

## BD [47 marks]

Mr Burke teaches a mathematics class with 15 students. In this class there are 6 female students and 9 male students.

Each day Mr Burke randomly chooses one student to answer a homework question.

- 1a. Find the probability that on any given day Mr Burke chooses a female student to answer a question. [1 mark]

In the first month, Mr Burke will teach his class 20 times.

- 1b. Find the probability he will choose a female student 8 times. [2 marks]

- 1c. Find the probability he will choose a male student at most 9 times. [3 marks]

A polygraph test is used to determine whether people are telling the truth or not, but it is not completely accurate. When a person tells the truth, they have a 20% chance of failing the test. Each test outcome is independent of any previous test outcome.

10 people take a polygraph test and all 10 tell the truth.

- 2a. Calculate the expected number of people who will pass this polygraph test. [2 marks]

- 2b. Calculate the probability that exactly 4 people will fail this polygraph test. [2 marks]

- 2c. Determine the probability that fewer than 7 people will pass this polygraph test. [3 marks]

The aircraft for a particular flight has 72 seats. The airline's records show that historically for this flight only 90% of the people who purchase a ticket arrive to board the flight. They assume this trend will continue and decide to sell extra tickets and hope that no more than 72 passengers will arrive.

The number of passengers that arrive to board this flight is assumed to follow a binomial distribution with a probability of 0.9.

3a. The airline sells 74 tickets for this flight. Find the probability that more than 72 passengers arrive to board the flight. [3 marks]

3b. Write down the expected number of passengers who will arrive to board the flight if 72 tickets are sold. [2 marks]

3c. Find the maximum number of tickets that could be sold if the expected number of passengers who arrive to board the flight must be less than or equal to 72. [2 marks]

Each passenger pays \$150 for a ticket. If too many passengers arrive, then the airline will give \$300 in compensation to each passenger that cannot board.

3d. Find, to the nearest integer, the expected increase or decrease in the money made by the airline if they decide to sell 74 tickets rather than 72. [8 marks]

In a large university the probability that a student is left handed is 0.08. A sample of 150 students is randomly selected from the university. Let  $k$  be the expected number of left-handed students in this sample.

4a. Find  $k$ . [2 marks]

4b. Hence, find the probability that exactly  $k$  students are left handed; [2 marks]

4c. Hence, find the probability that fewer than  $k$  students are left handed. [2 marks]

The following table shows a probability distribution for the random variable  $X$ , where  $E(X) = 1.2$ .

$x$	0	1	2	3
$P(X=x)$	$p$	$\frac{1}{2}$	$\frac{3}{10}$	$q$

5a. Find  $q$ . [2 marks]

5b. Find  $p$ . [2 marks]

A bag contains white and blue marbles, with at least three of each colour. Three marbles are drawn from the bag, without replacement. The number of blue marbles drawn is given by the random variable  $X$ .

5c. Write down the probability of drawing three blue marbles. [1 mark]

5d. Explain why the probability of drawing three white marbles is  $\frac{1}{6}$ . [1 mark]

5e. The bag contains a total of ten marbles of which  $w$  are white. Find  $w$ . [3 marks]

A game is played in which three marbles are drawn from the bag of ten marbles, without replacement. A player wins a prize if three white marbles are drawn.

5f. Grant plays the game until he wins two prizes. Find the probability that he wins his second prize on his eighth attempt. [4 marks]