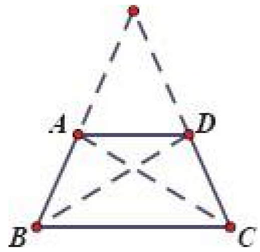
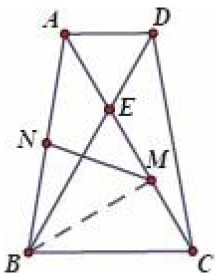


Example 1.4.16 *ABCD* is an isosceles trapezium where *AD* // *BC* and *AB* = *CD*. Its diagonals *AC*, *BD* intersect at *E* and $\angle AED = 60°$. Let *M*, *N* be the midpoints of *CE*, *AB* respectively. Show that $MN = \frac{1}{2}AB$.

***Proof*.** Refer to the diagram on the below. Since *ABCD* is an isosceles trapezium with *AB* = *CD* we must have $\angle ABC = \angle BCD$. Hence, $\Delta ABC \cong \angle DCB$ (S.A.S.), which implies $\angle BCE = \angle CBE$.



Since $\angle BEC = \angle AED = 60°$, ΔBCE must be an equilateral triangle. Since *M* is the midpoint of *CE*, we must have $BM \perp CE$.
Since *N* is the midpoint of *AB*, *MN* is the median on the hypotenuse of ΔAMB and hence, $MN = \frac{1}{2}AB$ (Theorem 1.4.6).

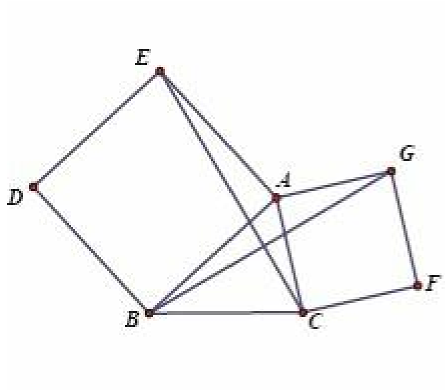
□

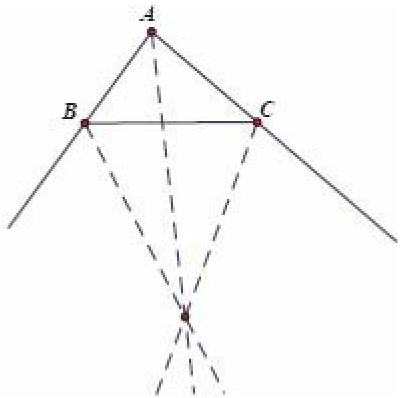
1.5 Exercises

1. In a right angled triangle ΔABC where $\angle A = 90°$, *P* is a point on *BC*. If *AP* = *BP*, show that *BP* = *CP*, i.e., *P* is the midpoint of *BC*.

2. Given ΔABC where $\angle B = 2\angle C$, *D* is a point on *BC* such that *AD* bisects $\angle A$. Show that *AC* = *AB* + *BD*.

3. Refer to the left diagram below. Given ΔABC, draw squares *ABDE* and *ACFG* outwards from *AB*, *AC* respectively. Show that *BG* = *CE* and $BG \perp CE$.





4. Refer to the right diagram above. Show that in ΔABC, the angle bisector of $\angle A$, the exterior angles bisectors of $\angle B$ and $\angle C$ are concurrent (i.e., they pass through the same point).

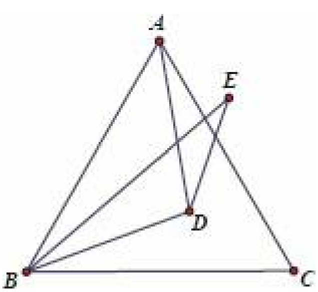
Note: This point is called the ex-center of ΔABC opposite A. One may see that each triangle has three ex-centers.

5. Given ΔABC, J_1 and J_2 are the ex-centers (refer to Exercise 1.4) opposite B and C respectively. Let I be the incenter of ΔABC. Show that $J_1J_2 \perp AI$.

6. Let $ABCD$ be a square. E, F are points on BC, CD respectively and $\angle EAF = 45°$. Show that $EF = BE + DF$.

7. In the acute angled triangle ΔABC, $BD \perp AC$ at D and $CE \perp AB$ at E. BD and CE intersect at Q. P is on BD extended such that $BP = AC$. If $CQ = AB$, find $\angle AQP$.

8. Refer to the diagram on the below. ΔABC is an equilateral triangle. D is a point inside ΔABC such that $AD = BD$. Choose E such that $BE = AB$ and BD bisects $\angle CBE$. Find $\angle BED$.



9. Let I be the incenter of ΔABC. AI extended intersects BC at D. Draw $IH \perp BC$ at H. Show that $\angle BID = \angle CIH$.

10. Given a quadrilateral $ABCD$, the diagonal AC bisects both $\angle A$ and $\angle C$. If AB extended and DC extended intersect at E, and AD extended and BC extended intersect at F, show that for any point P on line AC, $PE = PF$.

11. In ΔABC, $AB = AC$ and D is a point on AB. Let O be the circumcenter of ΔBCD and I be the incenter of ΔACD. Show that A, I, O are collinear.

12. Given a quadrilateral $ABCD$ where BD bisects $\angle B$, P is a point on BC such that PD bisects $\angle APC$. Show that $\angle BDP + \angle PAD = 90°$.

13. $ABCD$ is a quadrilateral where $AD \,//\, BC$. Show that if $BC - AB = AD - CD$, then $ABCD$ is a parallelogram.

14. Given a square $ABCD$, ℓ_1 is a straight line intersecting AB, AD at E, F respectively and ℓ_2 is a straight line intersecting BC, CD at G, H respectively. EH, FG intersect at I. If $\ell_1 \,//\, \ell_2$ and the distance between ℓ_1, ℓ_2 is equal to AB, find $\angle GIH$.