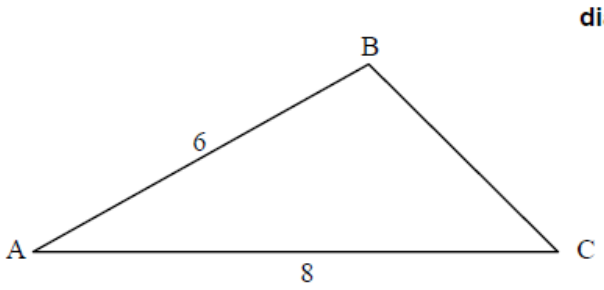
Sine and Cosine Rules *[134 marks]*

The following diagram shows triangle ABC, with $AB = 6$ and $AC = 8$.



1a. Given that $\cos \hat{A} = \frac{5}{6}$ find the value of $\sin \hat{A}$. *[3 marks]*

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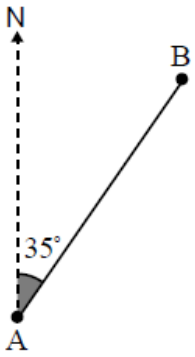
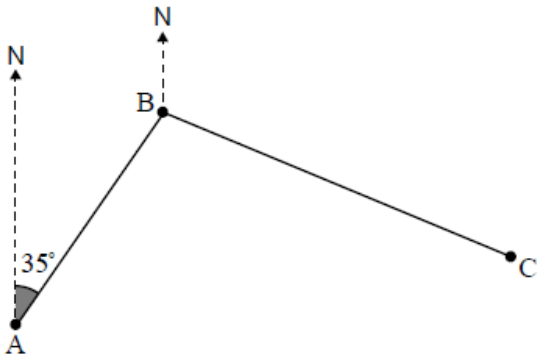
1b. Find the area of triangle ABC. *[2 marks]*

Adam sets out for a hike from his camp at point A. He hikes at an average speed of 4.2 km/h for 45 minutes, on a bearing of 035° from the camp, until he stops for a break at point B.

2a. Find the distance from point A to point B. *[2 marks]*



Adam leaves point B on a bearing of 114° and continues to hike for a distance of 4.6 km until he reaches point C.



2b. Show that $\hat{ABC}$ is 101°. *[2 marks]*

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2c. Find the distance from the camp to point C. *[3 marks]*

2d. Find $\mathrm{B}\hat{\mathrm{C}}\mathrm{A}$. *[3 marks]*

Adam’s friend Jacob wants to hike directly from the camp to meet Adam at point C .

2e. Find the bearing that Jacob must take to point C. *[3 marks]*

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2f. Jacob hikes at an average speed of 3.9 km/h. *[3 marks]*

Find, to the nearest minute, the time it takes for Jacob to reach point C.

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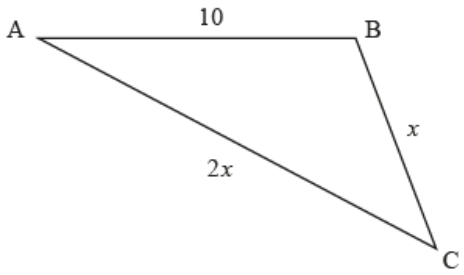
3. Consider a triangle ABC, where $\mathrm{AC} = 12,\ \mathrm{CB} = 7$ and $\mathrm{B\widehat{A}C} = 25^\circ$. *[5 marks]*

Find the smallest possible perimeter of triangle ABC.

4. The following diagram shows triangle ABC, with $\mathrm{AB} = 10$, $\mathrm{BC} = x$ and $\mathrm{AC} = 2x$. *[7 marks]*

diagram not to scale

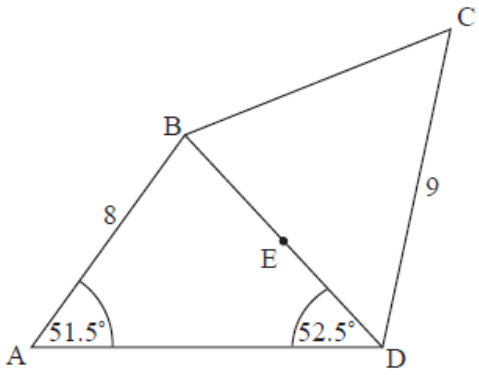


Given that $\cos \widehat{\mathrm{C}} = \frac{3}{4}$, find the area of the triangle.

Give your answer in the form $\frac{p\sqrt{q}}{2}$ where $p, q \in \mathbb{Z}^+$.

Using geometry software, Pedro draws a quadrilateral ABCD. $\mathrm{AB} = 8\ \mathrm{cm}$ and $\mathrm{CD} = 9\ \mathrm{cm}$. Angle $\mathrm{BAD} = 51.5^\circ$ and angle $\mathrm{ADB} = 52.5^\circ$. This information is shown in the diagram.

diagram not to scale



5a. Calculate the length of BD. *[3 marks]*

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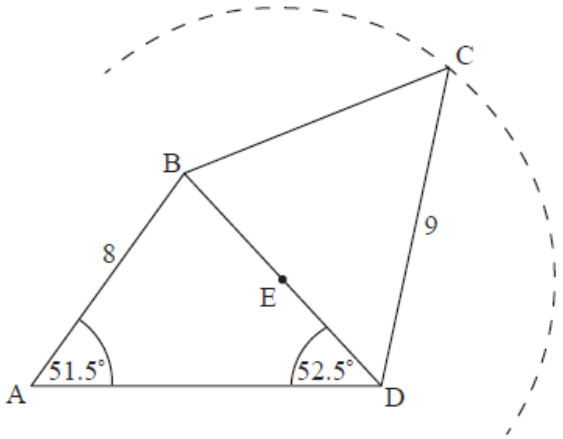
$\mathrm{CE} = 7\ \mathrm{cm}$, where point E is the midpoint of BD.

5b. Show that angle $\mathrm{EDC} = 48.0^{\circ}$, correct to three significant figures. *[4 marks]*

5c. Calculate the area of triangle BDC. *[3 marks]*

5d. Pedro draws a circle, with centre at point E, passing through point C. *[5 marks]*
Part of the circle is shown in the diagram.

diagram not to scale



Show that point A lies outside this circle. Justify your reasoning.

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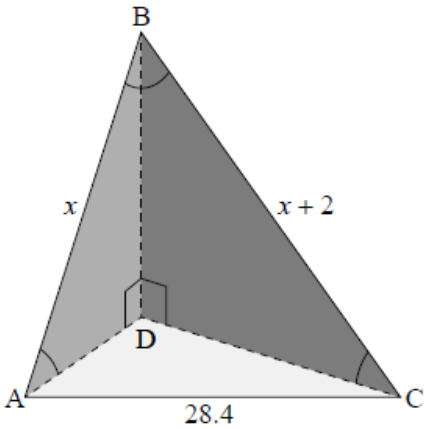
6. The diagram below shows a triangular-based pyramid with base ADC. *[6 marks]*
Edge BD is perpendicular to the edges AD and CD.

diagram not to scale



$\mathrm{AC} = 28.4\,\mathrm{cm},\ \mathrm{AB} = x\,\mathrm{cm},\ \mathrm{BC} = x + 2\,\mathrm{cm},\ \mathrm{A\widehat{B}C} = 0.667,\ \mathrm{B\widehat{A}D} = 0.611$

Calculate AD

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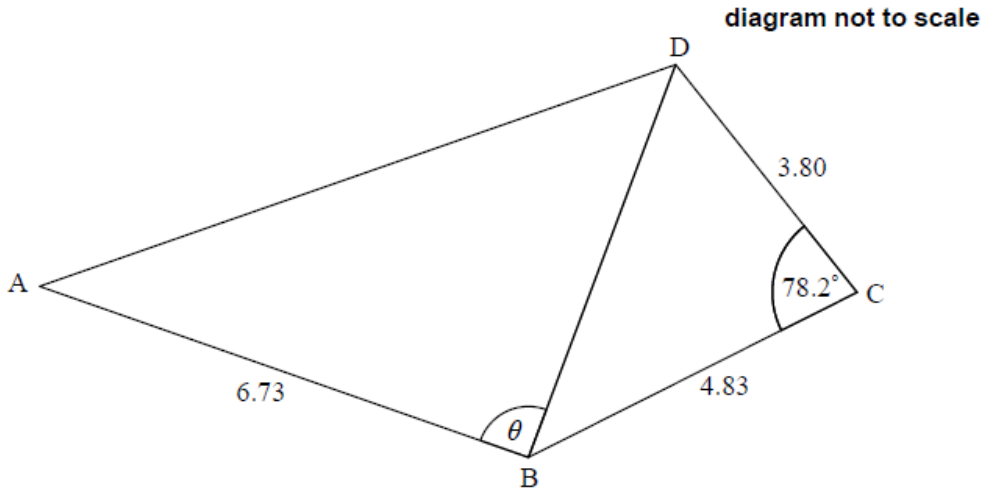
The following diagram shows the quadrilateral ABCD.



AB = 6.73 cm, BC = 4.83 cm, $B\hat{C}D = 78.2°$ and CD = 3.80 cm.

7a. Find BD. *[3 marks]*

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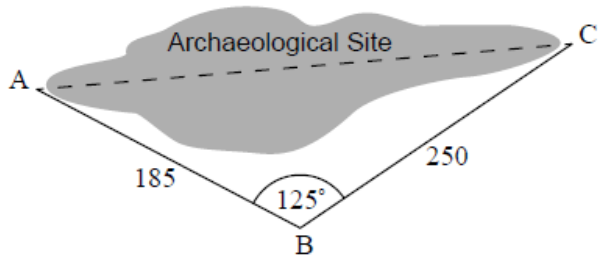
7b. The area of triangle ABD is 18.5 cm^2. Find the possible values of θ. *[4 marks]*

An archaeological site is to be made accessible for viewing by the public. To do this, archaeologists built two straight paths from point A to point B and from point B to point C as shown in the following diagram. The length of path AB is 185 m, the length of path BC is 250 m, and angle $\mathrm{A\hat{B}C}$ is 125°.

diagram not to scale



8a. Find the distance from A to C. *[3 marks]*

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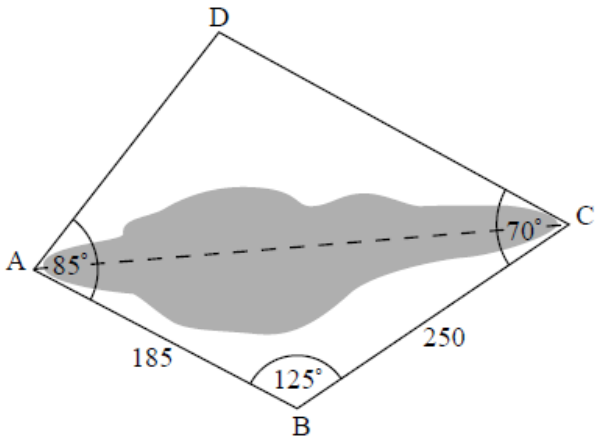
The archaeologists plan to build two more straight paths, AD and DC. For the paths to go around the site, angle $\mathrm{B\hat{A}D}$ is to be made equal to 85° and angle $\mathrm{B\hat{C}D}$ is to be made equal to 70° as shown in the following diagram.

diagram not to scale



8b. *[3 marks]*

Find the size of angle $\mathrm{B\hat{A}C}$.

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8c. *[1 mark]*

Find the size of angle $\mathrm{C\hat{A}D}$.

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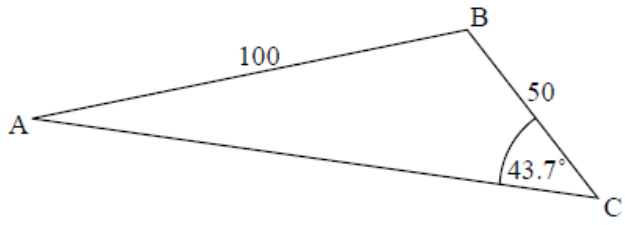
8d. Find the size of angle $\hat{A}CD$. *[2 marks]*

8e. The length of path AD is 287 m. *[4 marks]*

Find the area of the region ABCD.

A flat horizontal area, ABC, is such that AB = 100 m , BC = 50 m and angle AĈB = 43.7° as shown in the diagram.

diagram not to scale



9a. Show that the size of angle BÂC is 20.2°, correct to 3 significant figures. *[3 marks]*

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9b. Calculate the area of triangle ABC. *[4 marks]*

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9c. Find the length of AC. *[3 marks]*

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9d. A vertical pole, TB, is constructed at point B and has height 25 m. *[5 marks]*

Calculate the angle of elevation of T from, M, the midpoint of the side AC.

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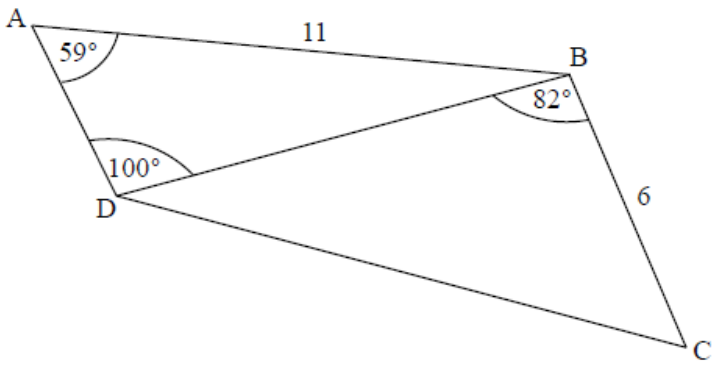
The following diagram shows quadrilateral ABCD.

diagram not to scale



$\mathrm{AB} = 11\,\mathrm{cm}$, $\mathrm{BC} = 6\,\mathrm{cm}$, $\mathrm{B\hat{A}D} = 100°$, and $\mathrm{C\hat{B}D} = 82°$

10a. Find DB. *[3 marks]*

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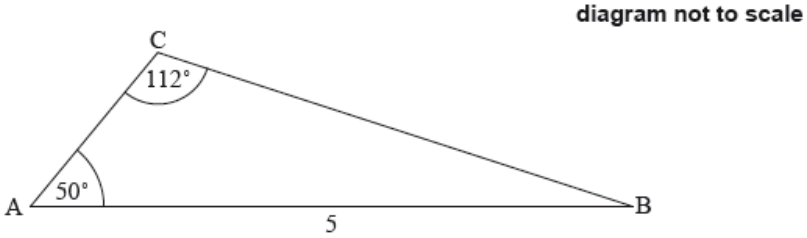
10b. Find DC. *[3 marks]*

The following diagram shows a triangle ABC.



$\mathrm{AB} = 5\mathrm{cm}, \mathrm{C\hat{A}B} = 50°$ and $\mathrm{A\hat{C}B} = 112°$

11a. Find BC. *[3 marks]*

11b. Find the area of triangle ABC. *[3 marks]*

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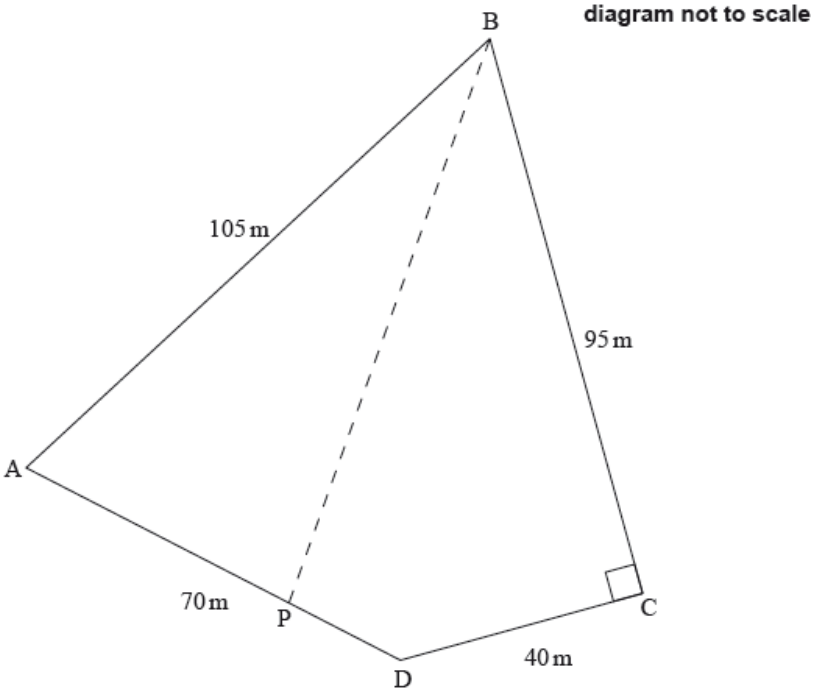
A farmer owns a plot of land in the shape of a quadrilateral ABCD.

$\mathrm{AB} = 105\mathrm{m}$, $\mathrm{BC} = 95\mathrm{m}$, $\mathrm{CD} = 40\mathrm{m}$, $\mathrm{DA} = 70\mathrm{m}$ and angle $\mathrm{DCB} = 90°$.



The farmer wants to divide the land into two equal areas. He builds a fence in a straight line from point B to point P on AD, so that the area of PAB is equal to the area of PBCD.

Calculate

12a. the length of BD; *[2 marks]*

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12b. the size of angle DAB; *[3 marks]*

12c. the area of triangle ABD; *[3 marks]*

12d. the area of quadrilateral ABCD; *[2 marks]*

12e. the length of AP; *[3 marks]*

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12f. the length of the fence, BP. *[3 marks]*

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13. In triangle ABC, AB = 5, BC = 14 and AC = 11. *[5 marks]*

Find all the interior angles of the triangle. Give your answers in degrees to one decimal place.

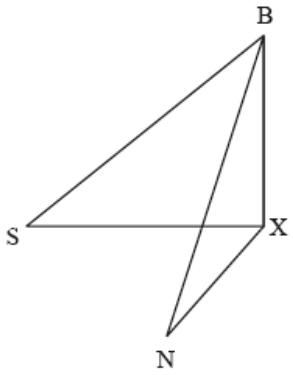
14. Barry is at the top of a cliff, standing 80 m above sea level, and observes *[6 marks]* two yachts in the sea.

"*Seaview*" (S) is at an angle of depression of 25°.

"*Nauti Buoy*" (N) is at an angle of depression of 35°.

The following three dimensional diagram shows Barry and the two yachts at S and N.

X lies at the foot of the cliff and angle $\mathrm{SXN} = 70°$.



Find, to 3 significant figures, the distance between the two yachts.

15a. Find the set of values of k that satisfy the inequality $k^2 - k - 12 < 0$. *[2 marks]*

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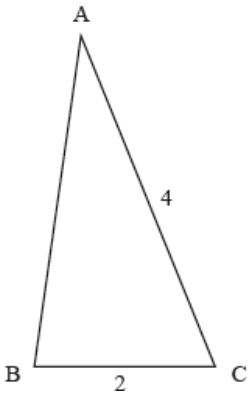
15b. The triangle ABC is shown in the following diagram. Given that $\cos B < \frac{1}{4}$, find the range of possible values for AB. *[4 marks]*



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