Compound angles *[18 marks]*

Let $\sin\theta = \frac{\sqrt{5}}{3}$, where θ is acute.

1a. Find $\cos\theta$. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

evidence of valid approach ***(M1)***

*eg*right triangle, $\cos^2\theta = 1 - \sin^2\theta$

correct working ***(A1)***

*eg*missing side is 2, $\sqrt{1-\left(\frac{\sqrt{5}}{3}\right)^2}$

$\cos\theta = \frac{2}{3}$ ***A1 N2***

[3 marks]

1b. Find $\cos 2\theta$. *[2 marks]*

Markscheme

correct substitution into formula for $\cos 2\theta$ ***(A1)***

eg$2\times\left(\frac{2}{3}\right)^2 - 1,\ 1-2\left(\frac{\sqrt{5}}{3}\right)^2,\ \left(\frac{2}{3}\right)^2-\left(\frac{\sqrt{5}}{3}\right)^2$

$\cos 2\theta = -\frac{1}{9}$ ***A1 N2***

[2 marks]

2. A and B are acute angles such that $\cos A = \frac{2}{3}$ and $\sin B = \frac{1}{3}$. *[7 marks]*

Show that $\cos(2A+B) = -\frac{2\sqrt{2}}{27} - \frac{4\sqrt{5}}{27}$.

Markscheme

attempt to use $\cos(2A+B) = \cos 2A \cos B - \sin 2A \sin B$ (may be seen later) ***M1***

attempt to use any double angle formulae (seen anywhere) ***M1***

attempt to find either $\sin A$ or $\cos B$ (seen anywhere) ***M1***

$\cos A = \frac{2}{3} \Rightarrow \sin A \left(= \sqrt{1 - \frac{4}{9}}\right) = \frac{\sqrt{5}}{3}$ ***(A1)***

$\sin B = \frac{1}{3} \Rightarrow \cos B \left(= \sqrt{1 - \frac{1}{9}} = \frac{\sqrt{8}}{3}\right) = \frac{2\sqrt{2}}{3}$ ***A1***

$\cos 2A \left(= 2\cos^2 A - 1\right) = -\frac{1}{9}$ ***A1***

$\sin 2A \left(= 2\sin A \cos A\right) = \frac{4\sqrt{5}}{9}$ ***A1***

So $\cos(2A+B) = \left(-\frac{1}{9}\right)\left(\frac{2\sqrt{2}}{3}\right) - \left(\frac{4\sqrt{5}}{9}\right)\left(\frac{1}{3}\right)$

$= -\frac{2\sqrt{2}}{27} - \frac{4\sqrt{5}}{27}$ ***AG***

[7 marks]

3. Given that $\sin x = \frac{1}{3}$, where $0 < x < \frac{\pi}{2}$, find the value of $\cos 4x$. *[6 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

METHOD 1

correct substitution into formula for $\cos(2x)$ or $\sin(2x)$ ***(A1)***

eg $1 - 2\left(\frac{1}{3}\right)^2,\ 2\left(\frac{\sqrt{8}}{3}\right)^2 - 1,\ 2\left(\frac{1}{3}\right)\left(\frac{\sqrt{8}}{3}\right),\ \left(\frac{\sqrt{8}}{3}\right)^2 - \left(\frac{1}{3}\right)^2$

$\cos(2x) = \frac{7}{9}$ or $\sin(2x) = \frac{2\sqrt{8}}{9}$ $\left(= \frac{\sqrt{32}}{9} = \frac{4\sqrt{2}}{9}\right)$ (may be seen in substitution) ***A2***

recognizing $4x$ is double angle of $2x$ (seen anywhere) ***(M1)***

eg $\cos(2(2x)),\ 2\cos^2(2\theta) - 1,\ 1 - 2\sin^2(2\theta),\ \cos^2(2\theta) - \sin^2(2\theta)$

correct substitution of **their** value of $\cos(2x)$ and/or $\sin(2x)$ into formula for $\cos(4x)$ ***(A1)*** *eg* $2\left(\frac{7}{9}\right)^2 - 1,\ \frac{98}{81} - 1,\ 1 - 2\left(\frac{2\sqrt{8}}{9}\right)^2,\ 1 - \frac{64}{81},\ \left(\frac{7}{9}\right)^2 - \left(\frac{2\sqrt{8}}{9}\right)^2,\ \frac{49}{81} - \frac{32}{81}$

$\cos\ (4x) = \frac{17}{81}$ ***A1 N2***

METHOD 2

recognizing $4x$ is double angle of $2x$ (seen anywhere) ***(M1)***

eg $\cos\ (2\ (2x))$

double angle identity for $2x$ ***(M1)***

eg $2\cos^2\ (2\theta) - 1,\ 1 - 2\sin^2\ (2x),\ \cos^2\ (2\theta) - \sin^2\ (2\theta)$

correct expression for $\cos\ (4x)$ in terms of $\sin x$ and/or $\cos x$ ***(A1)***

eg $2\left(1 - 2\sin^2\theta\right)^2 - 1,\ 1 - 2(2\sin x\cos x)^2,$
$\left(1 - 2\sin^2\theta\right)^2 - (2\sin\theta\cos\theta)^2$

correct substitution for $\sin x$ and/or $\cos x$ ***A1***

eg $2\left(1 - 2\left(\frac{1}{3}\right)^2\right)^2 - 1,\ 2\left(1 - 4\left(\frac{1}{3}\right)^2 + 4\left(\frac{1}{3}\right)^4\right) - 1,\ 1 - 2\left(2 \times \frac{1}{3} \times \frac{\sqrt{8}}{3}\right)^2$

correct working ***(A1)***

eg $2\left(\frac{49}{81}\right) - 1,\ 1 - 2\left(\frac{32}{81}\right),\ \frac{49}{81} - \frac{32}{81}$

$\cos\ (4x) = \frac{17}{81}$ ***A1 N2***

[6 marks]

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