BD *[47 marks]*

Mr Burke teaches a mathematics class with 15 students. In this class there are 6 female students and 9 male students.

Each day Mr Burke randomly chooses one student to answer a homework question.

1a. Find the probability that on any given day Mr Burke chooses a female student to answer a question. *[1 mark]*

Markscheme

$\frac{6}{15}\left(0.4,\ \frac{2}{5}\right)$ ***A1***

[1 mark]

In the first month, Mr Burke will teach his class 20 times.

1b. Find the probability he will choose a female student 8 times. *[2 marks]*

Markscheme

P(X = 8) ***(M1)***

Note: Award ***(M1)*** for evidence of recognizing binomial probability. eg P(X = 8), $X \sim \mathrm{B}\left(20,\ \frac{6}{15}\right)$.

0.180 (0.179705…) ***A1***

[2 marks]

1c. Find the probability he will choose a male student at most 9 times. *[3 marks]*

Markscheme

P(male) = $\frac{9}{15}(0.6)$ ***A1***

P($X \le 9$) = 0.128 (0.127521…) ***(M1)A1***

Note: Award ***(M1)*** for evidence of correct approach *eg*, P($X \le 9$).

[3 marks]

A polygraph test is used to determine whether people are telling the truth or not, but it is not completely accurate. When a person tells the truth, they have a 20% chance of failing the test. Each test outcome is independent of any previous test outcome.

10 people take a polygraph test and all 10 tell the truth.

2a. Calculate the expected number of people who will pass this polygraph test. *[2 marks]*

Markscheme

$(\mathrm{E}(X)=)\ 10 \times 0.8$ ***(M1)***

8 (people) ***A1***

[2 marks]

2b. Calculate the probability that exactly 4 people will fail this polygraph test. *[2 marks]*

Markscheme

recognition of binomial probability ***(M1)***

$0.0881 \quad (0.0880803\ldots)$ ***A1***

[2 marks]

2c. Determine the probability that fewer than 7 people will pass this polygraph test. *[3 marks]*

Markscheme

0.8 and 6 seen **OR** 0.2 and 3 seen ***(A1)***

attempt to use binomial probability ***(M1)***

$0.121 \quad (0.120873\ldots)$ ***A1***

[3 marks]

The aircraft for a particular flight has 72 seats. The airline's records show that historically for this flight only 90% of the people who purchase a ticket arrive to board the flight. They assume this trend will continue and decide to sell extra tickets and hope that no more than 72 passengers will arrive.

The number of passengers that arrive to board this flight is assumed to follow a binomial distribution with a probability of 0.9.

3a. The airline sells 74 tickets for this flight. Find the probability that more than 72 passengers arrive to board the flight. *[3 marks]*

Markscheme

(let T be the number of passengers who arrive)

$(\mathrm{P}(T > 72) =)\ \mathrm{P}(T \geq 73)$ **OR** $1 - \mathrm{P}(T \leq 72)$ ***(A1)***

$T \sim \mathrm{B}(74,\ 0.9)$ **OR** $n = 74$ ***(M1)***

$= 0.00379 \quad (0.00379124\ldots)$ ***A1***

Note: Using the distribution $\mathrm{B}(74,\ 0.1)$, to work with the 10% that do not arrive for the flight, here and throughout this question, is a valid approach.

[3 marks]

3b. Write down the expected number of passengers who will arrive to board the flight if 72 tickets are sold. *[2 marks]*

Markscheme

72×0.9 ***(M1)***

64.8 ***A1***

[2 marks]

3c. Find the maximum number of tickets that could be sold if the expected number of passengers who arrive to board the flight must be less than or equal to 72. *[2 marks]*

Markscheme

$n \times 0.9 = 72$ ***(M1)***

80 ***A1***

[2 marks]

Each passenger pays $\$150$ for a ticket. If too many passengers arrive, then the airline will give $\$300$ in compensation to each passenger that cannot board.

3d. Find, to the nearest integer, the expected increase or decrease in the money made by the airline if they decide to sell 74 tickets rather than 72. *[8 marks]*

Markscheme

METHOD 1

EITHER

when selling 74 tickets

	$T \leq 72$	$T = 73$	$T = 74$
Income minus compensation (I)	11100	10800	10500
Probability	0.9962…	0.003380…	0.0004110…

top row ***A1A1***

bottom row ***A1A1***

Note: Award ***A1A1*** for each row correct. Award ***A1*** for one correct entry and

A1 for the remaining entries correct.

$\mathrm{E}(I) = 11100 \times 0.9962\ldots + 10800 \times 0.00338\ldots + 10500 \times 0.000411 \approx 11099$ ***(M1)A1***

OR

income is $74 \times 150 = 11100$ ***(A1)***

expected compensation is

$0.003380\ldots \times 300 + 0.0004110\ldots \times 600 \ (= 1.26070\ldots)$ ***(M1)A1A1***

expected income when selling 74 tickets is $11100 - 1.26070\ldots$ ***(M1)***

$= 11098.73\ldots \ (= \$11099)$ ***A1***

THEN

income for 72 tickets $= 72 \times 150 = 10800$ ***(A1)***

so expected gain $\approx 11099 - 10800 = \$299$ ***A1***

METHOD 2

for 74 tickets sold, let C be the compensation paid out

$\mathrm{P}(T = 73) = 0.00338014\ldots, \ \mathrm{P}(T = 74) = 0.000411098\ldots$ ***A1A1***

$\mathrm{E}(C) = 0.003380\ldots \times 300 + 0.0004110\ldots \times 600 \ (= 1.26070\ldots)$ ***(M1)A1A1***

extra expected revenue
$= 300 - 1.01404\ldots - 0.246658\ldots \ (300 - 1.26070\ldots)$ ***(A1)(M1)***

Note: Award ***A1*** for the 300 and ***M1*** for the subtraction.

$= \$299$ (to the nearest dollar) ***A1***

METHOD 3

let D be the change in income when selling 74 tickets.

	$T \le 72$	$T = 73$	$T = 74$
Change in income	300	0	−300

(A1)(A1)

Note: Award ***A1*** for one error, however award ***A1A1*** if there is no explicit mention that $T = 73$ would result in $D = 0$ and the other two are correct.

$\mathrm{P}(T \le 73) = 0.9962\ldots, \ \mathrm{P}(T = 74) = 0.000411098\ldots$ ***A1A1***

$\mathrm{E}(D) = 300 \times 0.9962\ldots + 0 \times 0.003380\ldots - 300 \times 0.0004110$ ***(M1)A1A1***

$= \$299$ ***A1***

[8 marks]

In a large university the probability that a student is left handed is 0.08. A sample of 150 students is randomly selected from the university. Let k be the expected number of left-handed students in this sample.

4a. Find k. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

evidence of binomial distribution (may be seen in part (b)) ***(M1)***

$eg np,\ 150 \times 0.08$

$k = 12$ ***A1*** ***N2***

[2 marks]

4b. Hence, find the probability that exactly k students are left handed; *[2 marks]*

Markscheme

$\mathrm{P}\left(X = 12\right) = \begin{pmatrix} 150 \\ 12 \end{pmatrix} \left(0.08\right)^{12} \left(0.92\right)^{138}$ ***(A1)***

0.119231

probability $= 0.119$ ***A1*** ***N2***

[2 marks]

4c. Hence, find the probability that fewer than k students are left handed. *[2 marks]*

Markscheme

recognition that $X \leqslant 11$ ***(M1)***

0.456800

$\mathrm{P}(X < 12) = 0.457$ ***A1*** ***N2***

[2 marks]

The following table shows a probability distribution for the random variable X, where $\mathrm{E}(X) = 1.2$.

x	0	1	2	3
$\mathrm{P}(X=x)$	p	$\frac{1}{2}$	$\frac{3}{10}$	q

5a. Find q. *[2 marks]*

Markscheme

correct substitution into $\mathrm{E}(X)$ formula ***(A1)***

eg $0(p) + 1(0.5) + 2(0.3) + 3(q) = 1.2$

$q = \frac{1}{30}$, 0.0333 ***A1*** ***N2***

[2 marks]

5b. Find p. *[2 marks]*

Markscheme

evidence of summing probabilities to 1 ***(M1)***

eg $p + 0.5 + 0.3 + q = 1$

$p = \frac{1}{6},\ 0.167$ ***A1*** ***N2***

[2 marks]

A bag contains white and blue marbles, with at least three of each colour. Three marbles are drawn from the bag, without replacement. The number of blue marbles drawn is given by the random variable X.

5c. Write down the probability of drawing three blue marbles. *[1 mark]*

Markscheme

$\mathrm{P}\ (3\text{ blue}) = \frac{1}{30},\ 0.0333$ ***A1*** ***N1***

[1 mark]

5d. Explain why the probability of drawing three white marbles is $\frac{1}{6}$. *[1 mark]*

Markscheme

valid reasoning ***R1***

eg $\mathrm{P}\ (3\text{ white}) = \mathrm{P}(0\text{ blue})$

$\mathrm{P}(3\text{ white}) = \frac{1}{6}$ ***AG*** ***N0***

[1 mark]

5e. The bag contains a total of ten marbles of which w are white. Find w. *[3 marks]*

Markscheme

valid method ***(M1)***

eg $\mathrm{P}(3\text{ white}) = \frac{w}{10} \times \frac{w-1}{9} \times \frac{w-2}{8},\ \frac{{}_{w}C_3}{{}_{10}C_3}$

correct equation ***A1***

eg $\frac{w}{10} \times \frac{w-1}{9} \times \frac{w-2}{8} = \frac{1}{6},\ \frac{{}_{w}C_3}{{}_{10}C_3} = 0.167$

$w = 6$ ***A1*** ***N2***

[3 marks]

A game is played in which three marbles are drawn from the bag of ten marbles, without replacement. A player wins a prize if three white marbles are drawn.

5f. Grant plays the game until he wins two prizes. Find the probability that he wins his second prize on his eighth attempt. *[4 marks]*

Markscheme

recognizing one prize in first seven attempts ***(M1)***

eg $\binom{7}{1}$, $\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^6$

correct working ***(A1)***

eg $\binom{7}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^6,\ 0.390714$

correct approach ***(A1)***

eg $\binom{7}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^6 \times \frac{1}{6}$

0.065119

0.0651 ***A1*** ***N2***

[4 marks]