# Tuesday 13.12 *[51 marks]*

The flight times, $T$ minutes, between two cities can be modelled by a normal distribution with a mean of $75$ minutes and a standard deviation of $\sigma$ minutes.

1a. Given that $2\%$ of the flight times are longer than $82$ minutes, find the value of $\sigma$. *[3 marks]*

1b. Find the probability that a randomly selected flight will have a flight time of more than $80$ minutes. *[2 marks]*

1c. Given that a flight between the two cities takes longer than $80$ minutes, find the probability that it takes less than $82$ minutes. *[4 marks]*

On a particular day, there are $64$ flights scheduled between these two cities.

1d. Find the expected number of flights that will have a flight time of more than $80$ minutes. *[3 marks]*

1e. Find the probability that more than $6$ of the flights on this particular day will have a flight time of more than $80$ minutes. *[3 marks]*

The curve $C$ has equation $\mathrm{e}^{2y} = x^3 + y$.

2a. Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{2\mathrm{e}^{2y}-1}$. *[3 marks]*

2b. The tangent to $C$ at the point P is parallel to the $y$-axis. *[4 marks]*

Find the $x$-coordinate of P.

Consider the functions $f\left(x\right) = \sqrt{3}\sin x + \cos x$ where $0 \leq x \leq \pi$ and $g(x) = 2x$ where $x \in \mathbb{R}$.

3a. Find $(f \circ g)(x)$. *[2 marks]*

3b. Solve the equation $(f \circ g)(x) = 2\cos 2x$ where $0 \leq x \leq \pi$. *[5 marks]*

Consider the three planes

$\prod_1 : \ 2x - y + z = 4$

$\prod_2 : \ x - 2y + 3z = 5$

$\prod_3 : -9x + 3y - 2z = 32$

4a. Show that the three planes do not intersect. *[4 marks]*

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4b. Verify that the point $\mathrm{P}(1, \ -2, \ 0)$ lies on both $\prod_1$ and $\prod_2$. *[1 mark]*

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4c. Find a vector equation of $L$, the line of intersection of $\prod_1$ and $\prod_2$. *[4 marks]*

4d. Find the distance between $L$ and $\prod_3$. *[6 marks]*

Mary, three female friends, and her brother, Peter, attend the theatre. In the theatre there is a row of $10$ empty seats. For the first half of the show, they decide to sit next to each other in this row.

5a. Find the number of ways these five people can be seated in this row. *[3 marks]*

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For the second half of the show, they return to the same row of $10$ empty seats. The four girls decide to sit at least one seat apart from Peter. The four girls do not have to sit next to each other.

5b. Find the number of ways these five people can now be seated in this row. *[4 marks]*

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