BD *[13 marks]*

A six-sided biased die is weighted in such a way that the probability of obtaining a "six" is $\frac{7}{10}$.

1a. The die is tossed five times. Find the probability of obtaining at most three "sixes". *[3 marks]*

Markscheme

recognition of binomial ***(M1)***

$X \sim$ B(5, 0.7)

attempt to find P ($X \leq 3$) ***M1***

= 0.472 (= 0.47178) ***A1***

[3 marks]

1b. The die is tossed five times. Find the probability of obtaining the third "six" on the fifth toss. *[3 marks]*

Markscheme

recognition of 2 sixes in 4 tosses ***(M1)***

P (3rd six on the 5th toss)

$= \left[\binom{4}{2} \times (0.7)^2 \times (0.3)^2\right] \times 0.7 \, (= 0.2646 \times 0.7)$ ***A1***

= 0.185 (= 0.18522) ***A1***

[3 marks]

A factory manufactures lamps. It is known that the probability that a lamp is found to be defective is 0.05. A random sample of 30 lamps is tested.

2a. Find the probability that there is at least one defective lamp in the sample. *[3 marks]*

Markscheme

recognize that the variable has a Binomial distribution ***(M1)***

$X \sim \mathrm{B}(30, 0.05)$

attempt to find $\mathrm{P}(X \geq 1)$ ***(M1)***

$1 - \mathrm{P}(X = 0)$ OR $1 - 0.95^{30}$ OR $1 - 0.214638\ldots$ OR $0.785361\ldots$

Note: The two ***M*** marks are independent of each other.

$\mathrm{P}(X \geq 1) = 0.785$ ***A1***

[3 marks]

2b. Given that there is at least one defective lamp in the sample, find the probability that there are at most two defective lamps. *[4 marks]*

Markscheme

recognition of conditional probability ***(M1)***

$\mathrm{P}(X \leq 2 \mid X \geq 1)$ OR $\mathrm{P}(\text{at most 2 defective} \mid \text{at least 1 defective})$

Note: Recognition must be shown in context either in words or symbols but not just $\mathrm{P}(A \mid B)$.

$\frac{\mathrm{P}(1 \leq X \leq 2)}{\mathrm{P}(X \geq 1)}$ OR $\frac{\mathrm{P}(X=1)+\mathrm{P}(X=2)}{\mathrm{P}(X \geq 1)}$ ***(A1)***

$\frac{0.597540\ldots}{0.785361\ldots}$ OR $\frac{0.812178\ldots - 0.214638\ldots}{0.785361\ldots}$ OR $\frac{0.338903\ldots + 0.258636\ldots}{0.785361\ldots}$ ***(A1)***

$= 0.760847\ldots$

$\mathrm{P}(X \leq 2 \mid X \geq 1) = 0.761$ ***A1***

[4 marks]