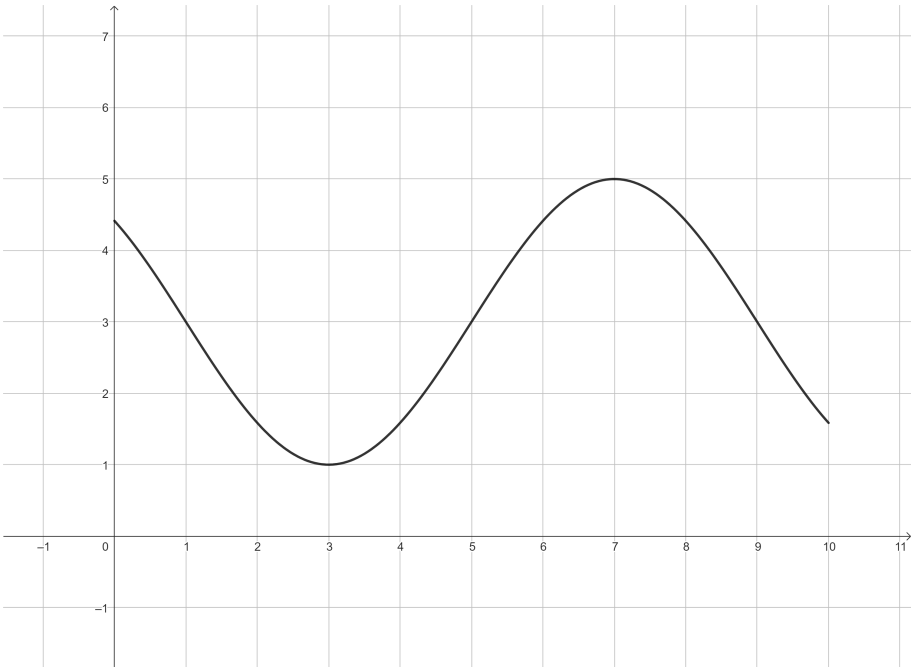
Name:
Group 1
Result:

1. The following diagram shows graph of $f(x) = A\sin(B(x-C)) + D$, with $0 \leqslant x \leqslant 10$.



The graph has a maximum at $(7, 5)$ and a minimum at $(3, 1)$

a) Find A, B, C and D, given that $C > 0$. [2 *points*]

b) Solve the inequality $f(x) < 2$. [2 *points*]

c) Write down a sequence of transformations that maps the graph of $y = \sin x$ into the graph of $y = f(x)$. [2 *points*]

2.

a) Show that: [2 *points*]

$$\csc(x) - \frac{\cos x}{\tan x} \equiv \sin x$$

where $x \neq \frac{k\pi}{2},\ k \in \mathbb{Z}$.

b) Hence, or otherwise solve the equation: [2 *points*]

$$\csc(2\theta) - \frac{\cos(2\theta)}{\tan(2\theta)} = \frac{1}{2}$$

for $0 \leqslant \theta \leqslant 2\pi$. Give your answers in terms of π.

c) Solve the equation: [2 *points*]

$$\cos^2 \alpha - 3\cos \alpha - 1 = \sin^2 \alpha$$

for $0 \leqslant \alpha \leqslant 2\pi$. Give your answers in terms of π.

3. Given that $0 < \alpha < \pi < \beta < 2\pi$ and $\cos\alpha = -\frac{1}{3}$ and $\tan\beta = 2$. **Without calculating** α **and** β, find the **exact** value of the expression: [6 *points*]

$$\frac{\sin(2\pi - \alpha) + \cos(\pi + \beta)}{\tan\frac{\pi}{6}}$$

4. Tomasz starts at point A and walks 4.5 km at a bearing of 025 to reach point B. He then walks another 3.5 km at a bearing of 100 and arrives at point C.

a) Find the direct distance from A to C. [2 *points*]

b) Find the bearing of A from C. [2 *points*]

c) Due to limited endurance Tomasz can only walk another 2 km. Find the area of the sector within the triangle ABC that Tomasz can reach. [2 *points*]

5. The average precipitation (in mm) in Otwock for each month is given in the table below:

Month	January	February	March	April	May	June
Precipitation $[mm]$	14.2	16.2	16.8	22.5	34.1	39.2

Month	July	August	September	October	November	December
Precipitation $[mm]$	42.2	34.8	25.3	18.9	18.5	16.2

a) Use the maximum and minimum precipitation and the periodicity of the seasons to create a *cosine* model for $P(t)$ in the form $P_1(t) = A\cos(B(t-C)) + D$, where $t = 1$ corresponds to January, $t = 2$ to February etc. [2 *points*]

b) Use technology to find *sine* model for $P_2(t)$. [2 *points*]

Another model for average precipitation in Otwock was developed based on daily data:

$$P_3(t) = 31 - 19\cos\left(\frac{2\pi}{365}(t-13)\right)$$

where $t = 1$ corresponds to 1st of Janauary, $t = 2$ to 2nd of January etc.

c) Use the third model to find the maximum precipitation in Otwock and the day at which it occurs. [2 *points*]