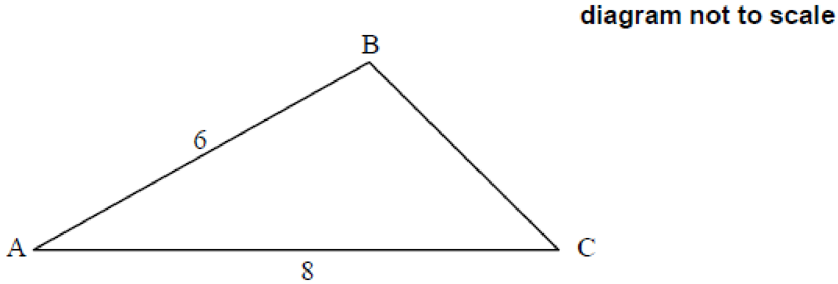
Sine and Cosine Rules *[134 marks]*

The following diagram shows triangle ABC, with $AB = 6$ and $AC = 8$.



1a. Given that $\cos \hat{A} = \frac{5}{6}$ find the value of $\sin \hat{A}$. *[3 marks]*

Markscheme

valid approach using Pythagorean identity ***(M1)***

$\sin^2 A + \left(\frac{5}{6}\right)^2 = 1$ (or equivalent) ***(A1)***

$\sin A = \frac{\sqrt{11}}{6}$ ***A1***

[3 marks]

1b. Find the area of triangle ABC. *[2 marks]*

Markscheme

$\frac{1}{2} \times 8 \times 6 \times \frac{\sqrt{11}}{6}$ (or equivalent) ***(A1)***

area $= 4\sqrt{11}$ ***A1***

[2 marks]

Adam sets out for a hike from his camp at point A. He hikes at an average speed of 4.2 km/h for 45 minutes, on a bearing of 035° from the camp, until he stops for a break at point B.

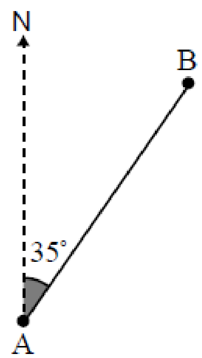
2a. Find the distance from point A to point B. *[2 marks]*

Markscheme

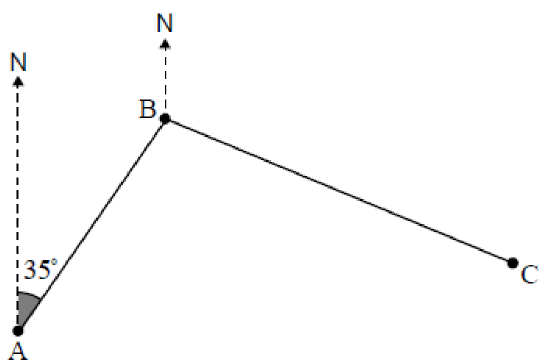
$\frac{4.2}{60} \times 45$ ***A1***

AB = 3.15 (km) ***A1***

[2 marks]



Adam leaves point B on a bearing of 114° and continues to hike for a distance of 4.6 km until he reaches point C.



2b. Show that $A\hat{B}C$ is 101°. *[2 marks]*

Markscheme

66° or (180 − 114) ***A1***

35 + 66 ***A1***

$A\hat{B}C$ = 101° ***AG***

[2 marks]

2c. Find the distance from the camp to point C. *[3 marks]*

Markscheme

attempt to use cosine rule ***(M1)***

$AC^2 = 3.15^2 + 4.6^2 - 2 \times 3.15 \times 4.6 \cos 101°$ (or equivalent) ***A1***

AC = 6.05 (km) ***A1***

[3 marks]

2d. Find $B\hat{C}A$. *[3 marks]*

Markscheme

valid approach to find angle BCA ***(M1)***

eg sine rule

correct substitution into sine rule ***A1***

eg $\frac{\sin\left(B\hat{C}A\right)}{3.15} = \frac{\sin 101}{6.0507\ldots}$

$B\hat{C}A$ = 30.7° ***A1***

[3 marks]

Adam’s friend Jacob wants to hike directly from the camp to meet Adam at point C .

2e. Find the bearing that Jacob must take to point C. *[3 marks]*

Markscheme

$\mathrm{B}\overset{\wedge}{\mathrm{A}}\mathrm{C}$ = 48.267 (seen anywhere) ***A1***

valid approach to find correct bearing ***(M1)***

eg 48.267 + 35

bearing = 83.3° (accept 083°) ***A1***

[3 marks]

2f. Jacob hikes at an average speed of 3.9 km/h. *[3 marks]*

Find, to the nearest minute, the time it takes for Jacob to reach point C.

Markscheme

attempt to use $\text{time} = \frac{\text{distance}}{\text{speed}}$ ***M1***

$\frac{6.0507}{3.9}$ or 0.065768 km/min ***(A1)***

t = 93 (minutes) ***A1***

[3 marks]

3. Consider a triangle ABC, where $\mathrm{AC} = 12,\ \mathrm{CB} = 7$ and $\mathrm{B}\widehat{\mathrm{A}}\mathrm{C} = 25^\circ$. *[5 marks]*

Find the smallest possible perimeter of triangle ABC.

Markscheme

EITHER

attempt to use cosine rule **_(M1)_**

$12^2 + \mathrm{AB}^2 - 2 \times 12 \times \cos 25^\circ \times \mathrm{AB} = 7^2$ OR
$\mathrm{AB}^2 - 21.7513\ldots \mathrm{AB} + 95 = 0$ **_(A1)_**

at least one correct value for AB **_(A1)_**

$\mathrm{AB} = 6.05068\ldots$ OR $\mathrm{AB} = 15.7007\ldots$

using their smaller value for AB to find minimum perimeter **_(M1)_**

$12 + 7 + 6.05068\ldots$

OR

attempt to use sine rule **_(M1)_**

$\frac{\sin \mathrm{B}}{12} = \frac{\sin 25^\circ}{7}$ OR $\sin \mathrm{B} = 0.724488\ldots$ OR $\widehat{\mathrm{B}} = 133.573\ldots^\circ$ OR
$\widehat{\mathrm{B}} = 46.4263\ldots^\circ$ **_(A1)_**

at least one correct value for C **_(A1)_**

$\widehat{C} = 21.4263\ldots^\circ$ OR $\widehat{C} = 108.573\ldots^\circ$

using their acute value for $\widehat{C}$ to find minimum perimeter **_(M1)_**

$12 + 7 + \sqrt{12^2 + 7^2 - 2 \times 12 \times 7 \cos 21.4263\ldots^\circ}$ OR
$12 + 7 + \frac{7 \sin 21.4263\ldots^\circ}{\sin 25^\circ}$

THEN

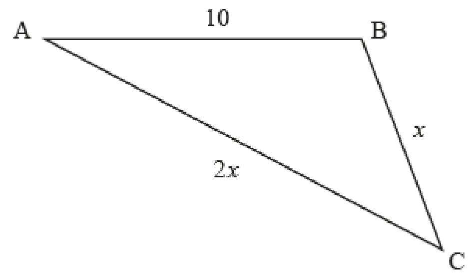
$25.0506\ldots$

minimum perimeter $= 25.1$. **_A1_**

[5 marks]

4. The following diagram shows triangle ABC, with $\mathrm{AB} = 10$, $\mathrm{BC} = x$ and $\mathrm{AC} = 2x$. *[7 marks]*

diagram not to scale



Given that $\cos \widehat{\mathrm{C}} = \frac{3}{4}$, find the area of the triangle.

Give your answer in the form $\frac{p\sqrt{q}}{2}$ where $p, q \in \mathbb{Z}^+$.

Markscheme

METHOD 1

attempt to use the cosine rule to find the value of x ***(M1)***

$100 = x^2 + 4x^2 - 2(x)(2x)\left(\frac{3}{4}\right)$ ***A1***

$2x^2 = 100$

$x^2 = 50$ OR $x = \sqrt{50}\left(= 5\sqrt{2}\right)$ ***A1***

attempt to find $\sin \widehat{C}$ (seen anywhere) ***(M1)***

$\sin^2 \widehat{C} + \left(\frac{3}{4}\right)^2 = 1$ OR $x^2 + 3^2 = 4^2$ or right triangle with side 3 and hypotenuse 4

$\sin \widehat{C} = \frac{\sqrt{7}}{4}$ ***(A1)***

Note: The marks for finding $\sin \widehat{C}$ may be awarded independently of the first three marks for finding x.

correct substitution into the area formula using their value of x (or x^2) and their value of $\sin \widehat{C}$ ***(M1)***

$A = \frac{1}{2} \times 5\sqrt{2} \times 10\sqrt{2} \times \frac{\sqrt{7}}{4}$ or $A = \frac{1}{2} \times \sqrt{50} \times 2\sqrt{50} \times \frac{\sqrt{7}}{4}$

$A = \frac{25\sqrt{7}}{2}$ ***A1***

METHOD 2

attempt to find the height, h, of the triangle in terms of x ***(M1)***

$h^2 + \left(\frac{3}{4}x\right)^2 = x^2$ OR $h^2 + \left(\frac{5}{4}x\right)^2 = 10^2$ OR $h = \frac{\sqrt{7}}{4}x$ ***A1***

equating their expressions for either h^2 or h ***(M1)***

$x^2 - \left(\frac{3}{4}x\right)^2 = 10^2 - \left(\frac{5}{4}x\right)^2$ OR $\sqrt{100 - \frac{25}{16}x^2} = \frac{\sqrt{7}}{4}x$ (or equivalent) ***A1***

$x^2 = 50$ OR $x = \sqrt{50}\left(= 5\sqrt{2}\right)$ ***A1***

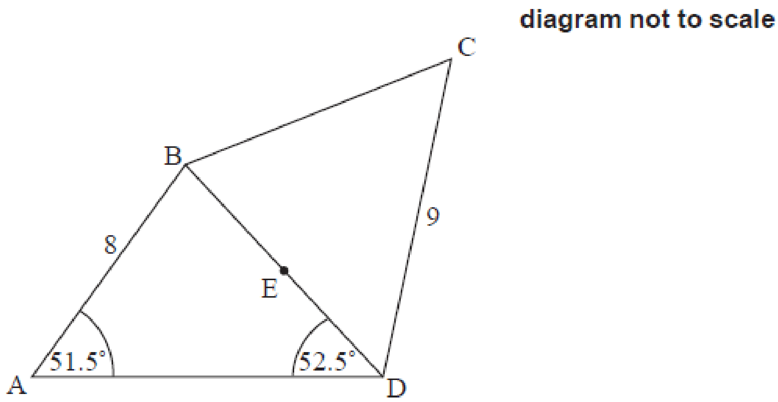
correct substitution into the area formula using their value of x (or x^2) ***(M1)***

$A = \frac{1}{2} \times 2\sqrt{50} \times \frac{\sqrt{7}}{4}\sqrt{50}$ OR $A = \frac{1}{2}\left(2 \times 5\sqrt{2}\right)\left(\frac{\sqrt{7}}{4}5\sqrt{2}\right)$

$A = \frac{25\sqrt{7}}{2}$ ***A1***

[7 marks]

Using geometry software, Pedro draws a quadrilateral ABCD. $\mathrm{AB} = 8\ \mathrm{cm}$ and $\mathrm{CD} = 9\ \mathrm{cm}$. Angle $\mathrm{BAD} = 51.5^\circ$ and angle $\mathrm{ADB} = 52.5^\circ$. This information is shown in the diagram.



5a. Calculate the length of BD. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\frac{\mathrm{BD}}{\sin 51.5^\circ} = \frac{8}{\sin 52.5^\circ}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted sine rule, ***(A1)*** for correct substitution.

$(\mathrm{BD} =)\ 7.89\ (\mathrm{cm})\ \ (7.89164\ldots)$ ***(A1)(G2)***

Note: If radians are used the answer is $9.58723\ldots$ award at most ***(M1)(A1)(A0)***.

[3 marks]

$\mathrm{CE} = 7\ \mathrm{cm}$, where point E is the midpoint of BD.

5b. Show that angle $\mathrm{EDC} = 48.0^\circ$, correct to three significant figures. *[4 marks]*

Markscheme

$\cos \mathrm{EDC} = \frac{9^2+3.94582\ldots^2-7^2}{2\times 9\times 3.94582\ldots}$ ***(A1)*****(ft)*****(M1)(A1)*****(ft)**

Note: Award ***(A1)*** for $3.94582\ldots$ or $\frac{7.89164\ldots}{2}$ seen, ***(M1)*** for substituted cosine rule, ***(A1)*****(ft)** for correct substitutions.

$(\mathrm{EDC} =)\ 47.9515\ldots^\circ$ ***(A1)***

48.0° (3 sig figures) ***(AG)***

Note: Both an unrounded answer that rounds to the given answer and the rounded value must be seen for the final ***(M1)*** to be awarded.
Award at most ***(A1)*****(ft)*****(M1)(A1)*****(ft)*****(A0)*** if the known angle 48.0° is used to validate the result. Follow through from their BD in part (a).

[4 marks]

5c. Calculate the area of triangle BDC. *[3 marks]*

Markscheme

Units are required in this question.

$(\text{area} =)\ \frac{1}{2} \times 7.89164\ldots \times 9 \times \sin 48.0^\circ$ ***(M1)(A1)*****(ft)**

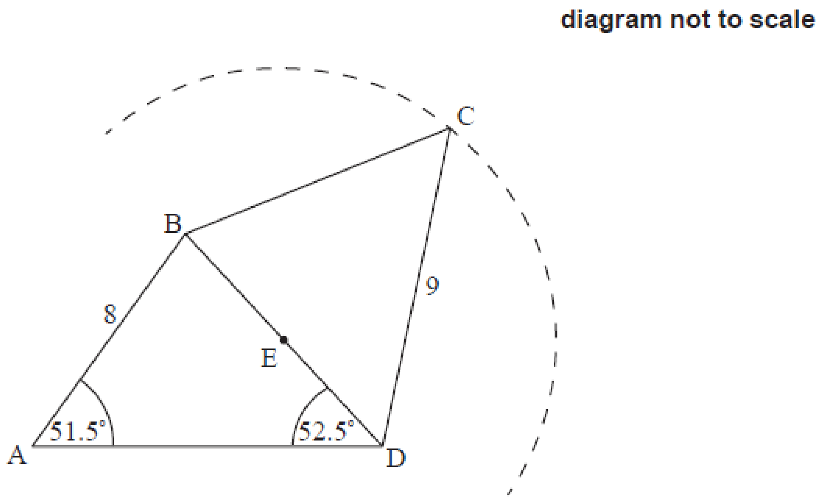
Note: Award ***(M1)*** for substituted area formula. Award ***(A1)*** for correct substitution.

$(\text{area} =)\ 26.4\ \mathrm{cm}^2\ \ (26.3908\ldots)$ ***(A1)(ft)(G3)***

Note: Follow through from part (a).

[3 marks]

5d. Pedro draws a circle, with centre at point E, passing through point C. *[5 marks]*
Part of the circle is shown in the diagram.



Show that point A lies outside this circle. Justify your reasoning.

Markscheme

$\mathrm{AE}^2 = 8^2 + (3.94582\ldots)^2 - 2 \times 8 \times 3.94582\ldots \cos(76^\circ)$ ***(A1)(M1)(A1)*(ft)**

Note: Award ***(A1)*** for 76° seen. Award ***(M1)*** for substituted cosine rule to find AE, ***(A1)*(ft)** for correct substitutions.

$(\mathrm{AE} =)\ 8.02\,(\mathrm{cm})\ \ (8.01849\ldots)$ ***(A1)(ft)(G3)***

Note: Follow through from part (a).

OR

$\mathrm{AE}^2 = 9.78424\ldots^2 + (3.94582\ldots)^2 - 2 \times 9.78424\ldots \times 3.94582\ldots \cos(52.5^\circ$ ***(A1)(M1)(A1)*(ft)**

Note: Award ***(A1)*** for $\mathrm{AD}\ (9.78424\ldots)$ or 76° seen. Award ***(M1)*** for substituted cosine rule to find AE (do not award ***(M1)*** for cosine or sine rule to find AD), ***(A1)*(ft)** for correct substitutions.

$(\mathrm{AE} =)\ 8.02\,(\mathrm{cm})\ \ (8.01849\ldots)$ ***(A1)(ft)(G3)***

Note: Follow through from part (a).

$8.02 > 7.$ ***(A1)(ft)***

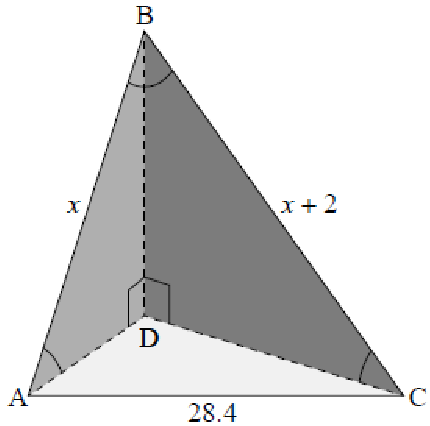
point A is outside the circle. ***(AG)***

Note: Award ***(A1)*** for a numerical comparison of AE and CE. Follow through for the final ***(A1)(ft)*** within the part for their 8.02. The final ***(A1)(ft)*** is contingent on a valid method to find the value of AE.
Do not award the final ***(A1)(ft)*** if the ***(AG)*** line is not stated.
Do not award the final ***(A1)(ft)*** if their point A is inside the circle.

[5 marks]

6. The diagram below shows a triangular-based pyramid with base ADC. *[6 marks]*
Edge BD is perpendicular to the edges AD and CD.

diagram not to scale



$\mathrm{AC} = 28.4\,\mathrm{cm},\ \mathrm{AB} = x\,\mathrm{cm},\ \mathrm{BC} = x + 2\,\mathrm{cm},\ \mathrm{A\hat{B}C} = 0.667,\ \mathrm{B\hat{A}D} = 0.611$

Calculate AD

Markscheme

evidence of choosing cosine rule ***(M1)***

eg $a^2 = b^2 + c^2 - 2bc\cos A$

correct substitution to find AB ***(A1)***

eg $28.4^2 = x^2 + (x+2)^2 - 2x(x+2)\cos(0.667)$

$x = 42.2822$ ***A2***

appropriate approach to find AD ***(M1)***

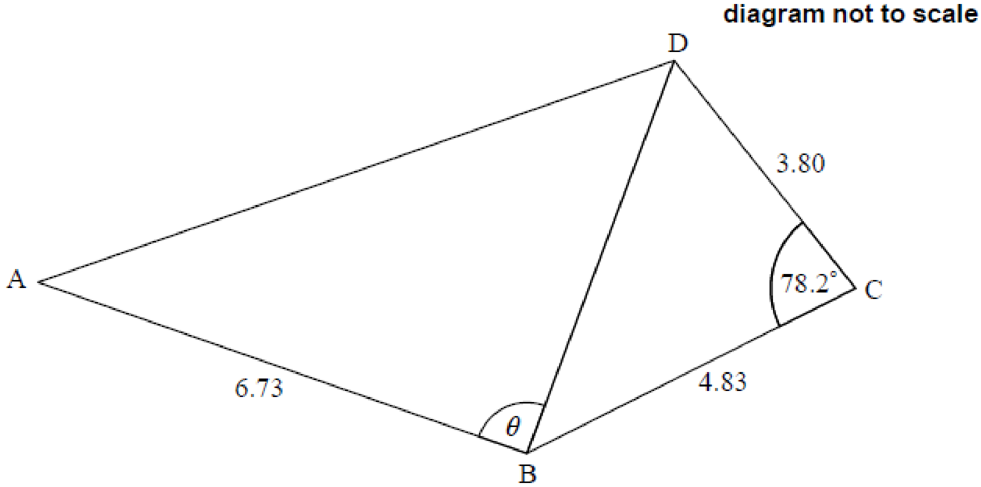
eg $\mathrm{AD} = x\cos(0.611),\ \cos(0.611) = \frac{\mathrm{AD}}{42.2822}$

34.6322

$\mathrm{AD} = 34.6$ ***A1 N3***

[6 marks]

The following diagram shows the quadrilateral ABCD.



AB = 6.73 cm, BC = 4.83 cm, $B\hat{C}D$ = 78.2° and CD = 3.80 cm.

7a. Find BD. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

choosing cosine rule ***(M1)***

eg $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution into RHS ***(A1)***

eg $4.83^2 + 3.80^2 - 2 \times 4.83 \times 3.80 \times \cos 78.2$, 30.2622,

$4.83^2 + 3.80^2 - 2\,(4.83)\,(3.80) \times \cos 1.36$

5.50111

5.50 (cm) ***A1 N2***

[3 marks]

7b. The area of triangle ABD is 18.5 cm^2. Find the possible values of θ. *[4 marks]*

Markscheme

correct substitution for area of triangle ABD ***(A1)***

eg $\frac{1}{2} \times 6.73 \times 5.50111 \sin\theta$

correct equation ***A1***

eg $\frac{1}{2} \times 6.73 \times 5.50111 \sin\theta = 18.5$, $\sin\theta = 0.999393$

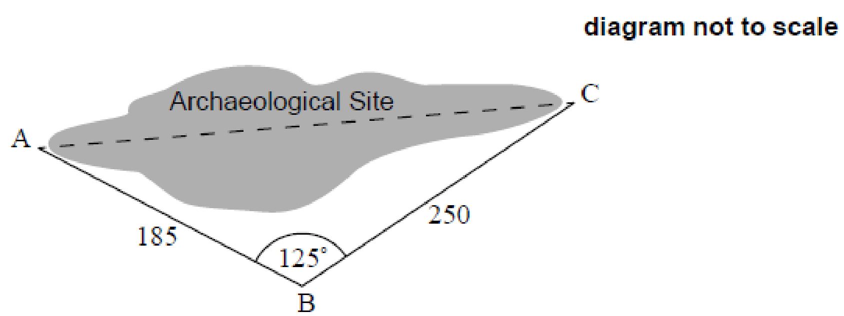
88.0023, 91.9976, 1.53593, 1.60566

θ = 88.0 (degrees) or 1.54 (radians)

θ = 92.0 (degrees) or 1.61 (radians) ***A1A1 N2***

[4 marks]

An archaeological site is to be made accessible for viewing by the public. To do this, archaeologists built two straight paths from point A to point B and from point B to point C as shown in the following diagram. The length of path AB is 185 m, the length of path BC is 250 m, and angle $\mathrm{A}\hat{\mathrm{B}}\mathrm{C}$ is 125°.



8a. Find the distance from A to C. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$AC^2 = 185^2 + 250^2 - 2 \times 185 \times 250 \times \cos(125°)$ ***(M1)(A1)***

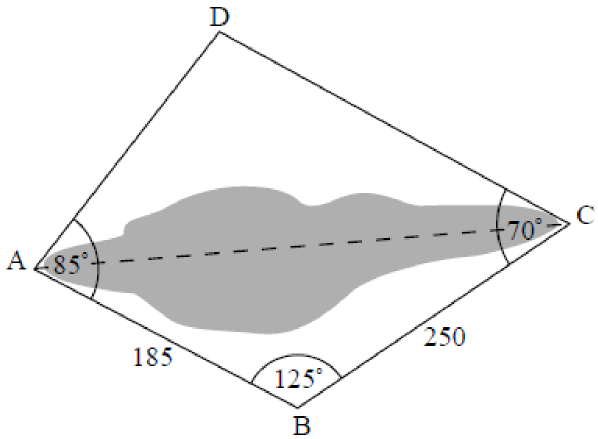
Note: Award ***(M1)*** for substitution in the cosine formula; ***(A1)*** for correct substitution.

387 (387.015…) (m) ***(A1)(G2)***

Note: If radians are used the answer is 154 (154.471…), award at most ***(M1)(A1)(A0)***.

[3 marks]

The archaeologists plan to build two more straight paths, AD and DC. For the paths to go around the site, angle $\mathrm{B\hat{A}D}$ is to be made equal to 85° and angle $\mathrm{B\hat{C}D}$ is to be made equal to 70° as shown in the following diagram.



8b. Find the size of angle $\mathrm{B\hat{A}C}$. *[3 marks]*

Markscheme

$\frac{250}{\sin \mathrm{B\hat{A}C}} = \frac{387.015\ldots}{\sin(125°)}$ ***(M1)(A1)*(ft)**

OR

$\cos^{-1}\left(\frac{185^2+387.015\ldots^2-250^2}{2\times185\times387.015\ldots}\right)$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substitution in the sine or cosine formulas; ***(A1)*(ft)** for correct substitution.

$\mathrm{B\hat{A}C} = 31.9°$ (31.9478...°) ***(A1)*(ft)*(G2)***

Note: Follow through from part (a).

[3 marks]

8c. Find the size of angle $\mathrm{C\hat{A}D}$. *[1 mark]*

Markscheme

(CAD =) 53.1° (53.0521...°) ***(A1)*****(ft)**

Note: Follow through from their part (b)(i) only if working seen.

[1 mark]

8d. Find the size of angle $\mathrm{A}\hat{\mathrm{C}}\mathrm{D}$. *[2 marks]*

Markscheme

(ACD =) 70° − (180° − 125° − 31.9478°...) ***(M1)***

Note: Award ***(M1)*** for subtracting their angle $\mathrm{A}\hat{\mathrm{C}}\mathrm{B}$ from 70°.

OR

(ADC =) 360 − (85 + 70 + 125) = 80

(ACD =) 180 − 80 − 53.0521... ***(M1)***

46.9° (46.9478...°) ***(A1)*****(ft)*****(G2)***

Note: Follow through from part (b)(i).

[2 marks]

8e. The length of path AD is 287 m. *[4 marks]*

Find the area of the region ABCD.

Markscheme

$$\frac{185 \times 250 \times \sin(125°)}{2} + \frac{287 \times 387.015\ldots \times \sin(53.0521\ldots°)}{2}$$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for substitution in the area formula for either triangle; ***(M1)*** for correct substitution for both areas; ***(M1)*** for adding their two areas;

18942.8… + 44383.9…

63300 (m^2) (63326.8… (m^2)) ***(A1)*(ft)*(G3)***

Note: Follow through from parts (a) and (b)(ii).

OR

$$\mathrm{DC} = \frac{287 \times \sin(53.0521\ldots)}{\sin(46.9478\ldots)} = 313.884\ldots$$

$$0.5 \times 287 \times 185 \times \sin 85° + 0.5 \times 250 \times 313.884\ldots \times \sin 70°$$ ***M1M1M1***

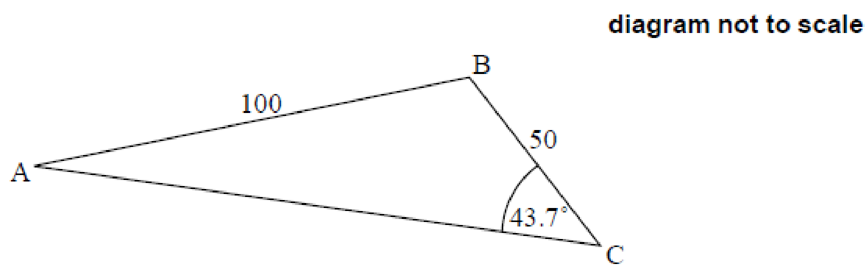
Note: Award ***(M1)*** for substitution in the area formula for either triangle; ***(M1)*** for correct substitution for both areas; ***(M1)*** for adding their two areas;

26446.4… + 36869.3…

63300 (63315.8…) (m^2) ***(A1)*(ft)*(G3)***

[4 marks]

A flat horizontal area, ABC, is such that AB = 100 m , BC = 50 m and angle AĈB = 43.7° as shown in the diagram.



9a. Show that the size of angle BÂC is 20.2°, correct to 3 significant figures. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\frac{\sin 43.7^\circ}{100} = \frac{\sin \text{BAC}}{50}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into sine rule formula, ***(A1)*** for correct substitution.

BAC = 20.2087... = 20.2° ***(A1)(AG)***

Note: Award ***(A1)*** only if both the correct unrounded and rounded answers are seen.

[3 marks]

9b. Calculate the area of triangle ABC. *[4 marks]*

Markscheme

units are required in part (b)

$\frac{1}{2}(100)(50)\sin(116.1)$ ***(A1)(M1)(A1)***

Note: Award ***(A1)*** for 116.1 or unrounded value or 116 seen, ***(M1)*** for substitution into area of triangle formula, ***(A1)*** for correct substitution.

= 2250 m^2 (2245.06... m^2) ***(A1)(G3)***

Note: The answer is 2250 m^2; the units are required. Use of 20.2087... gives 2245.23....

[4 marks]

9c. Find the length of AC. *[3 marks]*

Markscheme

$\frac{100}{\sin 43.7} = \frac{\text{AC}}{\sin(116.1)}$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substitution into sine rule formula, ***(A1)*(ft)** for their correct substitution. Follow through from their 116.1.

AC = 130 (m) (129.982… (m)) ***(A1)(ft)(G2)***

Note: Use of 20.2087… gives 129.992….

OR

$\text{AC}^2 = 100^2 + 50^2 - 2(100)(50)\cos(116.1)$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substitution into cosine rule formula, ***(A1)*(ft)** for their correct substitution. Follow through from their 116.1.

AC = 130 (m) (129.997… (m)) ***(A1)(ft)(G2)***

Note: Award ***(M1)*** for substitution into cosine rule formula, ***(A1)*(ft)** for their correct substitution.

[3 marks]

9d. A vertical pole, TB, is constructed at point B and has height 25 m. *[5 marks]*

Calculate the angle of elevation of T from, M, the midpoint of the side AC.

Markscheme

$BM^2 = 100^2 + 65^2 - 2(100)(65)\cos(20.2°)$ ***(M1)(A1)*****(ft)**

OR

$BM^2 = 50^2 + 65^2 - 2(50)(65)\cos(43.7°)$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substitution into cosine rule formula, ***(A1)*****(ft)** for correct substitution, including half their AC.

BM = 45.0 (44.9954… **OR** 45.0079…) ***(A1)*****(ft)**

Note: Use of 20.2052… gives 45. Award ***(G2)*** for 45.0 seen without working.

$\tan(\text{TMB}) = \frac{25}{\text{their BM}}$ ***(M1)***

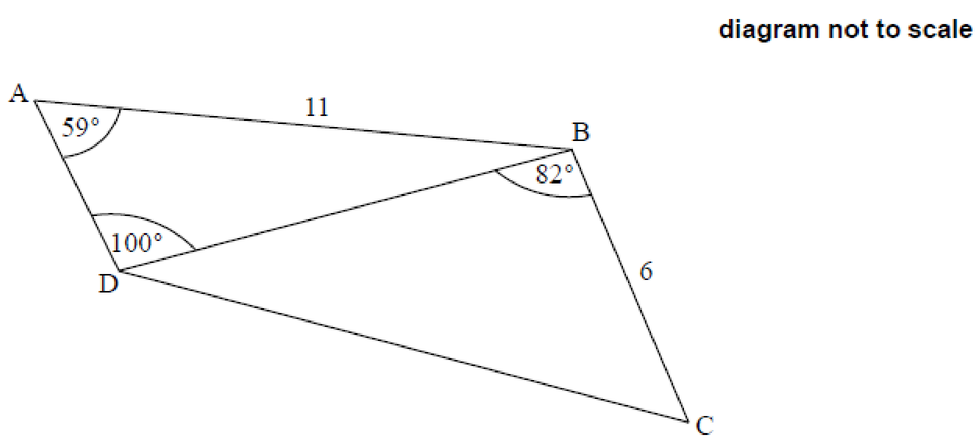
Note: Award ***(M1)*** for correct substitution into tangent formula.

$\text{T}\hat{\text{M}}\text{B} = 29.1°$ (29.0546…°) ***(A1)*****(ft)*****(G4)***

Note: Follow through within part (d) provided their BM **is seen**. Use of 44.9954 gives 29.0570… and use of 45.0079… gives 29.0503…. Follow through from their AC in part (c).

[5 marks]

The following diagram shows quadrilateral ABCD.



$\text{AB} = 11\,\text{cm},\ \text{BC} = 6\,\text{cm},\ \text{B}\hat{\text{A}}\text{D} = 100°,\ \text{and}\ \text{C}\hat{\text{B}}\text{D} = 82°$

10a. Find DB. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

evidence of choosing sine rule ***(M1)***

eg $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

correct substitution ***(A1)***

eg $\frac{\text{DB}}{\sin 59^\circ} = \frac{11}{\sin 100^\circ}$

9.57429

DB = 9.57 (cm) ***A1 N2***

[3 marks]

10b. Find DC. *[3 marks]*

Markscheme

evidence of choosing cosine rule ***(M1)***

eg

$a^2 = b^2 + c^2 - 2bc \cos (A),\ \text{DC}^2 = \text{DB}^2 + \text{BC}^2 - 2\text{DB} \times \text{BC} \times \cos \left(\text{D}\hat{\text{B}}\text{C}\right.$

correct substitution into RHS ***(A1)***

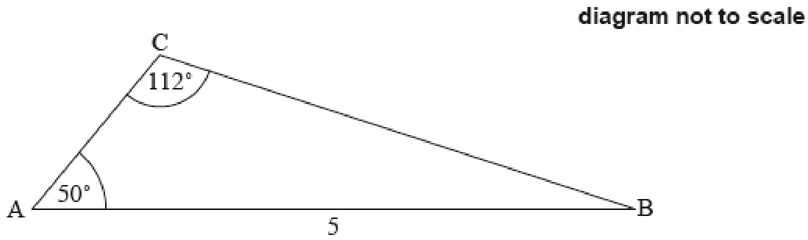
eg $9.57^2 + 6^2 - 2 \times 9.57 \times 6 \times \cos 82^\circ,\ 111.677$

10.5677

DC = 10.6 (cm) ***A1 N2***

[3 marks]

The following diagram shows a triangle ABC.



$\mathrm{AB} = 5\mathrm{cm}, \mathrm{C\hat{A}B} = 50°$ and $\mathrm{A\hat{C}B} = 112°$

11a. Find BC. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

evidence of choosing sine rule ***(M1)***

eg $\frac{\sin A}{a} = \frac{\sin B}{b}$

correct substitution ***(A1)***

eg $\frac{\mathrm{BC}}{\sin 50} = \frac{5}{\sin 112}$

4.13102

$\mathrm{BC} = 4.13\ (\mathrm{cm})$ ***A1*** ***N2***

[3 marks]

11b. Find the area of triangle ABC. *[3 marks]*

Markscheme

correct working ***(A1)***

eg $\hat{\mathrm{B}} = 180 - 50 - 112$, 18°, $\mathrm{AC} = 1.66642$

correct substitution into area formula ***(A1)***

eg $\frac{1}{2} \times 5 \times 4.13 \times \sin 18,\ 0.5(5)(1.66642) \sin 50,\ \frac{1}{2}(4.13)(1.66642) \sin 112$

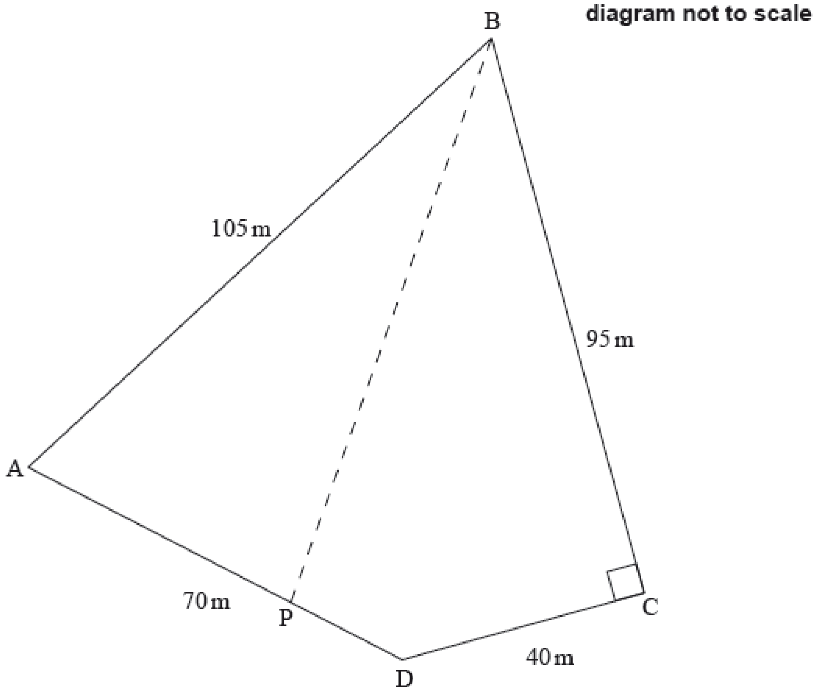
3.19139

$\mathrm{area} = 3.19\ (\mathrm{cm}^2)$ ***A1*** ***N2***

[3 marks]

A farmer owns a plot of land in the shape of a quadrilateral ABCD.

$\text{AB} = 105\text{m}$, $\text{BC} = 95\text{m}$, $\text{CD} = 40\text{m}$, $\text{DA} = 70\text{m}$ and angle $\text{DCB} = 90°$.



The farmer wants to divide the land into two equal areas. He builds a fence in a straight line from point B to point P on AD, so that the area of PAB is equal to the area of PBCD.

Calculate

12a. the length of BD; *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$(\text{BD} =)\ \sqrt{95^2 + 40^2}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras' theorem.

$= 103\ (\text{m})\ \left(103.077\ldots,\ 25\sqrt{17}\right)$ ***(A1)(G2)***

[2 marks]

12b. the size of angle DAB; *[3 marks]*

Markscheme

$\cos \mathrm{B\hat{A}D} = \frac{105^2+70^2-(103.077\ldots)^2}{2\times105\times70}$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substitution into cosine rule, ***(A1)*****(ft)** for their correct substitutions. Follow through from part (a).

$(\mathrm{B\hat{A}D}) = 68.9^\circ\ (68.8663\ldots)$ ***(A1)*****(ft)*****(G2)***

Note: If their 103 used, the answer is $68.7995\ldots$

[3 marks]

12c. the area of triangle ABD; *[3 marks]*

Markscheme

$(\text{Area of ABD} =)\frac{1}{2} \times 105 \times 70 \times \sin(68.8663\ldots)$ ***(M1)(A1)*****(ft)**

Notes: Award ***(M1)*** for substitution into the trig form of the area of a triangle formula.

Award ***(A1)*****(ft)** for their correct substitutions.

Follow through from part (b).

If 68.8° is used the area $= 3426.28\ldots\ \mathrm{m}^2$.

$= 3430\ \mathrm{m}^2\ (3427.82\ldots)$ ***(A1)*****(ft)*****(G2)***

[3 marks]

12d. the area of quadrilateral ABCD; *[2 marks]*

Markscheme

$\text{area of ABCD} = \frac{1}{2} \times 40 \times 95 + 3427.82\ldots$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted area of triangle formula **added** to their answer to part (c).

$= 5330 \text{ m}^2 \ (5327.83\ldots)$ ***(A1)*(ft)*(G2)***

[2 marks]

12e. the length of AP; *[3 marks]*

Markscheme

$\frac{1}{2} \times 105 \times \text{AP} \times \sin(68.8663\ldots) = 0.5 \times 5327.82\ldots$ ***(M1)(M1)***

Notes: Award ***(M1)*** for the correct substitution into triangle formula.

Award ***(M1)*** for equating their triangle area to half their part (d).

$(\text{AP} =)\ 54.4\ (\text{m})\ (54.4000\ldots)$ ***(A1)*(ft)*(G2)***

Notes: Follow through from parts (b) and (d).

[3 marks]

12f. the length of the fence, BP. *[3 marks]*

Markscheme

$\mathrm{BP}^2 = 105^2 + (54.4000\ldots)^2 - 2 \times 105 \times (54.4000\ldots) \times \cos(68.8663\ldots)$
(M1)(A1)**(ft)**

Notes: Award ***(M1)*** for substituted cosine rule formula.

Award ***(A1)*****(ft)** for their correct substitutions. Accept the exact fraction $\frac{53}{147}$ in place of $\cos(68.8663\ldots)$.

Follow through from parts (b) and (e).

$(\mathrm{BP} =)\ 99.3\ (\mathrm{m})\ (99.3252\ldots)$ ***(A1)*****(ft)*****(G2)***

Notes: If 54.4 and $\cos(68.9)$ are used the answer is $99.3567\ldots$

[3 marks]

13. In triangle ABC, AB = 5, BC = 14 and AC = 11. *[5 marks]*

Find all the interior angles of the triangle. Give your answers in degrees to one decimal place.

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

attempt to apply cosine rule ***M1***

$\cos \mathrm{A} = \frac{5^2+11^2-14^2}{2\times 5\times 11} = -0.4545\ldots$

$\Rightarrow \mathrm{A} = 117.03569\ldots^\circ$

$\Rightarrow \mathrm{A} = 117.0^\circ$ ***A1***

attempt to apply sine rule or cosine rule: ***M1***

$\frac{\sin 117.03569\ldots^\circ}{14} = \frac{\sin \mathrm{B}}{11}$

$\Rightarrow \mathrm{B} = 44.4153\ldots^\circ$

$\Rightarrow \mathrm{B} = 44.4^\circ$ ***A1***

$\mathrm{C} = 180^\circ - \mathrm{A} - \mathrm{B}$

$\mathrm{C} = 18.5^\circ$ ***A1***

Note: Candidates may attempt to find angles in any order of their choosing.

[5 marks]

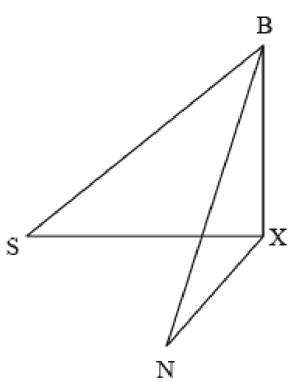
14. Barry is at the top of a cliff, standing 80 m above sea level, and observes two yachts in the sea. *[6 marks]*

"*Seaview*" (S) is at an angle of depression of 25°.

"*Nauti Buoy*" (N) is at an angle of depression of 35°.

The following three dimensional diagram shows Barry and the two yachts at S and N.

X lies at the foot of the cliff and angle $\mathrm{SXN} = $ 70°.



Find, to 3 significant figures, the distance between the two yachts.

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

attempt to use tan, or sine rule, in triangle BXN or BXS ***(M1)***

$\mathrm{NX} = 80\tan 55^\circ \left(= \frac{80}{\tan 35^\circ} = 114.25\right)$ ***(A1)***

$\mathrm{SX} = 80\tan 65^\circ \left(= \frac{80}{\tan 25^\circ} = 171.56\right)$ ***(A1)***

Attempt to use cosine rule ***M1***

$\mathrm{SN}^2 = 171.56^2 + 114.25^2 - 2 \times 171.56 \times 114.25\cos 70^\circ$ ***(A1)***

$\mathrm{SN} = 171\ (\mathrm{m})$ ***A1***

Note: Award final ***A1*** only if the correct answer has been given to 3 significant figures.

[6 marks]

15a. Find the set of values of k that satisfy the inequality $k^2 - k - 12 < 0$. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

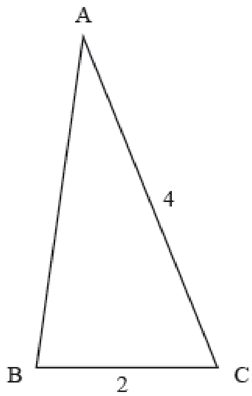
$k^2 - k - 12 < 0$

$(k-4)(k+3) < 0$ ***(M1)***

$-3 < k < 4$ ***A1***

[2 marks]

15b. The triangle ABC is shown in the following diagram. Given that $\cos B < \frac{1}{4}$, find the range of possible values for AB. *[4 marks]*



Markscheme

$\cos B = \frac{2^2+c^2-4^2}{4c}$ (or $16 = 2^2 + c^2 - 4c\cos B$) ***M1***

$\Rightarrow \frac{c^2-12}{4c} < \frac{1}{4}$ ***A1***

$\Rightarrow c^2 - c - 12 < 0$

from result in (a)

$0 < \mathrm{AB} < 4$ or $-3 < \mathrm{AB} < 4$ ***(A1)***

but AB must be at least 2

$\Rightarrow 2 < \mathrm{AB} < 4$ ***A1***

Note: Allow $\leqslant \mathrm{AB}$ for either of the final two ***A*** marks.

[4 marks]