Further trig review *[107 marks]*

1. Find the least positive value of x for which $\cos\left(\frac{x}{2}+\frac{\pi}{3}\right)=\frac{1}{\sqrt{2}}$. *[5 marks]*

Let $\sin\theta=\frac{\sqrt{5}}{3}$, where θ is acute.

2a. Find $\cos\theta$. *[3 marks]*

2b. Find $\cos 2\theta$. *[2 marks]*

3a. Show that $\log_9(\cos 2x+2)=\log_3\sqrt{\cos 2x+2}$. *[3 marks]*

3b. Hence or otherwise solve $\log_3(2\sin x)=\log_9(\cos 2x+2)$ for $0<x<\frac{\pi}{2}$. *[5 marks]*

Consider the functions $f\left(x\right)=\sqrt{3}\sin x+\cos x$ where $0\le x\le\pi$ and $g(x)=2x$ where $x\in\mathbb{R}$.

4a. Find $(f\circ g)(x)$. *[2 marks]*

4b. Solve the equation $(f\circ g)(x)=2\cos 2x$ where $0\le x\le\pi$. *[5 marks]*

5a. Show that $\sin 2x+\cos 2x-1=2\sin x(\cos x-\sin x)$. *[2 marks]*

5b. Hence or otherwise, solve $\sin 2x+\cos 2x-1+\cos x-\sin x=0$ for $0<x<2\pi$. *[6 marks]*

6a. Show that the equation $2\cos^2 x+5\sin x=4$ may be written in the form $2\sin^2 x-5\sin x+2=0$. *[1 mark]*

6b. Hence, solve the equation $2\cos^2 x + 5\sin x = 4, 0 \leq x \leq 2\pi$. *[5 marks]*

7. It is given that $\operatorname{cosec}\theta = \frac{3}{2}$, where $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$. Find the exact value of $\cot\theta$. *[4 marks]*

8. Solve the equation $2\cos^2 x + 5\sin x = 4, 0 \leq x \leq 2\pi$. *[7 marks]*

9. Let $f(x) = 4\cos\left(\frac{x}{2}\right) + 1$, for $0 \leqslant x \leqslant 6\pi$. Find the values of x for which $f(x) > 2\sqrt{2} + 1$. *[8 marks]*

10. A and B are acute angles such that $\cos A = \frac{2}{3}$ and $\sin B = \frac{1}{3}$. *[7 marks]*

Show that $\cos(2A + B) = -\frac{2\sqrt{2}}{27} - \frac{4\sqrt{5}}{27}$.

11. Given that $\sin x = \frac{1}{3}$, where $0 < x < \frac{\pi}{2}$, find the value of $\cos 4x$. *[6 marks]*

12. Let $f(x) = \tan(x + \pi)\cos\left(x - \frac{\pi}{2}\right)$ where $0 < x < \frac{\pi}{2}$. *[5 marks]*

Express $f(x)$ in terms of $\sin x$ and $\cos x$.

13. Solve $\log_2(2\sin x) + \log_2(\cos x) = -1$, for $2\pi < x < \frac{5\pi}{2}$. *[7 marks]*

14. Solve the equation $\sec^2 x + 2\tan x = 0,\ 0 \leqslant x \leqslant 2\pi$. *[5 marks]*

15a. Show that $\cot 2\theta = \frac{1-\tan^2\theta}{2\tan\theta}$. *[1 mark]*

15b. Verify that $x = \tan\theta$ and $x = -\cot\theta$ satisfy the equation $x^2 + (2\cot 2\theta)x - 1 = 0$. *[7 marks]*

15c. Hence, or otherwise, show that the exact value of $\tan\frac{\pi}{12} = 2 - \sqrt{3}$. *[5 marks]*

15d. Using the results from parts (b) and (c) find the exact value of $\tan\frac{\pi}{24} - \cot\frac{\pi}{24}$. *[6 marks]*

Give your answer in the form $a + b\sqrt{3}$ where $a, b \in \mathbb{Z}$.