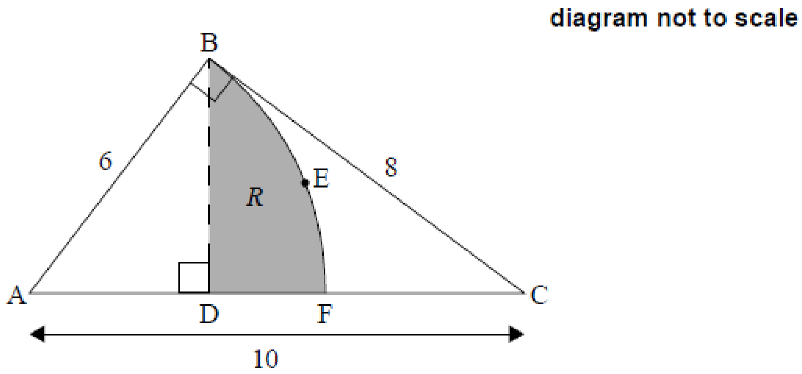
12e. If the observer stands directly under point C for one minute, point D will pass over his head n times. *[3 marks]*

Find the value of n.

13. Let $f(x) = 4\cos\left(\frac{x}{2}\right) + 1$, for $0 \leqslant x \leqslant 6\pi$. Find the values of x for which $f(x) > 2\sqrt{2} + 1$. *[8 marks]*

The following diagram shows a right-angled triangle, ABC, with $\mathrm{AC} = 10\,\mathrm{cm}$, $\mathrm{AB} = 6\,\mathrm{cm}$ and $\mathrm{BC} = 8\,\mathrm{cm}$.

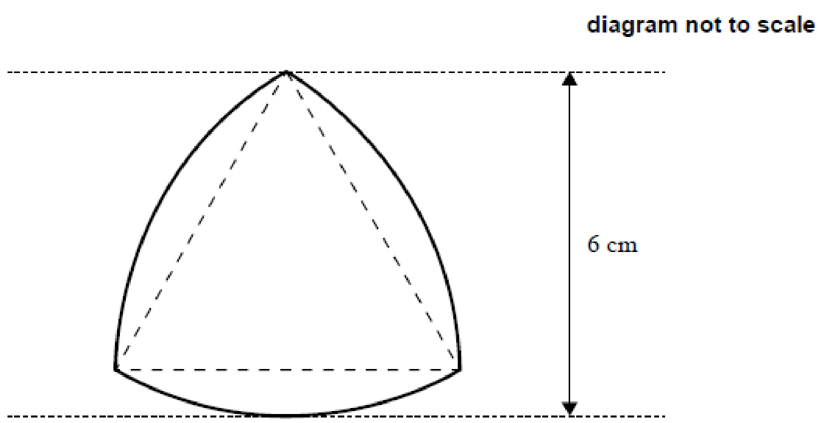
The points D and F lie on $[\mathrm{AC}]$.
$[\mathrm{BD}]$ is perpendicular to $[\mathrm{AC}]$.
BEF is the arc of a circle, centred at A.
The region R is bounded by $[\mathrm{BD}]$, $[\mathrm{DF}]$ and arc BEF.



14a. Find $\mathrm{B\hat{A}C}$. *[2 marks]*

14b. Find the area of R. *[5 marks]*

The following shape consists of three arcs of a circle, each with centre at the opposite vertex of an equilateral triangle as shown in the diagram.



For this shape, calculate

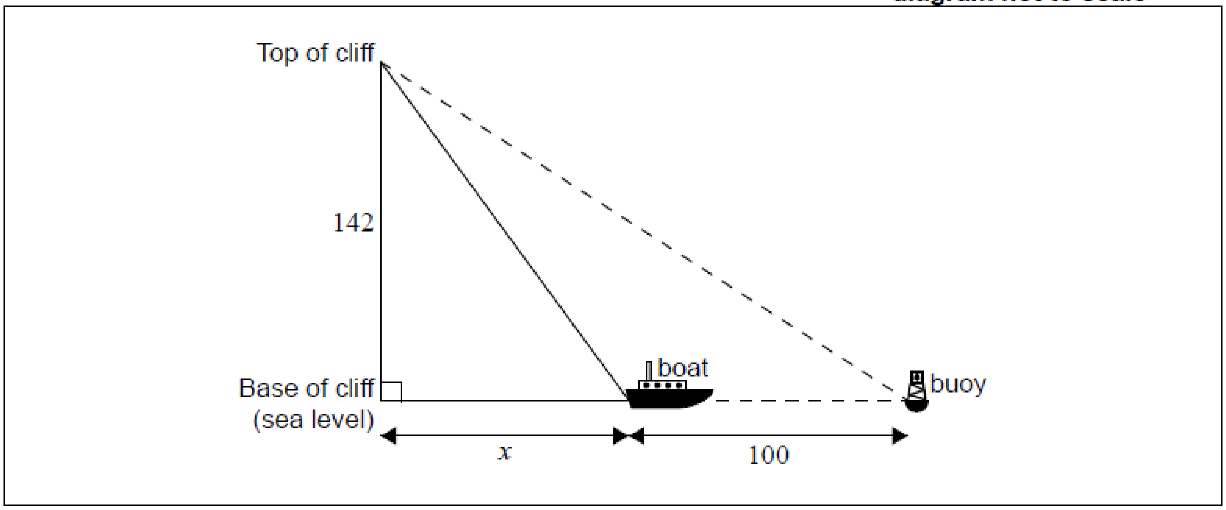
15a. the perimeter. *[2 marks]*

15b. the area. *[5 marks]*

A buoy is floating in the sea and can be seen from the top of a vertical cliff. A boat is travelling from the base of the cliff directly towards the buoy.

The top of the cliff is 142 m above sea level. Currently the boat is 100 metres from the buoy and the angle of depression from the top of the cliff to the boat is 64°.

diagram not to scale



16. Draw and label the angle of depression on the diagram. *[1 mark]*

The lengths of two of the sides in a triangle are 4 cm and 5 cm. Let θ be the angle between the two given sides. The triangle has an area of $\frac{5\sqrt{15}}{2}$ cm^2.

17a. Show that $\sin\theta = \frac{\sqrt{15}}{4}$. *[1 mark]*

17b. Find the two possible values for the length of the third side. *[6 marks]*

Let $f(x) = 2\sin(3x) + 4$ for $x \in \mathbb{R}$.

18a. The range of f is $k \le f(x) \le m$. Find k and m. *[3 marks]*

Let $g(x) = 5f(2x)$.

18b. Find the range of g. *[2 marks]*

The function g can be written in the form $g(x) = 10\sin(bx) + c$.

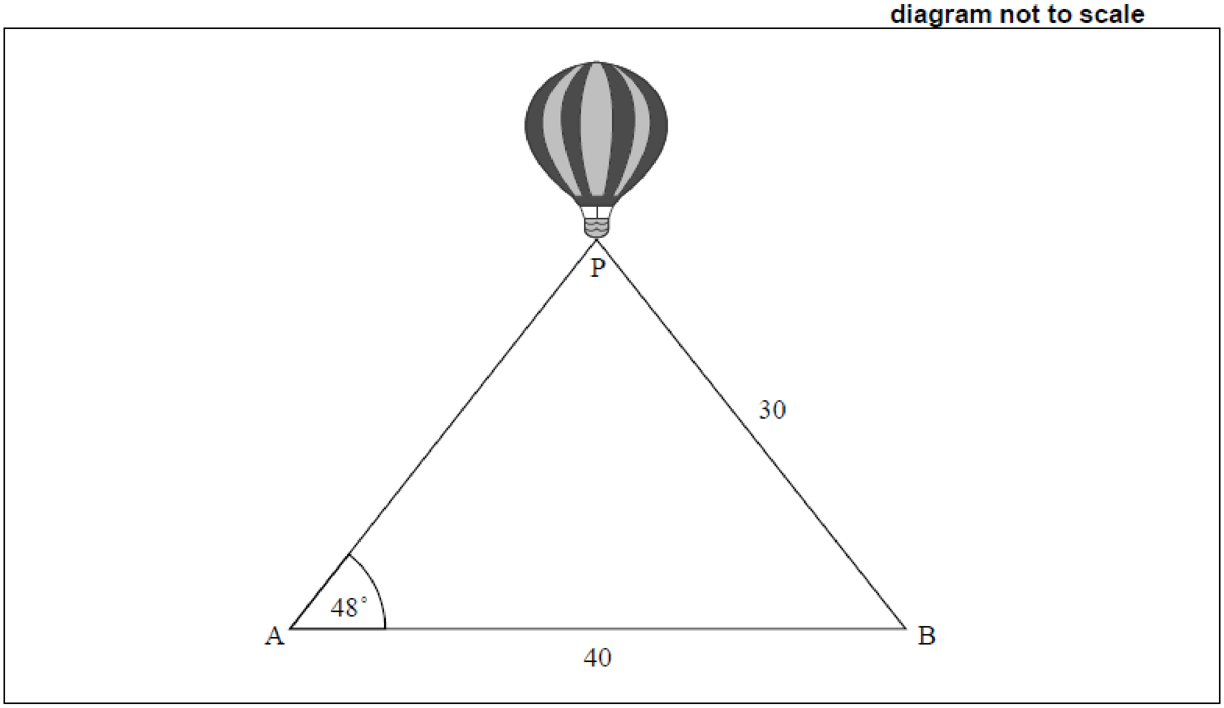
18c. Find the value of b and of c. *[3 marks]*

18d. Find the period of g. *[2 marks]*

18e. The equation $g\left(x\right) = 12$ has two solutions where $\pi \le x \le \frac{4\pi}{3}$. Find both solutions. *[3 marks]*

Two fixed points, A and B, are 40 m apart on horizontal ground. Two straight ropes, AP and BP, are attached to the same point, P, on the base of a hot air balloon which is vertically above the line AB. The length of BP is 30 m and angle BAP is 48°.

diagram not to scale



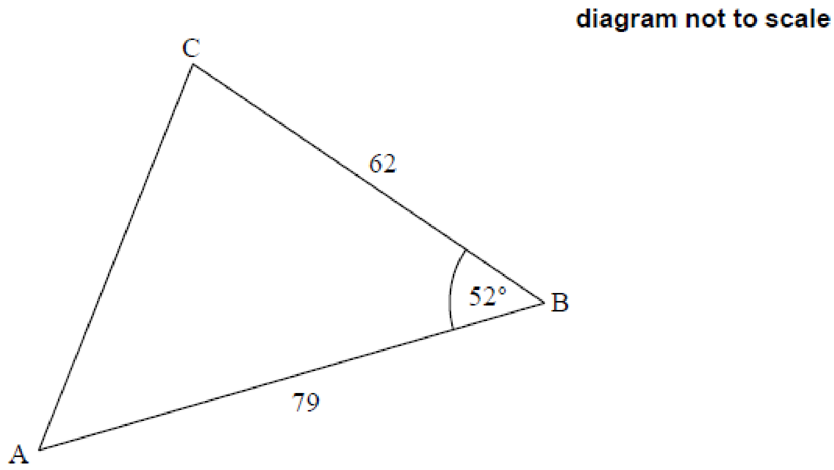
19a. On the diagram, draw and label with an *x* the angle of depression of B from P. *[1 mark]*

Angle APB is acute.

19b. Find the size of angle APB. *[3 marks]*

19c. Find the size of the angle of depression of B from P. *[2 marks]*

A park in the form of a triangle, ABC, is shown in the following diagram. AB is 79 km and BC is 62 km. Angle $A\hat{B}C$ is 52°.

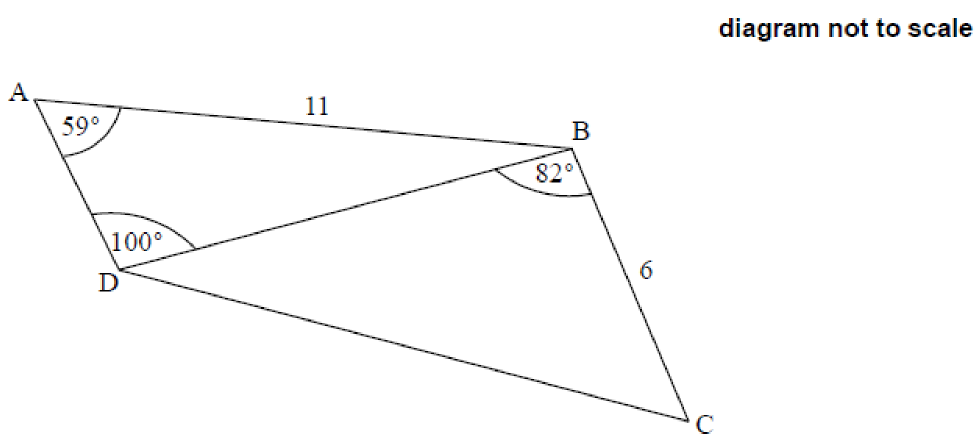


20a. Calculate the length of side AC in km. *[3 marks]*

20b. Calculate the area of the park. *[3 marks]*

21. Triangle ABC has *a* = 8.1 cm, *b* = 12.3 cm and area 15 cm^2. Find the largest possible perimeter of triangle ABC. *[7 marks]*

The following diagram shows quadrilateral ABCD.

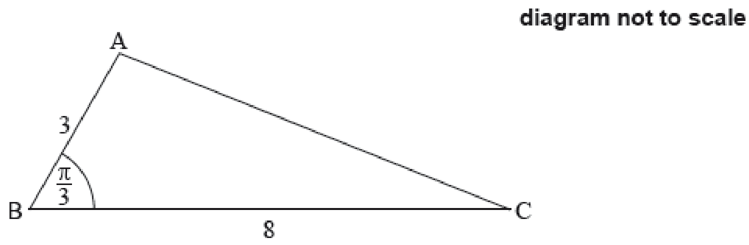


$$\mathrm{AB} = 11\,\mathrm{cm},\ \mathrm{BC} = 6\,\mathrm{cm},\ \mathrm{B}\hat{\mathrm{A}}\mathrm{D} = 100^\circ,\ \text{and}\ \mathrm{C}\hat{\mathrm{B}}\mathrm{D} = 82^\circ$$

22a. Find DB. *[3 marks]*

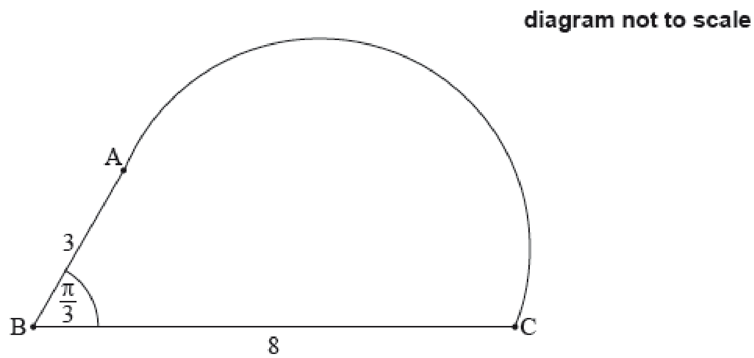
22b. Find DC. *[3 marks]*

The following diagram shows triangle ABC, with $\mathrm{AB} = 3\mathrm{cm}$, $\mathrm{BC} = 8\mathrm{cm}$, and $\mathrm{A\hat{B}C} = \frac{\pi}{3}$.



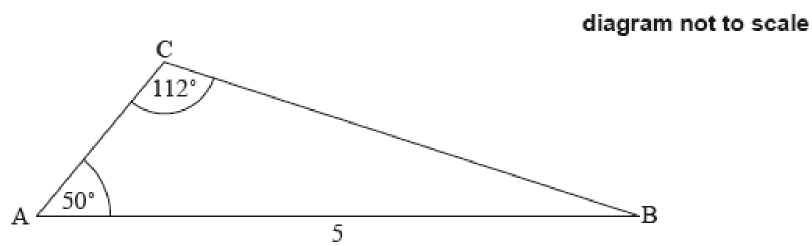
23a. Show that $\mathrm{AC} = 7\ \mathrm{cm}$. *[4 marks]*

23b. The shape in the following diagram is formed by adding a semicircle with diameter [AC] to the triangle. *[3 marks]*



Find the exact perimeter of this shape.

The following diagram shows a triangle ABC.

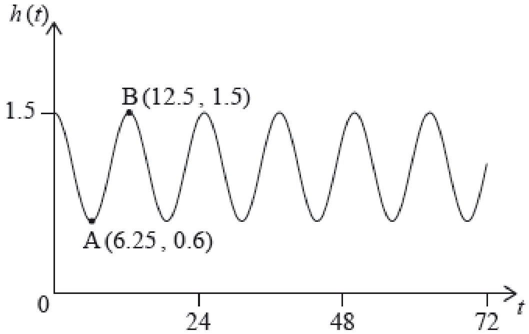


$\mathrm{AB} = 5\mathrm{cm}, \mathrm{C\hat{A}B} = 50°$ and $\mathrm{A\hat{C}B} = 112°$

24a. Find BC. *[3 marks]*

24b. Find the area of triangle ABC. *[3 marks]*

At Grande Anse Beach the height of the water in metres is modelled by the function $h(t) = p\cos(q \times t) + r$, where t is the number of hours after 21:00 hours on 10 December 2017. The following diagram shows the graph of h , for $0 \leqslant t \leqslant 72$.



The point $\mathrm{A}(6.25, 0.6)$ represents the first low tide and $\mathrm{B}(12.5, 1.5)$ represents the next high tide.

25a. How much time is there between the first low tide and the next high tide? *[2 marks]*

25b. Find the difference in height between low tide and high tide. *[2 marks]*

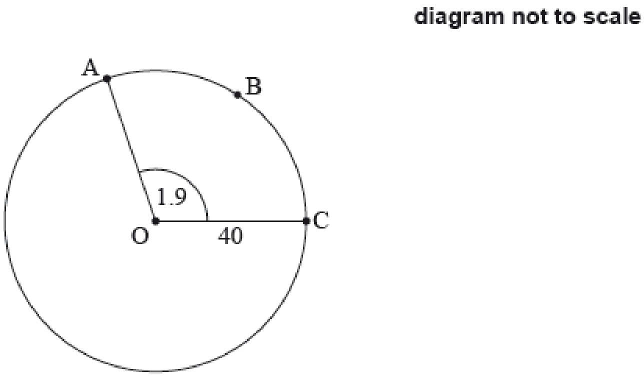
25c. Find the value of p; *[2 marks]*

25d. Find the value of q; *[3 marks]*

25e. Find the value of r. *[2 marks]*

25f. There are two high tides on 12 December 2017. At what time does the second high tide occur? *[3 marks]*

The following diagram shows a circle with centre O and radius 40 cm.



The points A, B and C are on the circumference of the circle and $\text{A}\hat{\text{O}}\text{C} = 1.9\text{radians}$.

26a. Find the length of arc ABC. *[2 marks]*

26b. Find the perimeter of sector OABC. *[2 marks]*

26c. Find the area of sector OABC. *[2 marks]*

The depth of water in a port is modelled by the function $d(t) = p\cos qt + 7.5$, for $0 \leqslant t \leqslant 12$, where t is the number of hours after high tide.

At high tide, the depth is 9.7 metres.

At low tide, which is 7 hours later, the depth is 5.3 metres.

27a. Find the value of p. *[2 marks]*

27b. Find the value of q. *[2 marks]*

27c. Use the model to find the depth of the water 10 hours after high tide. *[2 marks]*