

Financial Maths review [67 marks]

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €14 000, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of 14 % **compounded quarterly**. No deposit required and repayments are made each quarter.

1a. Find the repayment made each quarter.

[3 marks]

Markscheme

$$N = 24$$

$$I\% = 14$$

$$PV = -14000$$

$$FV = 0$$

$$P/Y = 4$$

$$C/Y = 4 \quad (M1)(A1)$$

Note: Award **M1** for an attempt to use a financial app in their technology, award **A1** for all entries correct. Accept $PV = 14000$.

$$(\text{€})871.82 \quad A1$$

[3 marks]

1b. Find the total amount paid for the car.

[2 marks]

Markscheme

$$4 \times 6 \times 871.82 \quad (M1)$$

$$(\text{€}) 20923.68 \quad A1$$

[2 marks]

1c. Find the interest paid on the loan.

[2 marks]

Markscheme

20923.68 – 14000 (M1)

(€) 6923.68 A1

[2 marks]

Finance option B:

A 6 year loan at a nominal annual interest rate of $r\%$ **compounded monthly**. Terms of the loan require a 10 % deposit and monthly repayments of €250.

1d. Find the amount to be borrowed for this option.

[2 marks]

Markscheme

$0.9 \times 14000 (= 14000 - 0.10 \times 14000)$ M1

(€) 12600.00 A1

[2 marks]

1e. Find the annual interest rate, r .

[3 marks]

Markscheme

$N = 72$

$PV = 12600$

$PMT = -250$

$FV = 0$

$P/Y = 12$

$C/Y = 12$ (M1)(A1)

Note: Award **M1** for an attempt to use a financial app in their technology, award **A1** for all entries correct. Accept $PV = -12600$ provided $PMT = 250$.

12.56(%) A1

[3 marks]

1f. State which option Bryan should choose. Justify your answer.

[2 marks]

Markscheme

EITHER

Bryan should choose Option A **A1**

no deposit is required **R1**

Note: Award **R1** for stating that no deposit is required. Award **A1** for the correct choice from that fact. Do not award **ROA1**.

OR

Bryan should choose Option B **A1**

cost of Option A (6923.69) > cost of Option B ($72 \times 250 - 12600 = 5400$)

R1

Note: Award **R1** for a correct comparison of costs. Award **A1** for the correct choice from that comparison. Do not award **ROA1**.

[2 marks]

1g. Bryan's car depreciates at an annual rate of 25 % per year.

[3 marks]

Find the value of Bryan's car six years after it is purchased.

Markscheme

$$14\,000 \left(1 - \frac{25}{100}\right)^6 \quad \text{(M1)(A1)}$$

Note: Award **M1** for substitution into compound interest formula.
Award **A1** for correct substitutions.

$$= (\text{€})2491.70 \quad \text{A1}$$

OR

$$N = 6$$

$$I\% = -25$$

$$PV = \pm 14\,000$$

$$P/Y = 1$$

$$C/Y = 1 \quad \text{(A1)(M1)}$$

Note: Award **A1** for $PV = \pm 14\,000$, **M1** for other entries correct.

$$(\text{€})2491.70 \quad \text{A1}$$

[3 marks]

Sophia pays \$200 into a bank account at the end of each month. The annual interest paid on money in the account is 3.1% which is compounded monthly.

2a. Find the value of her investment after a period of 5 years.

[3 marks]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

Number of time periods $12 \times 5 = 60$ **(A1)**

$N = 60$

$I\% = 3.1$

$PV = 0$

$PMT = 200$

$P/Y = 12$

$C/Y = 12$

Value (\$) 12,961.91 **(M1)A1**

[3 marks]

The average rate of inflation per year over the 5 years was 2%.

2b. Find an approximation for the real interest rate for the money invested in the account. [2 marks]

Markscheme

METHOD 1

Real interest rate = $3.1 - 2.0 = 1.1\%$ **(M1)A1**

METHOD 2

$\frac{1+0.031}{1+0.02} = 1.01078\dots$ **(M1)**

1.08% (accept 1.1%) **A1**

[2 marks]

2c. Hence find the real value of Sophia's investment at the end of 5 years. [2 marks]

Markscheme

$N = 60$
 $I\% = 1.1$
 $PV = 0$
 $PMT = 200$
 $P/Y = 12$
 $C/Y = 12$
 $(\$)12,300(12,330.33\dots)$ **(M1)A1**

Note: Award **A1** for \$12,300 only.

[2 marks]

Give your answers to this question correct to two decimal places.

Gen invests \$2400 in a savings account that pays interest at a rate of 4% per year, compounded annually. She leaves the money in her account for 10 years, and she does not invest or withdraw any money during this time.

3a. Calculate the value of her savings after 10 years.

[2 marks]

Markscheme

$$2400(1.04)^{10} = \$3552.59 \quad \mathbf{M1A1}$$

[2 marks]

3b. The rate of inflation during this 10 year period is 1.5% per year.

[3 marks]

Calculate the real value of her savings after 10 years.

Markscheme

$$\text{real interest rate} = 4 - 1.5 = 2.5\% \quad \mathbf{A1}$$

$$2400(1.025)^{10} = \$3072.20 \quad \mathbf{M1A1}$$

[3 marks]

Yejin plans to retire at age 60. She wants to create an annuity fund, which will pay her a monthly allowance of \$4000 during her retirement. She wants to save enough money so that the payments last for 30 years. A financial advisor has told her that she can expect to earn 5% interest on her funds, compounded annually.

- 4a. Calculate the amount Yejin needs to have saved into her annuity fund, in [3 marks] order to meet her retirement goal.

Markscheme

Use of finance solver **M1**

$N = 360$, $I = 5\%$, $Pmt = 4000$, $FV = 0$, $PpY = 12$, $CpY = 1$ **A1**

\$755000 (correct to 3 s.f.) **A1**

[3 marks]

- 4b. Yejin has just turned 28 years old. She currently has no retirement savings. She wants to save part of her salary each month into her annuity fund. [3 marks]

Calculate the amount Yejin needs to save each month, to meet her retirement goal.

Markscheme

$N = 384$, $I = 5\%$, $PV = 0$, $FV = 754638$, $PpY = 12$, $CpY = 1$ **M1A1**

\$817 per month (correct to 3 s.f.) **A1**

[3 marks]

Paul wants to buy a car. He needs to take out a loan for \$7000. The car salesman offers him a loan with an interest rate of 8%, compounded annually. Paul considers two options to repay the loan.

Option 1: Pay \$200 each month, until the loan is fully repaid

Option 2: Make 24 equal monthly payments.

Use option 1 to calculate

- 5a. the number of months it will take for Paul to repay the loan. [3 marks]

Markscheme

evidence of using Finance solver on GDC **M1**

$$N = 39.8 \quad \mathbf{A1}$$

It will take 40 months **A1**

[3 marks]

5b. the total amount that Paul has to pay.

[2 marks]

Markscheme

$$40 \times 200 = \$8000 \quad \mathbf{M1A1}$$

[2 marks]

Use option 2 to calculate

5c. the amount Paul pays each month.

[2 marks]

Markscheme

$$\text{Monthly payment} = \$316 \text{ } (\$315.70) \quad \mathbf{M1A1}$$

[2 marks]

5d. the total amount that Paul has to pay.

[2 marks]

Markscheme

$$24 \times 315.7 = \$7580 \text{ } (\$7576.80) \quad \mathbf{M1A1}$$

[2 marks]

Give a reason why Paul might choose

5e. option 1.

[1 mark]

Markscheme

The monthly repayment is lower, he might not be able to afford \$316 per month. **R1**

[1 mark]

5f. option 2.

[1 mark]

Markscheme

the total amount to repay is lower. **R1**

[1 mark]

Sophie is planning to buy a house. She needs to take out a mortgage for \$120000. She is considering two possible options.

Option 1: Repay the mortgage over 20 years, at an annual interest rate of 5%, compounded annually.

Option 2: Pay \$1000 every month, at an annual interest rate of 6%, compounded annually, until the loan is fully repaid.

6a. Calculate the monthly repayment using option 1.

[2 marks]

Markscheme

evidence of using Finance solver on GDC **M1**

Monthly payment = \$785 (\$784.60) **A1**

[2 marks]

6b. Calculate the total amount Sophie would pay, using option 1.

[2 marks]

Markscheme

$240 \times 785 = \$188000$ **M1A1**

[2 marks]

6c. Calculate the number of months it will take to repay the mortgage using [3 marks]
option 2.

Markscheme

$$N = 180.7 \quad \mathbf{M1A1}$$

It will take 181 months $\mathbf{A1}$

[3 marks]

6d. Calculate the total amount Sophie would pay, using option 2. [2 marks]

Markscheme

$$181 \times 1000 = \$ 181000 \quad \mathbf{M1A1}$$

[2 marks]

Give a reason why Sophie might choose

6e. option 1. [1 mark]

Markscheme

The monthly repayment is lower, she might not be able to afford \$1000 per month. $\mathbf{R1}$

[1 mark]

6f. option 2. [1 mark]

Markscheme

the total amount to repay is lower. $\mathbf{R1}$

[1 mark]

Sophie decides to choose option 1. At the end of 10 years, the interest rate is changed to 7%, compounded annually.

- 6g. Use your answer to part (a)(i) to calculate the amount remaining on her [2 marks] mortgage after the first 10 years.

Markscheme

\$74400 (accept \$74300) **M1A1**

[2 marks]

- 6h. Hence calculate her monthly repayment for the final 10 years. [2 marks]

Markscheme

Use of finance solver with $N = 120$, $PV = \$74400$, $I = 7\%$ **A1**

\$855 (accept \$854 – \$856) **A1**

[2 marks]

Juliana plans to invest money for 10 years in an account paying 3.5% interest, compounded annually. She expects the annual inflation rate to be 2% per year throughout the 10-year period.

Juliana would like her investment to be worth a real value of \$4000, compared to current values, at the end of the 10-year period. She is considering two options.

Option 1: Make a one-time investment at the start of the 10-year period.

Option 2: Invest \$1000 at the start of the 10-year period and then invest \$ x into the account at the end of each year (including the first and last years).

- 7a. For option 1, determine the minimum amount Juliana would need to invest. Give your answer to the nearest dollar. [3 marks]

Markscheme

METHOD 1 - (with $FV = 4000$)

EITHER

$$N = 10$$

$$I = 1.5$$

$$FV = 4000$$

$$P/Y = 1$$

$$C/Y = 1 \quad \textbf{(A1)(M1)}$$

Note: Award **A1** for $(3.5 - 2 =) 1.5$ seen and **M1** for all other entries correct.

OR

$$4000 = A(1 + 0.015)^{10} \quad \textbf{(A1)(M1)}$$

Note: Award **A1** for 1.5 or 0.015 seen, **M1** for attempt to substitute into compound interest formula **and** equating to 4000.

THEN

$$(PV =) \$3447 \quad \textbf{A1}$$

Note: Award **A0** if not rounded to a whole number or a negative sign given.

METHOD 2 - (With FV including inflation)

calculate FV with inflation

$$4000 \times 1.02^{10} \quad \textbf{(A1)}$$

$$(= 4875.977\dots)$$

EITHER

$$4000 \times 1.02^{10} = PV \times 1.035^{10} \quad \textbf{(M1)}$$

OR

$$N = 10$$

$$I = 3.5$$

$$FV = 4875.977\dots$$

$$P/Y = 1$$

$$C/Y = 1 \quad \textbf{(M1)}$$

Note: Award **M1** for *their* FV and all other entries correct.

THEN

$$(PV =) \$3457 \quad \textbf{A1}$$

Note: Award **A0** if not rounded to a whole number or a negative sign given.

METHOD 3 - (Using formula to calculate real rate of return)

(real rate of return =) $1.47058\dots(\%)$ **(A1)**

EITHER

$$4000 = PV \times 1.0147058\dots^{10} \quad \mathbf{(A1)}$$

OR

$$N = 10$$

$$I = 1.47058\dots$$

$$FV = 4000$$

$$P/Y = 1$$

$$C/Y = 1 \quad \mathbf{(M1)}$$

Note: Award **M1** for all entries correct.

THEN

$$(PV =) \$3457 \quad \mathbf{A1}$$

[3 marks]

7b. For option 2, find the minimum value of x that Juliana would need to invest each year. Give your answer to the nearest dollar. **[3 marks]**

Markscheme

METHOD 1 - (Finding the future value of the investment using *PV* from part (a))

$$N = 10$$

$$I = 3.5$$

$$PV = 3446.66\dots \text{ (from Method 1) OR } 3456.67\dots \text{ (from Methods 2, 3)}$$

$$P/Y = 1$$

$$C/Y = 1 \quad \textbf{(M1)}$$

Note: Award **M1** for interest rate 3.5 **and** answer to part (a) as *PV*.

$$(\text{FV} =) \$4861.87 \quad \textbf{OR} \quad \$4875.97 \quad \textbf{(A1)}$$

$$\text{so payment required (from TVM) will be } \$294 \quad \textbf{OR} \quad \$295 \quad \textbf{A1}$$

Note: Award **A0** if a negative sign given, unless already penalized in part (a).

METHOD 2 - (Using *FV*)

$$N = 10$$

$$I = 3.5$$

$$PV = -1000$$

$$FV = 4875.977\dots$$

$$P/Y = 1$$

$$C/Y = 1 \quad \textbf{(A1)(M1)}$$

Note: Award **A1** for $I = 3.5$ **and** $FV = \pm 4875.977\dots$, **M1** for all other entries correct and opposite *PV* and *FV* signs.

$$(\text{PMT} =) \$295 \quad (295.393) \quad \textbf{A1}$$

Note: Correct 3sf answer is 295, however accept an answer of 296 given that the context supports rounding up. Award **A0** if a negative sign given, unless already penalized in part (a).

[3 marks]

