3 **a** Draw the graph of $y = f(x) = 3^x$.
b On the same set of axes, sketch the graph of $y = [f(x)]^2$.
c Describe a stretch which transforms $y = f(x)$ to $y = [f(x)]^2$.

4 **a** Draw the graph of $y = f(x) = \log_3 x$.
b On the same set of axes, sketch the graph of $y = [f(x)]^2$.
c Find the invariant points when $y = f(x)$ is transformed to $y = [f(x)]^2$.

5 Show that if $f(x)$ is odd, then $[f(x)]^2$ is even.

6 Suppose $f(x)$ has domain $0 \leqslant x \leqslant 5$ and range $-4 \leqslant y \leqslant 3$. Find the domain and range of $[f(x)]^2$.

7 Consider the function $f(x) = \dfrac{2x+4}{x-1}$

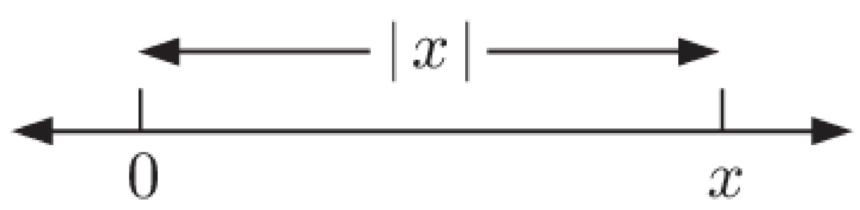
a Find the axes intercepts and asymptotes of the function.
b Hence find the axes intercepts and asymptotes of $y = [f(x)]^2$.
c Which points are invariant when $y = f(x)$ is transformed to $y = [f(x)]^2$?
d Sketch $y = f(x)$ and $y = [f(x)]^2$ on the same set of axes.

C ABSOLUTE VALUE FUNCTIONS

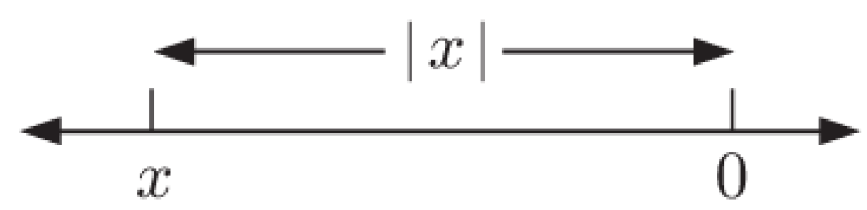
The **absolute value** or **modulus** of a real number x is its distance from 0 on the number line. We write the absolute value of x as $|x|$.

Because the absolute value is a distance, it cannot be negative.

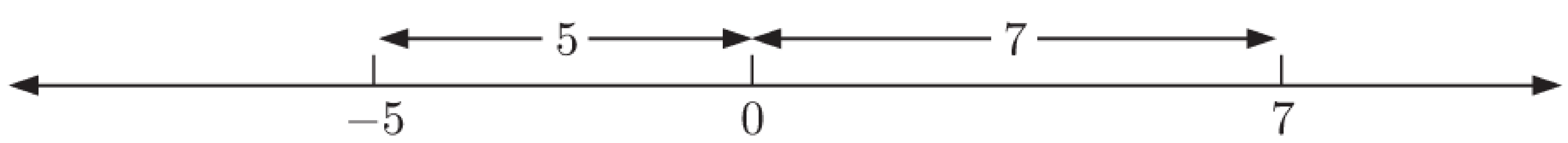
- If $x > 0$, $|x| = x$.



- If $x < 0$, $|x| = -x$.



For example: $|7| = 7$ and $|-5| = 5$.

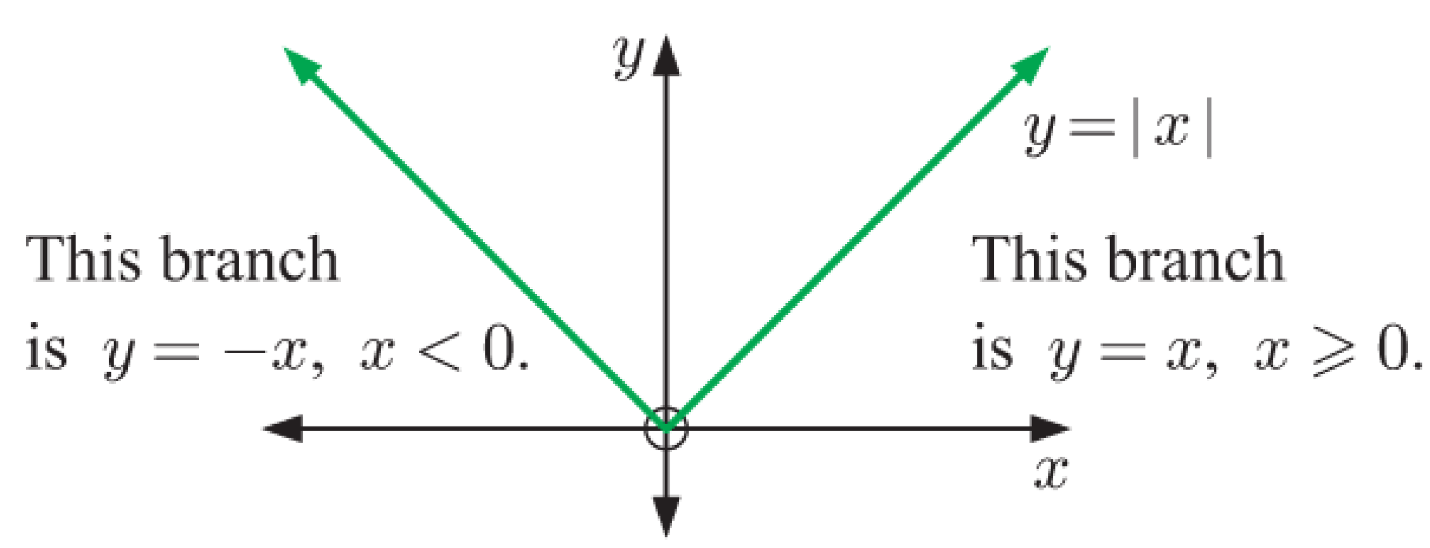


This leads us to the **algebraic definition**:

The absolute value of x is $|x| = \begin{cases} x & \text{if } x \geqslant 0 \\ -x & \text{if } x < 0 \end{cases}$

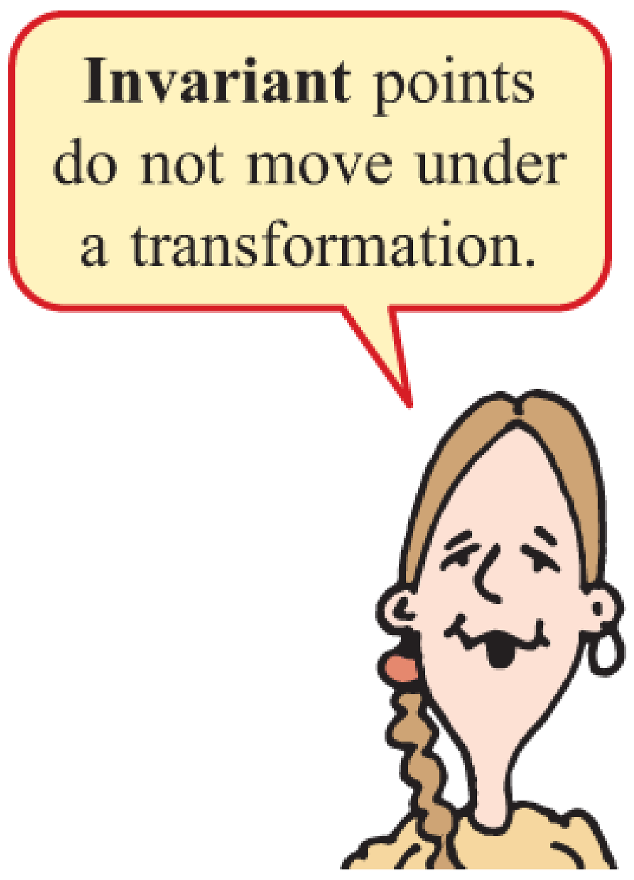
or alternatively $|x| = \sqrt{x^2}$.

The relation $y = |x|$ is in fact a function. We call it the **absolute value function**, and it has the graph shown.



THE GRAPH $y = |\,f(x)\,|$

The absolute value of the function $f(x)$ is $|\,f(x)\,| = \begin{cases} f(x) & \text{if } f(x) \geqslant 0 \\ -f(x) & \text{if } f(x) < 0 \end{cases}$.



To obtain the graph of $y = |\,f(x)\,|$ from the graph of $y = f(x)$:
- keep the graph for $f(x) \geqslant 0$
- reflect the graph in the x-axis for $f(x) < 0$, discarding what was there
- points on the x-axis are invariant.

THE GRAPH $y = f(|\,x\,|)$

We know that $|\,x\,| = \begin{cases} x & \text{if } x \geqslant 0 \\ -x & \text{if } x < 0 \end{cases}$, so $f(|\,x\,|) = \begin{cases} f(x) & \text{if } x \geqslant 0 \\ f(-x) & \text{if } x < 0 \end{cases}$.

To obtain the graph of $y = f(|\,x\,|)$ from the graph of $y = f(x)$:
- discard the graph for $x < 0$
- reflect the graph for $x \geqslant 0$ in the y-axis, keeping what was there
- points on the y-axis are invariant.

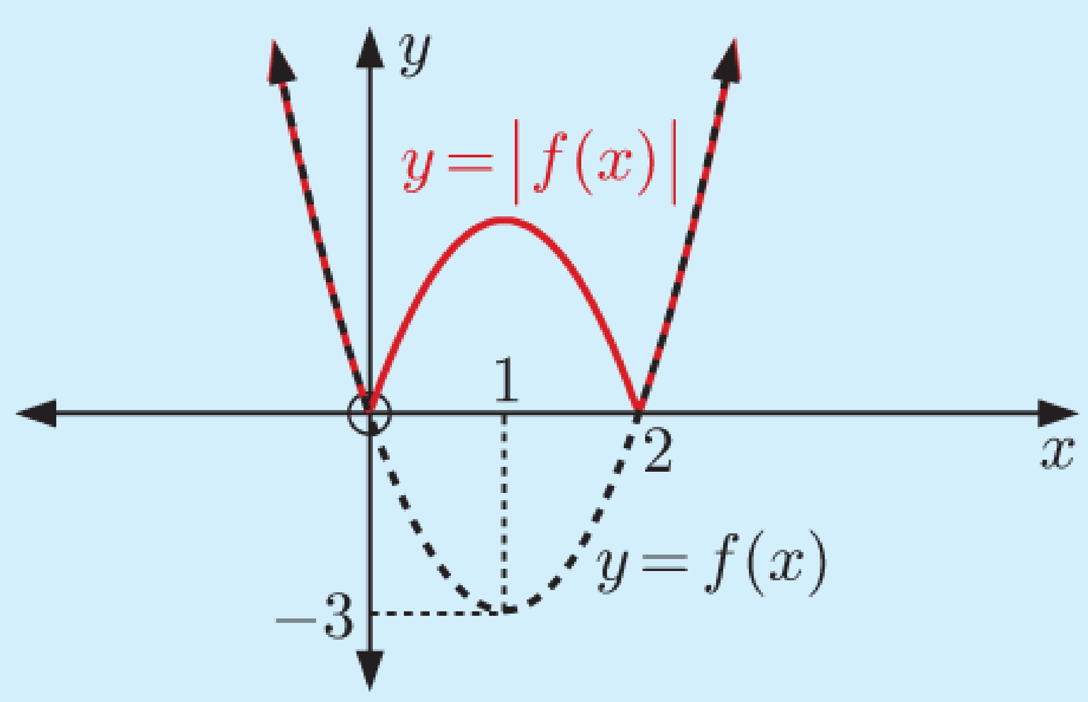
Example 4 — Self Tutor

Draw the graph of $f(x) = 3x(x-2)$, and on the same set of axes draw the graph of:

a $y = |\,f(x)\,|$ **b** $y = f(|\,x\,|)$

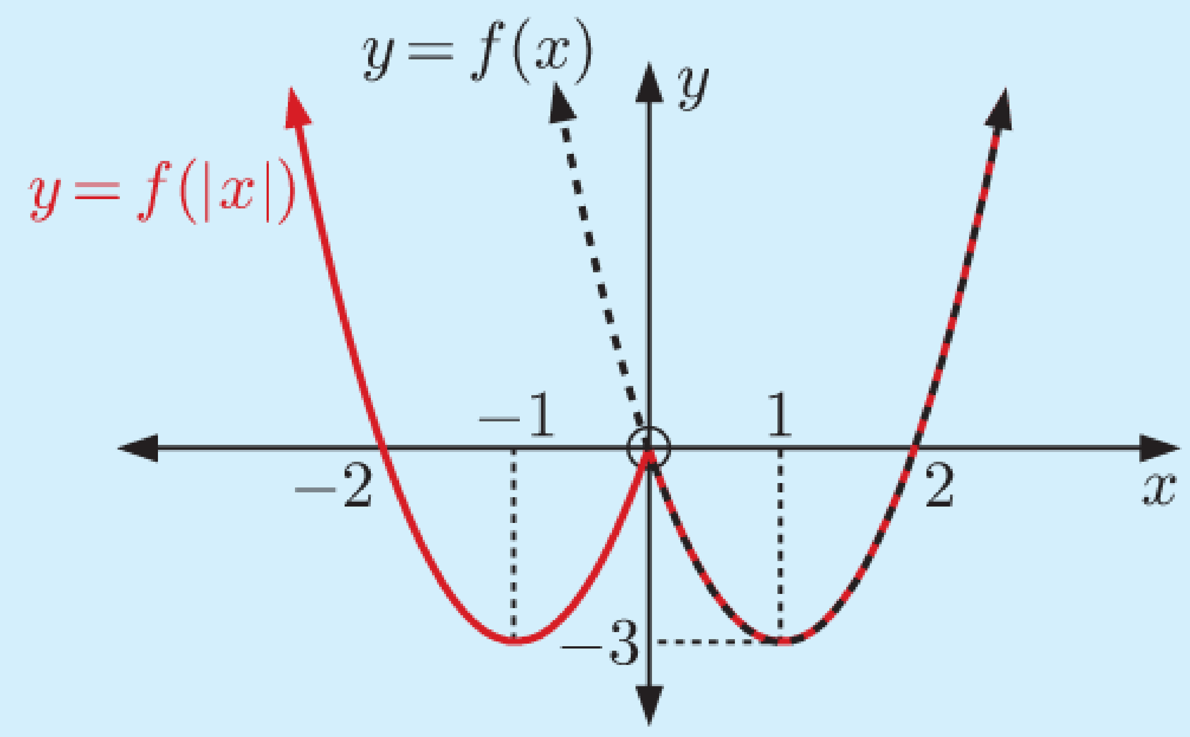
a $y = |\,f(x)\,| = \begin{cases} f(x) & \text{if } f(x) \geqslant 0 \\ -f(x) & \text{if } f(x) < 0 \end{cases}$

The graph is unchanged for $f(x) \geqslant 0$ and reflected in the x-axis for $f(x) < 0$.



b $y = f(|\,x\,|) = \begin{cases} f(x) & \text{if } x \geqslant 0 \\ f(-x) & \text{if } x < 0 \end{cases}$

The graph is unchanged for $x \geqslant 0$ and reflected in the y-axis for $x < 0$.



EXERCISE 6C.1

1 Draw $y = f(x) = 2x - 4$, and on the same set of axes draw the graph of:

a $y = |\,f(x)\,|$ **b** $y = f(|\,x\,|)$

2 Draw $y = f(x) = x(x+2)$, and on the same set of axes draw the graph of:

a $y = |\,f(x)\,|$ **b** $y = f(|\,x\,|)$

3 Draw $y = f(x) = -x^2 + 6x - 8$, and on the same set of axes draw the graph of:

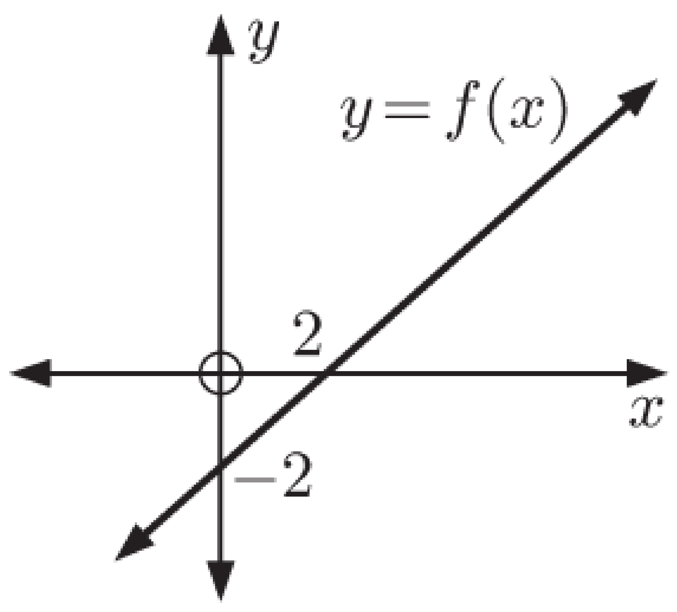
a $y = |\,f(x)\,|$ **b** $y = f(|\,x\,|)$

4 Draw $y = f(x) = 3^x - 3$, and on the same set of axes draw the graph of:

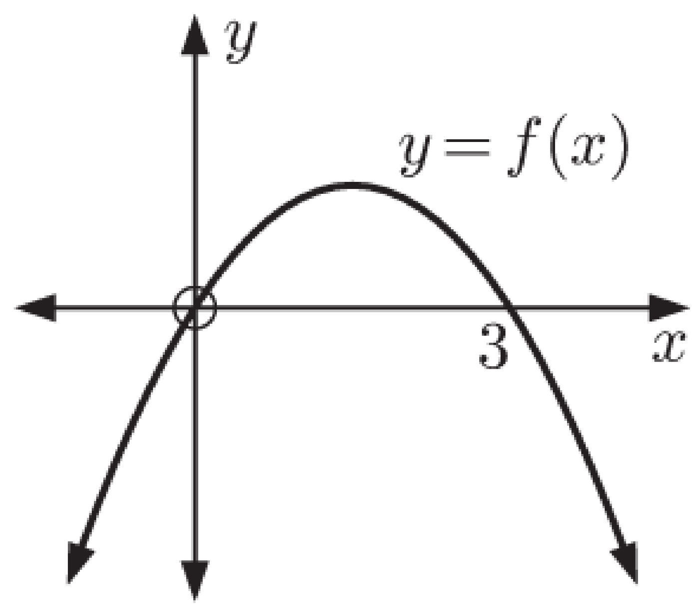
a $y = |\,f(x)\,|$ **b** $y = f(|\,x\,|)$

5 **a** Copy the following graphs for $y = f(x)$ and on the same set of axes draw the graph of $y = |\,f(x)\,|$:

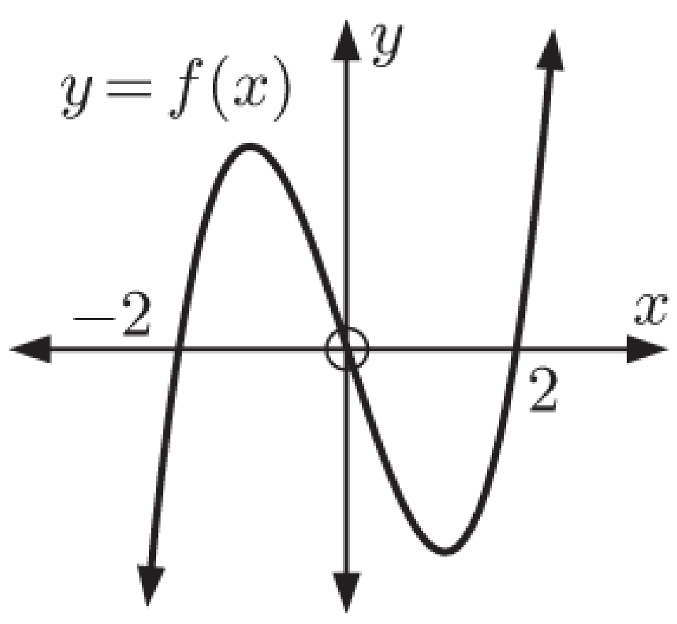
i



ii



iii



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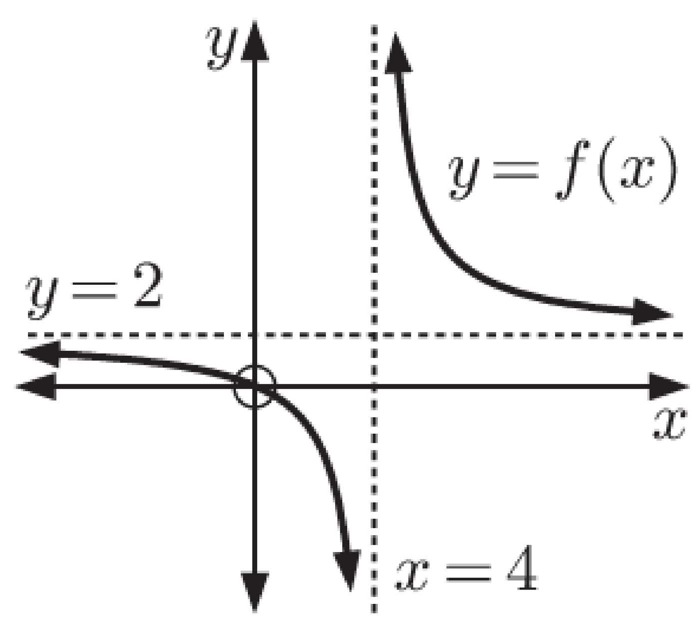


b Repeat **a**, but this time draw the graphs of $y = f(x)$ and $y = f(|\,x\,|)$ on the same set of axes.

6 Show that if $f(x)$ is an odd function, then $|\,f(x)\,|$ is an even function.

7 Let $f(x) = \sqrt{4-x}$.

a Draw the graph of $y = f(x)$ and state its domain and range.

b Draw the graph of $y = f(|\,x\,|)$ and state its domain and range.

8 Suppose $f(x)$ has domain $-2 \leqslant x \leqslant 6$ and range $-7 \leqslant y \leqslant 5$.

a Find:

i the domain of $f(|\,x\,|)$ **ii** the range of $|\,f(x)\,|$.

b Can we determine the range of $f(|\,x\,|)$? Explain your answer.

9 Suppose $f(x)$ has x-intercepts -3 and 4, and y-intercept -2. Find the axes intercepts of:

a $|\,f(x)\,|$ **b** $f(|\,x\,|)$

10 Suppose $f(x) = \log_2(x+4)$. Sketch the graphs of $y = f(x)$ and $y = |\,f(|\,x\,|)\,|$ on the same set of axes.

11 Use the definition $|x| = \sqrt{x^2}$ to prove that:

a $|-x| = |x|$ for all x

b $|x|^2 = x^2$ for all x

c $|xy| = |x||y|$ for all x, y

d $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$ for all x and y, $y \neq 0$

e $|x - y| = |y - x|$ for all x, y

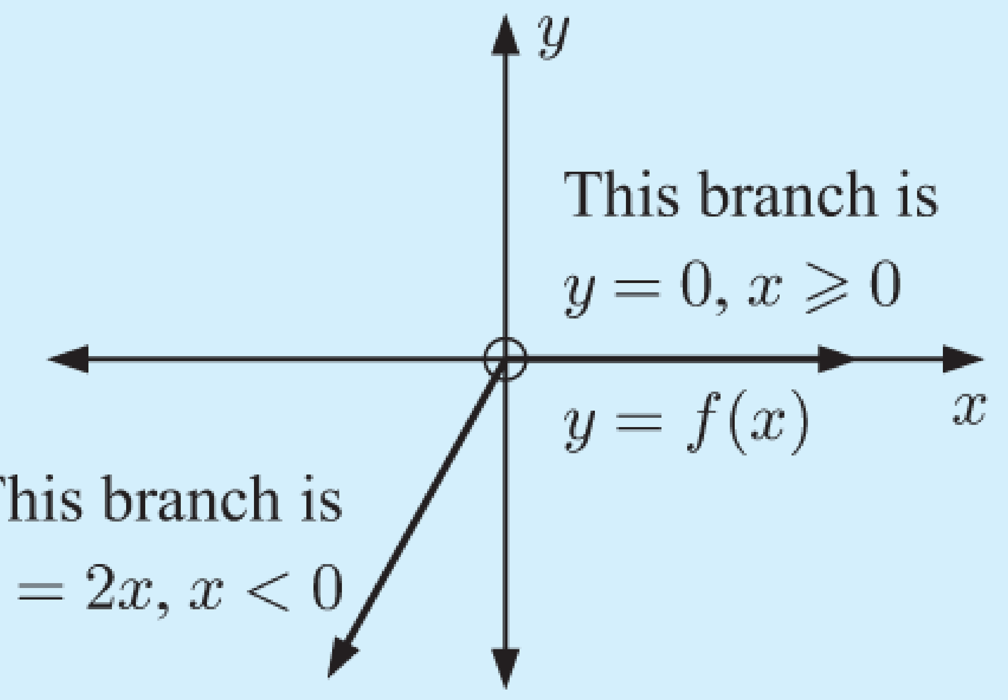
Example 5 **Self Tutor**

Use the definition $|x| = \begin{cases} x & \text{if } x \geqslant 0 \\ -x & \text{if } x < 0 \end{cases}$ to write the following functions without the modulus sign. Hence graph each function:

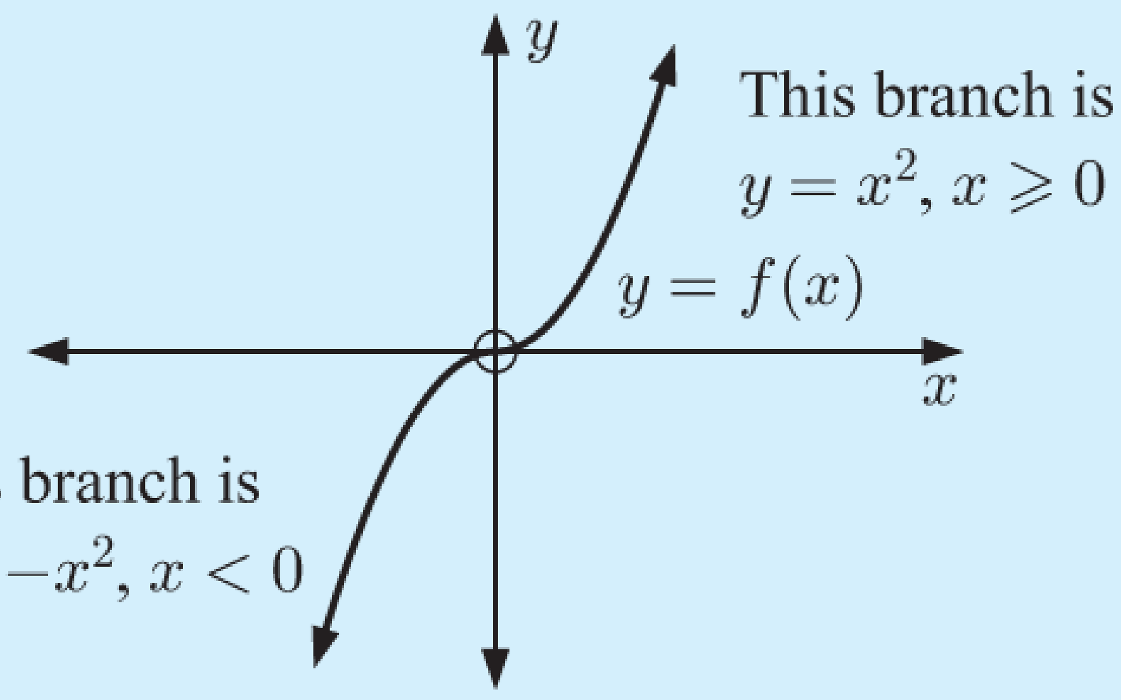
a $f(x) = x - |x|$

b $f(x) = x|x|$

a If $x < 0$, $f(x) = x - (-x) = 2x$.
If $x \geqslant 0$, $f(x) = x - x = 0$.



b If $x \geqslant 0$, $f(x) = x(x) = x^2$.
If $x < 0$, $f(x) = x(-x) = -x^2$.



12 Use $|x| = \begin{cases} x & \text{if } x \geqslant 0 \\ -x & \text{if } x < 0 \end{cases}$ to write the following functions without modulus signs.

Hence graph each function.

a $y = |x - 2|$

b $y = |x + 1|$

c $y = -|x|$

d $y = |x| + x$

e $y = \frac{|x|}{x}$

f $y = x - 2|x|$

g $y = |x| + |x - 2|$

h $y = |x| - |x - 1|$

MODULUS EQUATIONS

A modulus equation is an equation which involves the absolute value function.

To solve modulus equations we use the definitions of absolute value and the properties found in the previous Exercise:

- $|x| \geqslant 0$ for all x
- $|-x| = |x|$ for all x
- $|x|^2 = x^2$ for all x
- $|xy| = |x||y|$ for all x and y
- $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$ for all x and y, $y \neq 0$
- $|x - y| = |y - x|$ for all x and y.

We notice in particular that if $|x| = a, \ a > 0$

$$\begin{aligned} \text{then} \quad |x|^2 &= a^2 \\ \therefore \quad x^2 &= a^2 \qquad \{|x|^2 = x^2\} \\ \therefore \quad x &= \pm\sqrt{a^2} \\ \therefore \quad x &= \pm a \end{aligned}$$

If $|x| = a$ where $a > 0$, then $x = \pm a$.

Example 6 **Self Tutor**

Solve for x:

a $|2x+3| = 7$ **b** $|3-2x| = -1$

a

$$\begin{aligned} |2x+3| &= 7 \\ \therefore \quad 2x+3 &= \pm 7 \\ \therefore \quad 2x+3 = 7 \quad &\text{or} \quad 2x+3 = -7 \\ \therefore \quad 2x = 4 \quad &\text{or} \quad 2x = -10 \\ \therefore \quad x = 2 \quad &\text{or} \quad x = -5 \end{aligned}$$

So, $x = 2$ or -5.

b $|3-2x| = -1$

has no solution as LHS is never negative.

Example 7 **Self Tutor**

Solve for x:

a $\left|\dfrac{3x+2}{1-x}\right| = 4$ **b** $|x+1| = |2x-3|$

a If $\left|\dfrac{3x+2}{1-x}\right| = 4$ then $\dfrac{3x+2}{1-x} = \pm 4$.

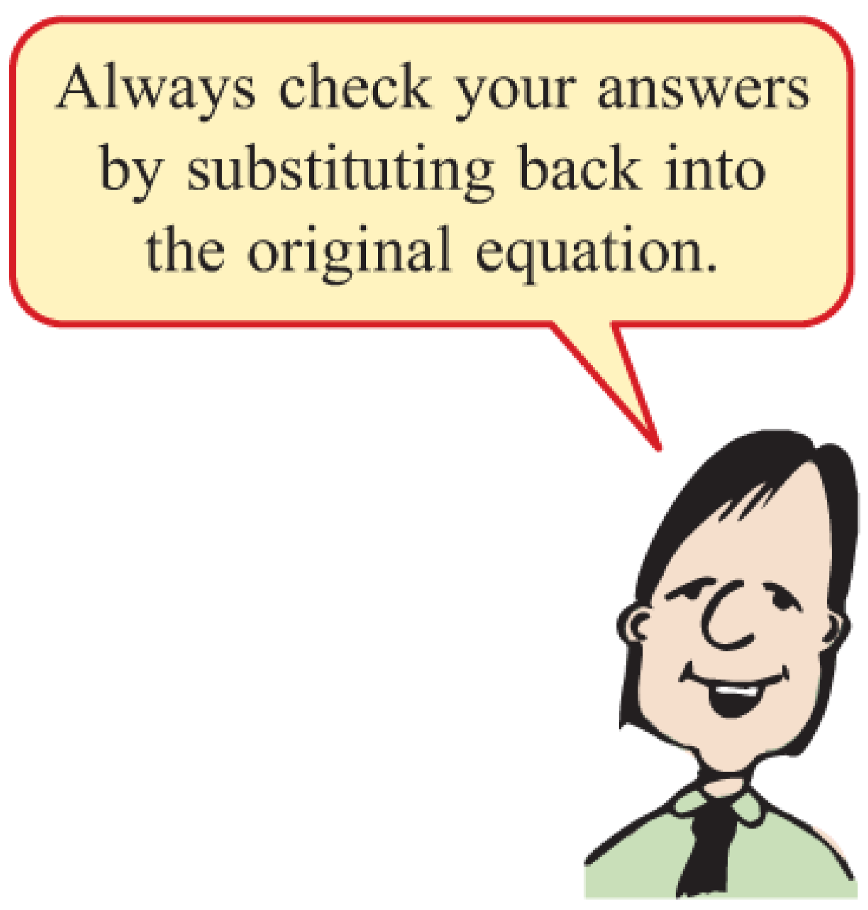
$$\begin{aligned} \therefore \quad \frac{3x+2}{1-x} = 4 \quad &\text{or} \quad \frac{3x+2}{1-x} = -4 \\ \therefore \quad 3x+2 = 4(1-x) \quad &\text{or} \quad 3x+2 = -4(1-x) \\ \therefore \quad 3x+2 = 4-4x \quad &\text{or} \quad 3x+2 = -4+4x \\ \therefore \quad 7x = 2 \quad &\text{or} \quad 6 = x \\ \therefore \quad x = \tfrac{2}{7} \quad &\text{or} \quad x = 6 \end{aligned}$$

So, $x = \frac{2}{7}$ or 6.

b If $|x+1| = |2x-3|$, then $x+1 = \pm(2x-3)$

$$\begin{aligned} \therefore \quad x+1 = 2x-3 \quad &\text{or} \quad x+1 = -(2x-3) \\ \therefore \quad 4 = x \quad &\text{or} \quad x+1 = -2x+3 \\ \therefore \quad x = 4 \quad &\text{or} \quad 3x = 2 \\ \therefore \quad x = 4 \quad &\text{or} \quad x = \tfrac{2}{3} \end{aligned}$$

So, $x = \frac{2}{3}$ or 4.



EXERCISE 6C.2

1 Solve for x:

a $|x| = 3$ **b** $|x| = -5$ **c** $|x| = 0$

d $|x - 1| = 3$ **e** $|3 - x| = 4$ **f** $|x + 5| = -1$

g $|3x - 2| = 1$ **h** $|3 - 2x| = 3$ **i** $|2 - 5x| = 12$

2 Solve for x:

a $\left|\frac{x}{x-1}\right| = 3$ **b** $\left|\frac{2x-1}{x+1}\right| = 5$ **c** $\left|\frac{x+3}{1-3x}\right| = \frac{1}{2}$

3 **a** Explain why the equation $\left|\frac{3x+1}{x-1}\right| = 3$ has only one solution.

b Find the solution.

4 Prove that if $|x| = |a|$ then $x = \pm a$.

5 Solve for x:

a $|3x - 1| = |x + 2|$ **b** $|2x + 5| = |1 - x|$ **c** $|x + 1| = |2 - x|$

d $|x| = |5 - x|$ **e** $|1 - 4x| = 2|x - 1|$ **f** $|3x + 2| = 2|2 - x|$

Example 8 Self Tutor

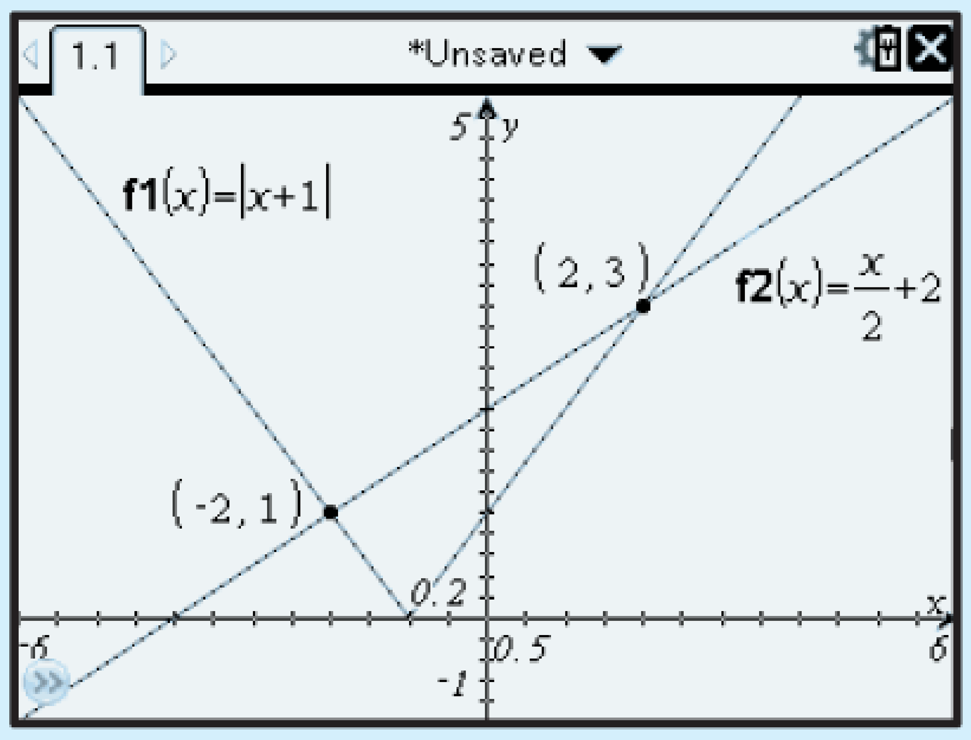
Solve graphically: $|x + 1| = \frac{x}{2} + 2$

We graph $y = |x + 1|$ and $y = \frac{x}{2} + 2$ on the same set of axes.

The graphs intersect at $(-2, 1)$ and $(2, 3)$

$\therefore$ the solutions are $x = -2$ or 2.

GRAPHICS CALCULATOR INSTRUCTIONS



6 **a** Sketch the graphs of $y = |x + 3|$ and $y = 7 - \frac{x}{3}$ on the same set of axes.

b Solve the equation $|x + 3| = 7 - \frac{x}{3}$.

7 Solve graphically:

a $|x + 2| = 2x + 1$ **b** $|2x + 3| = 3|x| - 1$ **c** $|x - 2| = \frac{2}{5}x + 1$

d $x^2 - 1 = |5x - x^2|$ **e** $|e^x - 6| = 2^x$ **f** $x^3 - 4x = \ln|x|$

MODULUS INEQUALITIES

There are several methods we can use to solve modulus inequalities, depending on the situation:

- For some modulus inequalities it is easiest to separate the inequality into two.

Example 9 **Self Tutor**

Solve for x:

a $|2x-1| < 3$

b $|4-x| \geqslant 5$

a
$$|2x-1| < 3$$
$$\therefore\ -3 < 2x-1 < 3$$
$$\therefore\ -2 < 2x < 4$$
$$\therefore\ -1 < x < 2$$

b
$$|4-x| \geqslant 5$$
$$\therefore\ 4-x \leqslant -5 \quad \text{or} \quad 4-x \geqslant 5$$
$$\therefore\ x \geqslant 9 \quad \text{or} \quad x \leqslant -1$$

- For modulus inequalities where we know each side of the inequality is positive, we can square both sides.

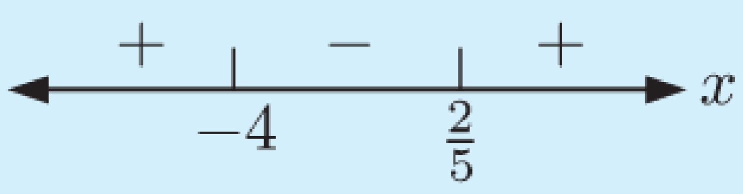
Example 10 **Self Tutor**

Solve for x:

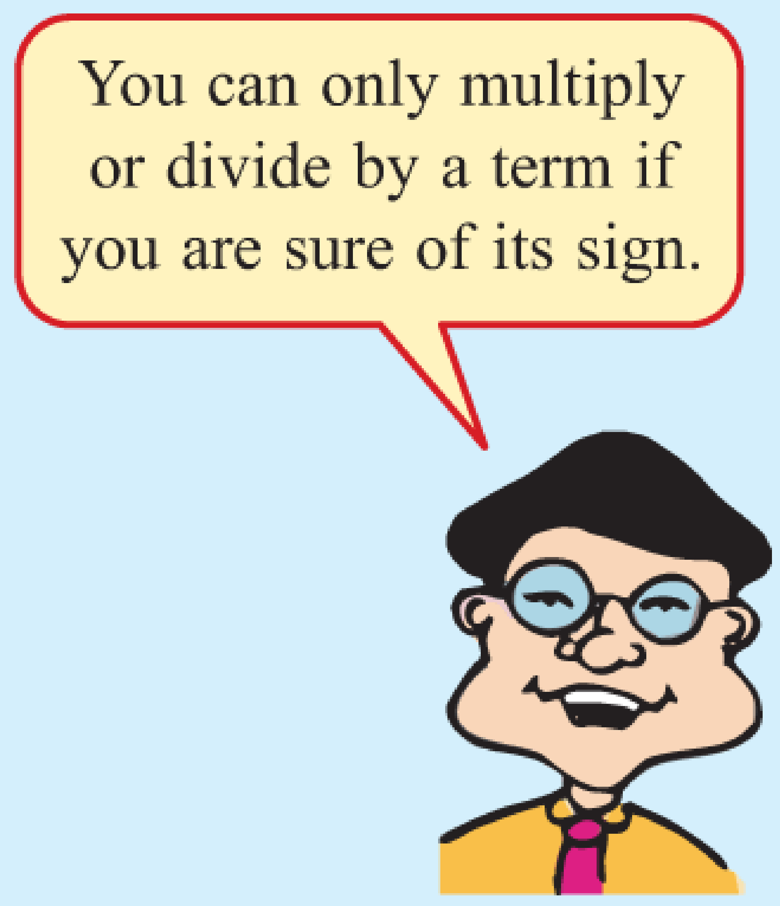
a $|3x+1| \geqslant |2x-3|$

b $\left|\dfrac{x}{x+1}\right| \geqslant \dfrac{1}{2}$

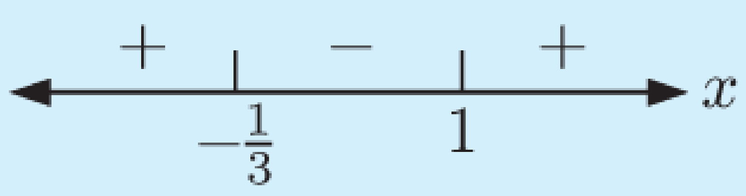
a
$$|3x+1| \geqslant |2x-3|$$
$$\therefore\ |3x+1|^2 \geqslant |2x-3|^2 \qquad \{\text{squaring both sides}\}$$
$$\therefore\ (3x+1)^2 \geqslant (2x-3)^2 \qquad \{|a|^2 = a^2\}$$
$$\therefore\ 9x^2+6x+1 \geqslant 4x^2-12x+9$$
$$\therefore\ 5x^2+18x-8 \geqslant 0$$
$$\therefore\ (5x-2)(x+4) \geqslant 0$$



$$\therefore\ x \leqslant -4 \quad \text{or} \quad x \geqslant \tfrac{2}{5}$$



b
$$\left|\frac{x}{x+1}\right| \geqslant \frac{1}{2}$$
$$\therefore\ \left|\frac{x}{x+1}\right|^2 \geqslant \left(\frac{1}{2}\right)^2 \qquad \{\text{squaring both sides}\}$$
$$\therefore\ \left(\frac{x}{x+1}\right)^2 \geqslant \frac{1}{4} \qquad \{|a|^2 = a^2\}$$
$$\therefore\ \frac{x^2}{(x+1)^2} \geqslant \frac{1}{4}$$
$$\therefore\ 4x^2 \geqslant (x+1)^2 \quad \text{provided } x \neq -1 \quad \{\text{since } (x+1)^2 \geqslant 0\}$$
$$\therefore\ 4x^2 \geqslant x^2+2x+1 \quad \text{provided } x \neq -1$$
$$\therefore\ 3x^2-2x-1 \geqslant 0 \quad \text{provided } x \neq -1$$
$$\therefore\ (3x+1)(x-1) \geqslant 0 \quad \text{provided } x \neq -1$$
$$\therefore\ x \leqslant -\tfrac{1}{3},\ x \neq -1 \quad \text{or} \quad x \geqslant 1$$



EXERCISE 6C.3

1 Solve for x:

a $|\,x-1\,| \leqslant 4$ **b** $|\,x+2\,| > 7$ **c** $|\,2x-3\,| < 1$

d $|\,3x+5\,| \geqslant 2$ **e** $|\,1-2x\,| < 4$ **f** $|\,-5x-4\,| > \frac{1}{2}$

2 Solve for x:

a $|\,x+5\,| < |\,x-1\,|$ **b** $|\,2x+1\,| > |\,x+2\,|$ **c** $|\,x-3\,| \leqslant |\,2x-6\,|$

d $|\,3x+4\,| \geqslant |\,x+3\,|$ **e** $2\,|\,x-4\,| < |\,3x+2\,|$ **f** $\frac{1}{3}\,|\,2x+5\,| \geqslant |\,4-x\,|$

3 Solve for x:

a $\left|\dfrac{x}{x-2}\right| \geqslant 3$ **b** $\left|\dfrac{2x+3}{x-1}\right| \geqslant 2$ **c** $\left|\dfrac{x-4}{1-2x}\right| < \dfrac{2}{3}$

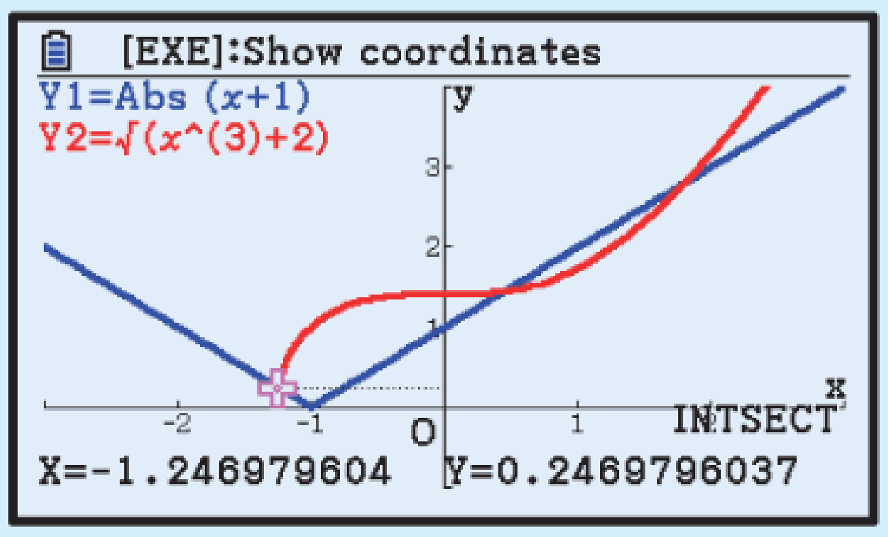
Example 11

Self Tutor

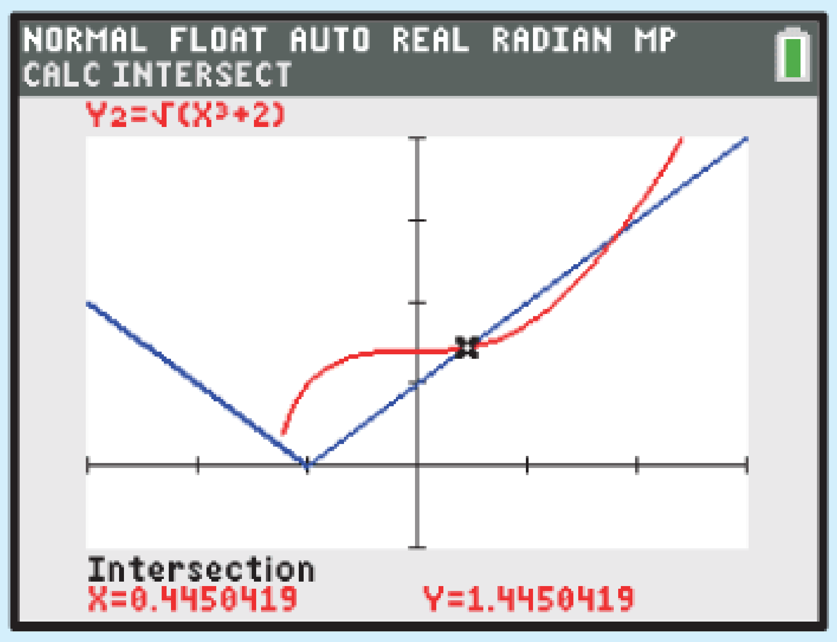
Solve graphically: $|\,x+1\,| > \sqrt{x^3+2}$.

We draw graphs of $y = |\,x+1\,|$ and $y = \sqrt{x^3+2}$ on the same set of axes.

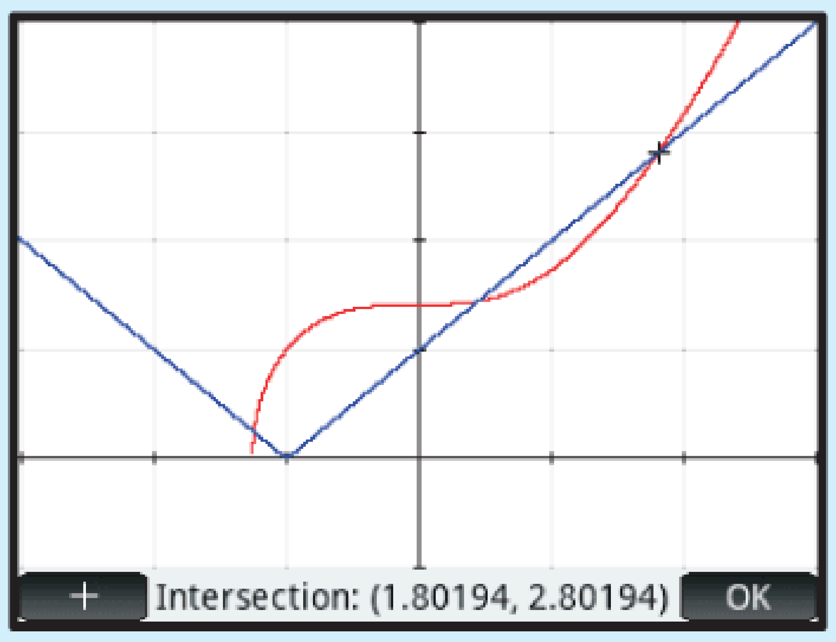
Casio fx-CG50



TI-84 Plus CE

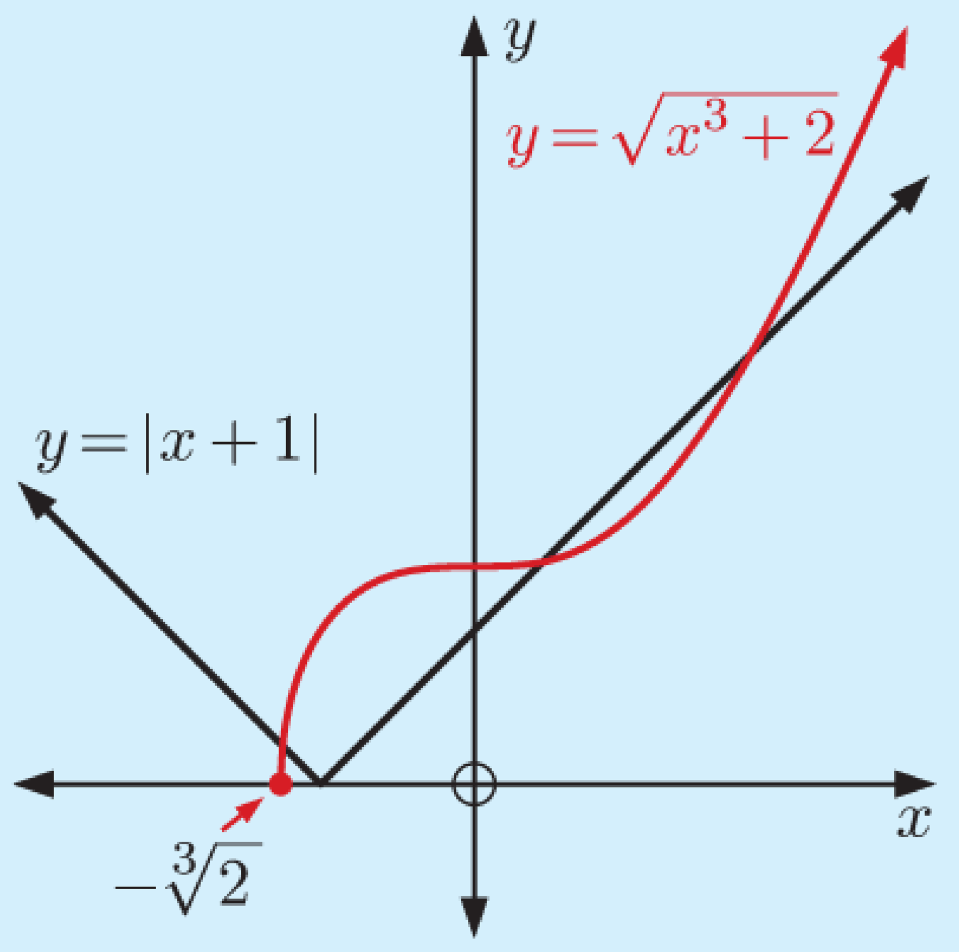


HP Prime

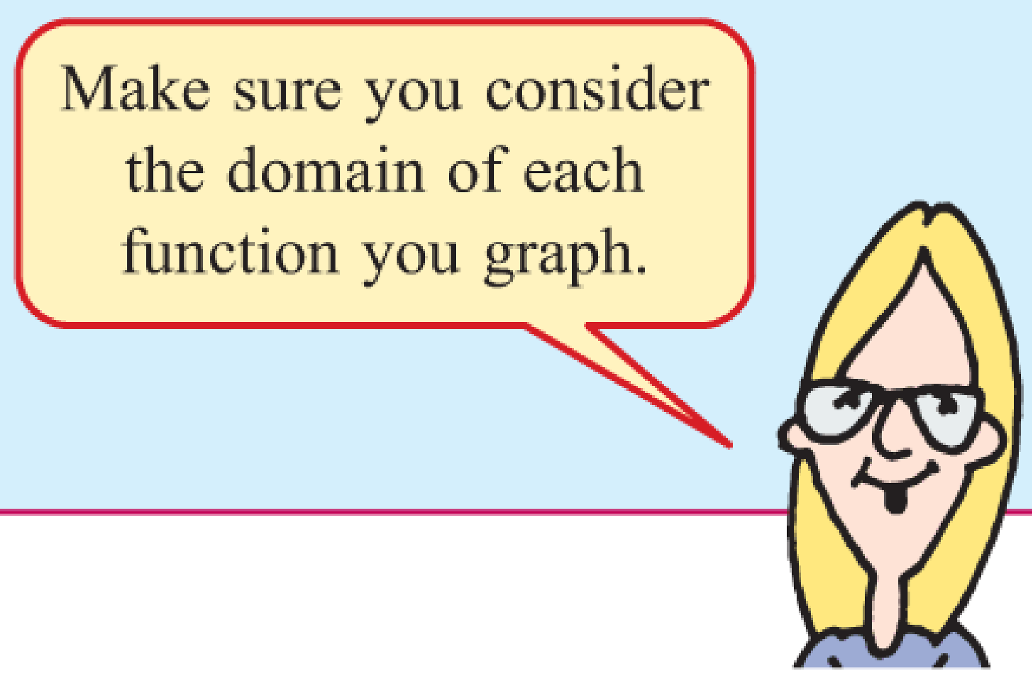


The graphs intersect at $x \approx -1.2470$, $x \approx 0.4450$, and $x \approx 1.8019$.

GRAPHICS CALCULATOR INSTRUCTIONS



$\therefore$ $|\,x+1\,| > \sqrt{x^3+2}$ when $-\sqrt[3]{2} \leqslant x < -1.247$ and when $0.445 < x < 1.802$.



4 Solve graphically:

a $|\,2x-3\,| < x$ **b** $|\,x\,| - 2 \geqslant |\,4-x\,|$

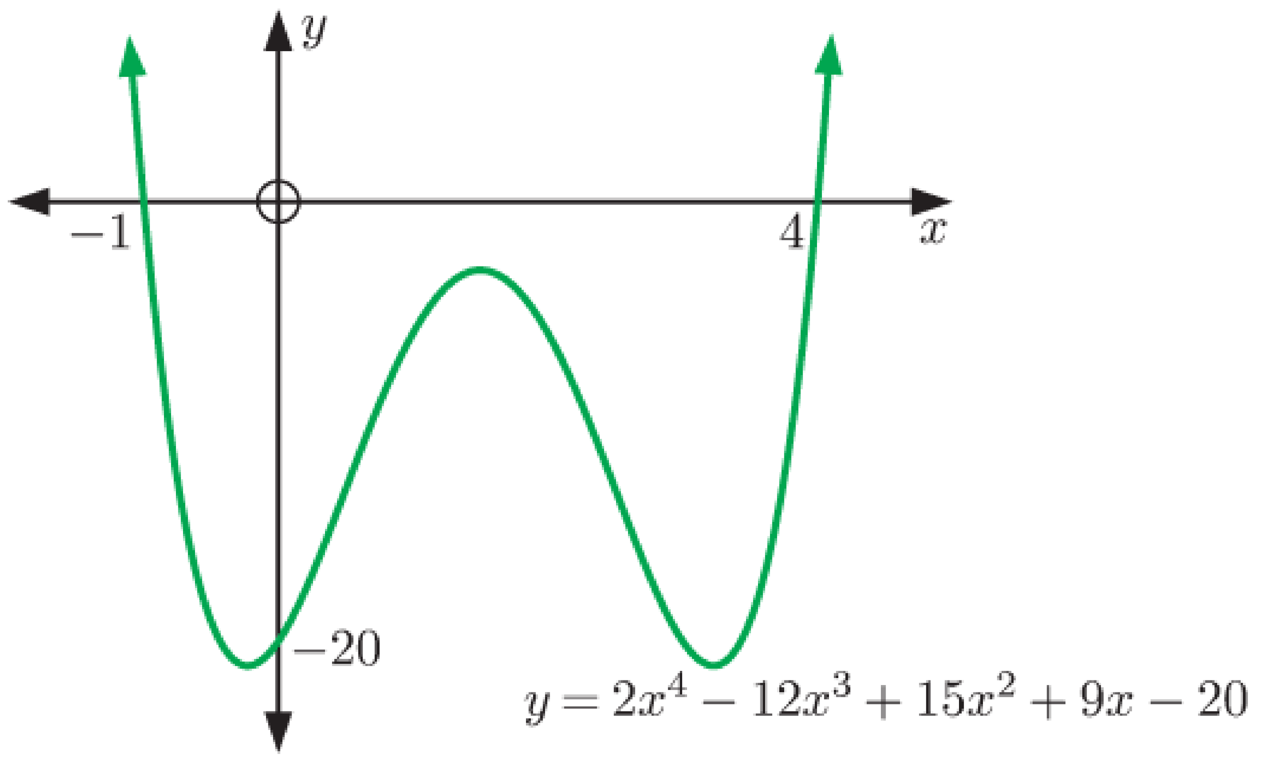
c $x^2 - 4 > |\,x-1\,|$ **d** $|\,2x-1\,| + |\,x-4\,| \leqslant 10 - x$

e $\ln\left|\,x^2-x+5\,\right| \geqslant 0.1x^4$ **f** $7\,|\,x\,| - e^{|\,x\,|} \geqslant \log_2(x^2-x-2)$

g $|\,3x\arccos x\,| > 1$ **h** $0.2\ln(7-2x) < \left|\,\sqrt{15-x^2} - 3\,\right|$

27 $(x+1)(x-4)(2x^2-6x+5)$

$2x^2-6x+5$ has $\Delta < 0$, so $2x^2-6x+5$ has no real zeros.



28 **a** $-\frac{1}{2}$ and $\pm i\sqrt{5}$ **b** -7, -1, and 2

29 **a** $x \leqslant -1.10$ or $0.854 \leqslant x \leqslant 4.25$ **b** $x > 2.33$

EXERCISE 6A

4 **a** odd **b** neither **c** even **d** odd **e** even **f** neither

5 $(-1, 3)$, $(5, -2)$ **6** $(-4, -6)$, $(1, -2)$ **7** $a = -\frac{3}{2}$

8 $b = -1$ **9** **a** $b = 0$, $d = 0$ **c** $b = 0$, $d = 0$

10 **a** Even, the graph of the function is symmetric about the y-axis.

b Odd, the graph of the function has rotational symmetry about the origin.

c Odd, the graph has rotational symmetry about the origin.

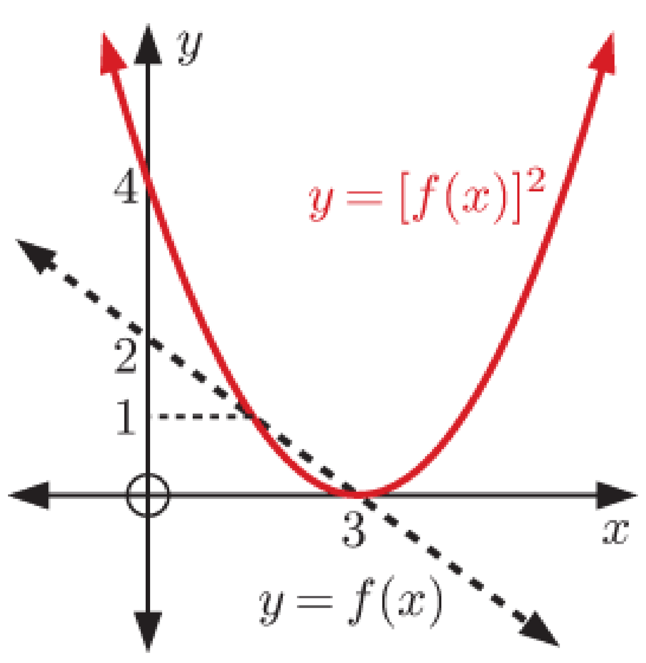
11 **a** $k = n\pi$, $n \in \mathbb{Z}$ **b** $k = \frac{\pi}{2} + n\pi$, $n \in \mathbb{Z}$

c $k \neq \frac{n\pi}{2}$, $n \in \mathbb{Z}$

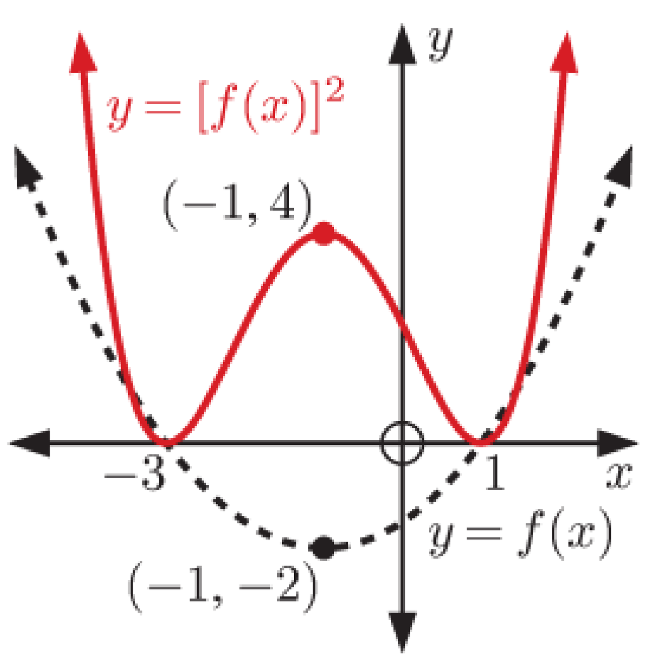
13 even **14** odd

EXERCISE 6B

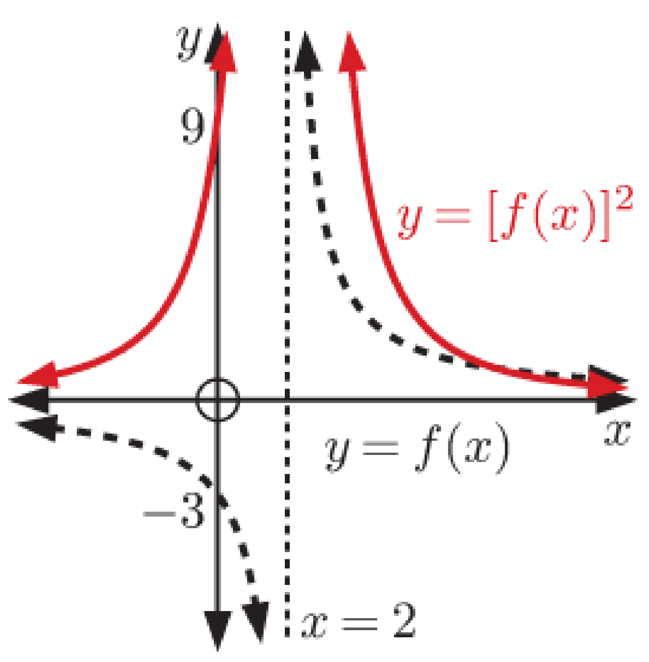
1 **a**



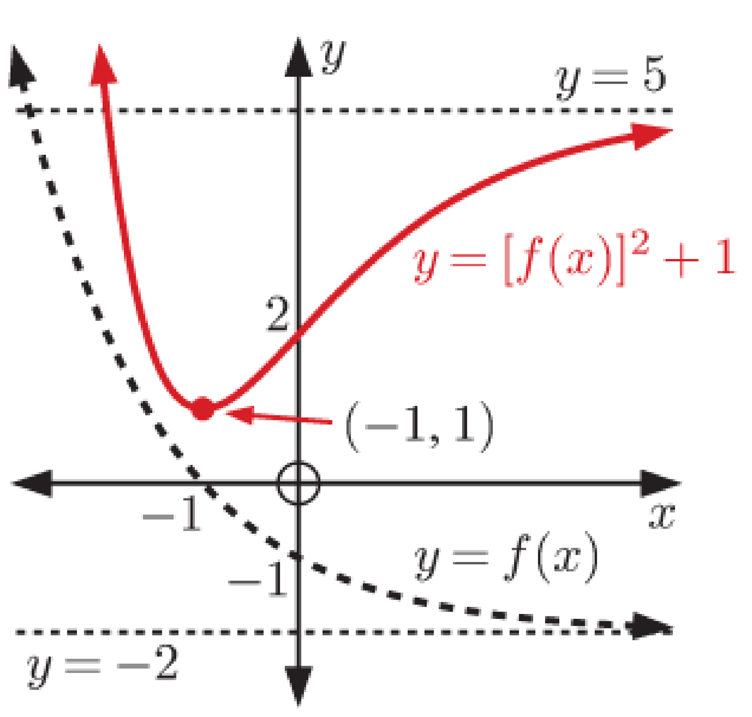
b



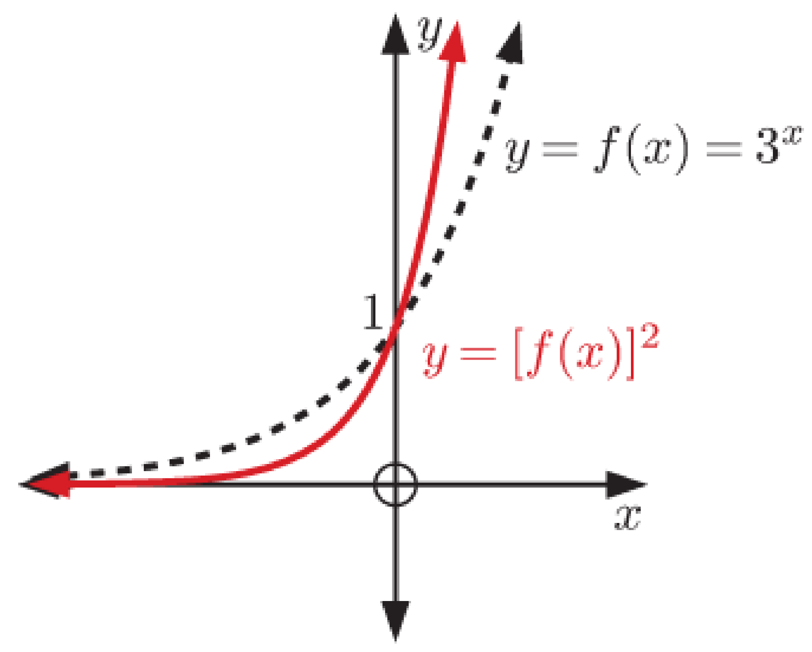
c



2

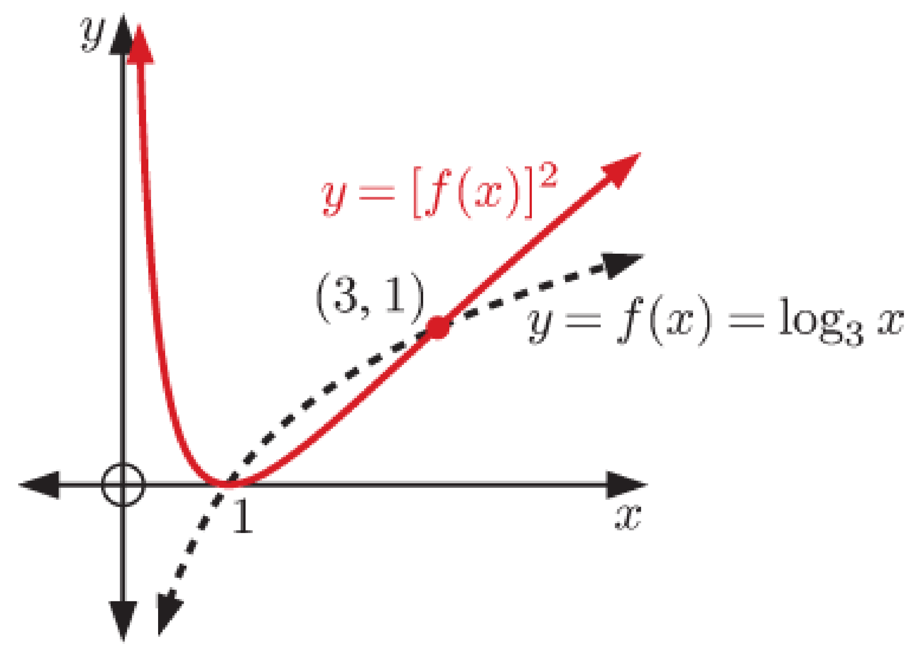


3 **a, b**



c A horizontal stretch with scale factor $\frac{1}{2}$.

4 **a, b**



c $(1, 0)$ and $(3, 1)$

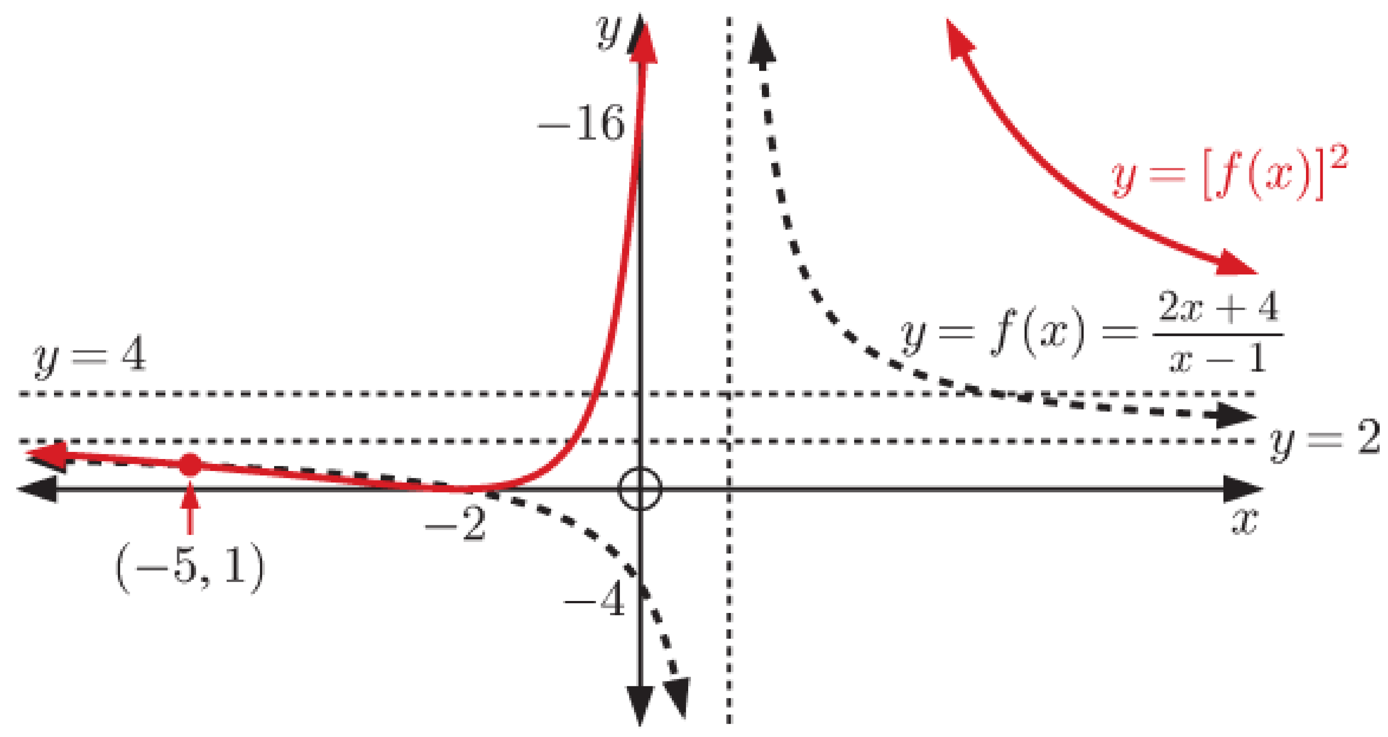
6 Domain is $\{x \mid 0 \leqslant x \leqslant 5\}$, Range is $\{y \mid 0 \leqslant y \leqslant 16\}$

7 **a** x-int. -2, y-int. -4, VA $x = 1$, HA $y = 2$

b x-int. -2, y-int. 16, VA $x = 1$, HA $y = 4$

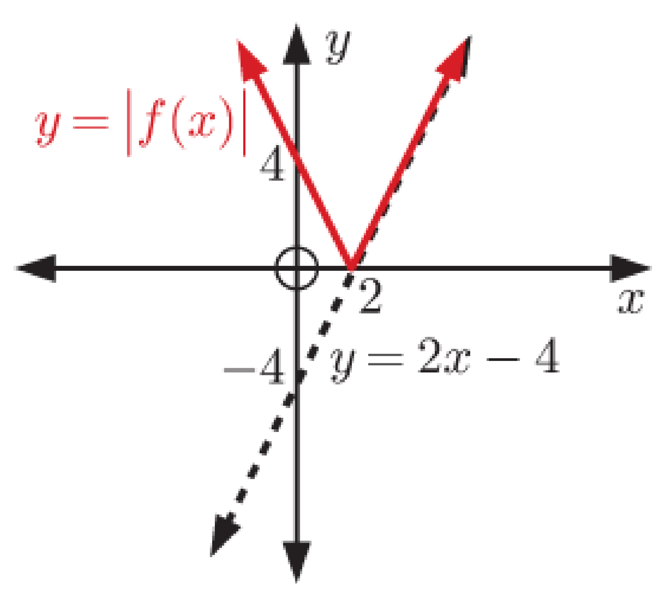
c $(-5, 1)$ and $(-2, 0)$

d

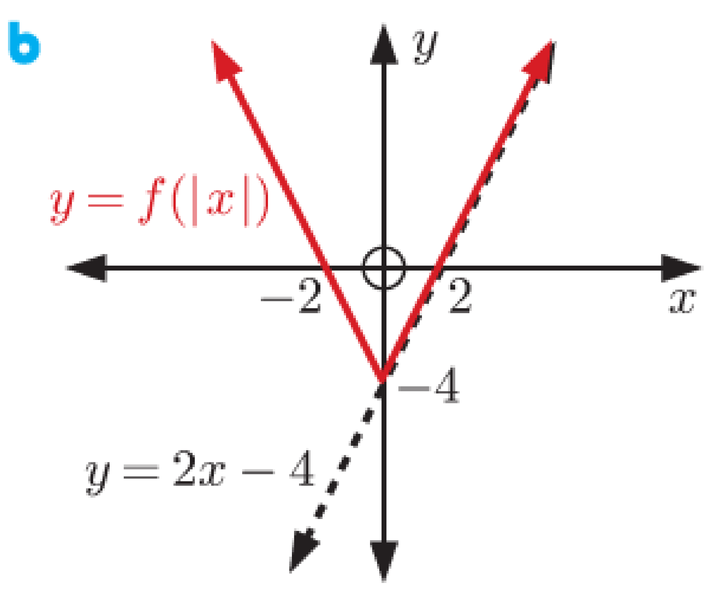


EXERCISE 6C.1

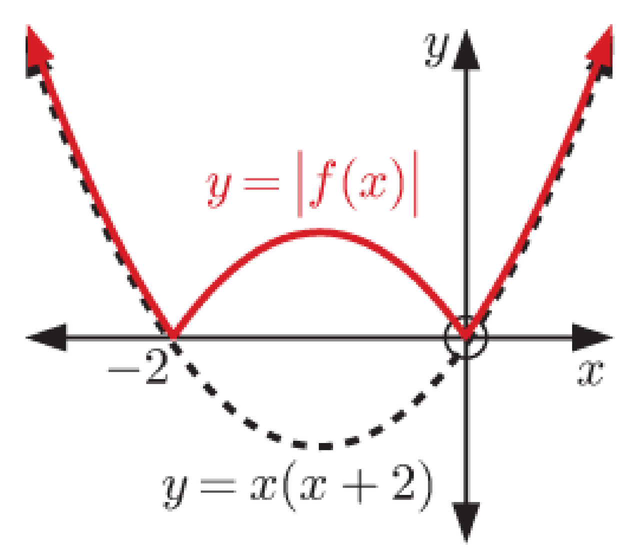
1 **a**



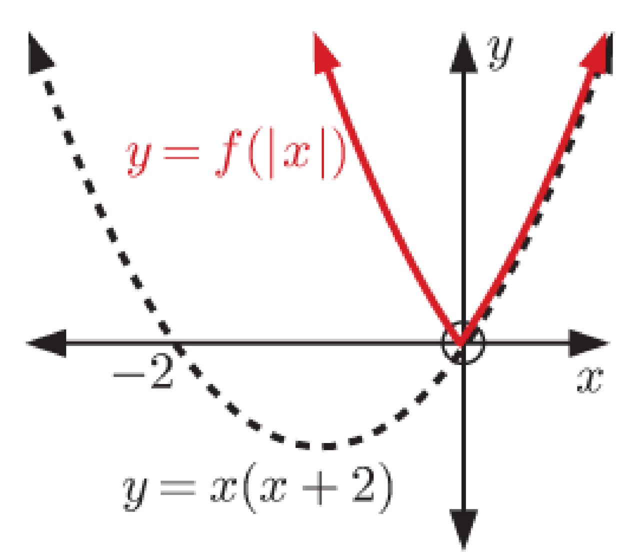
b



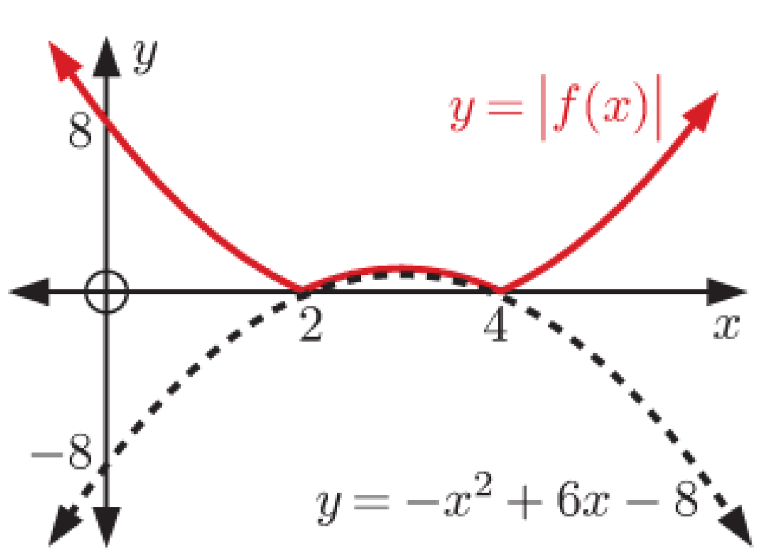
2 **a**



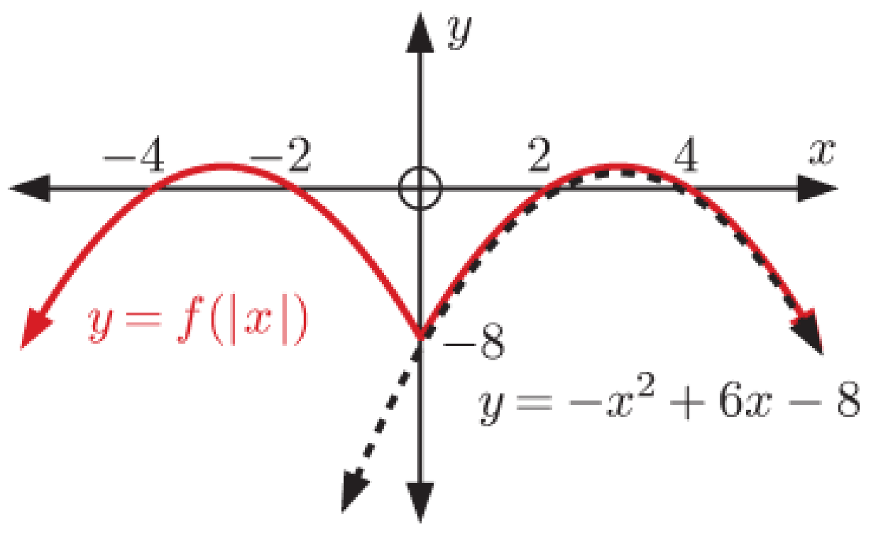
b



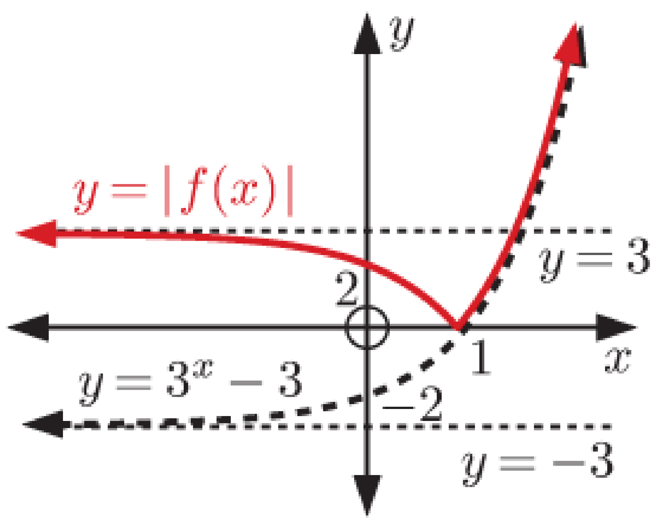
3 **a**



b



4 **a**



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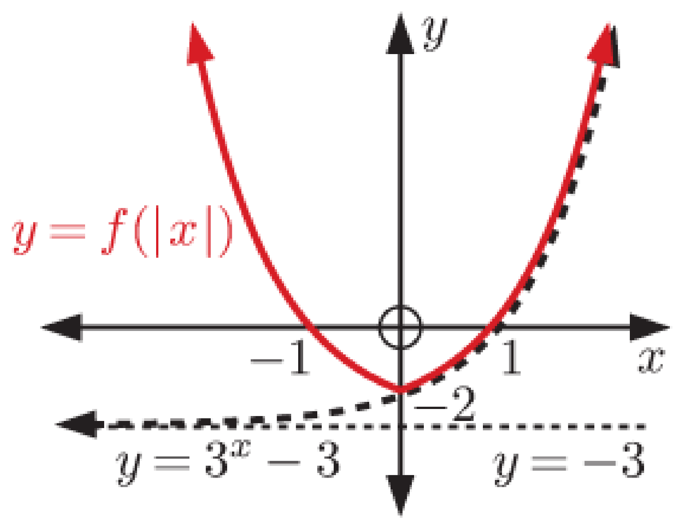
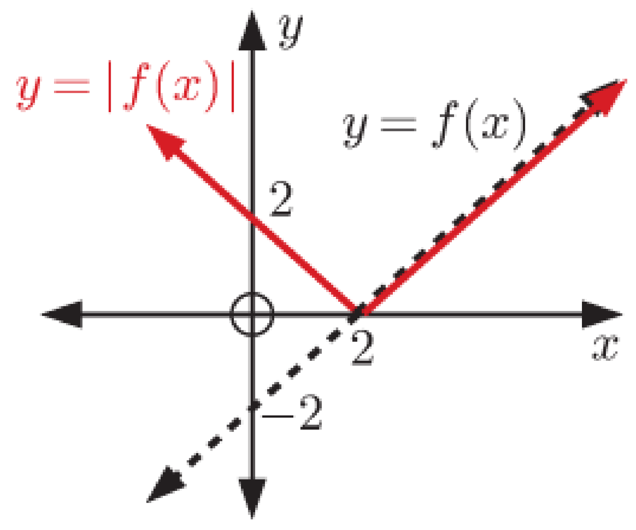
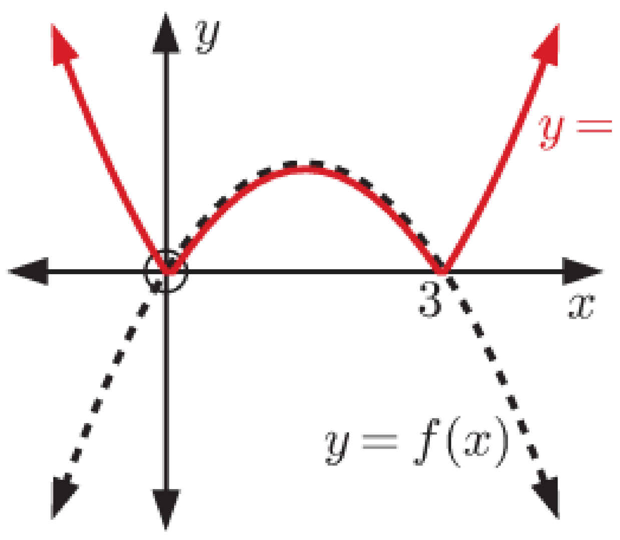


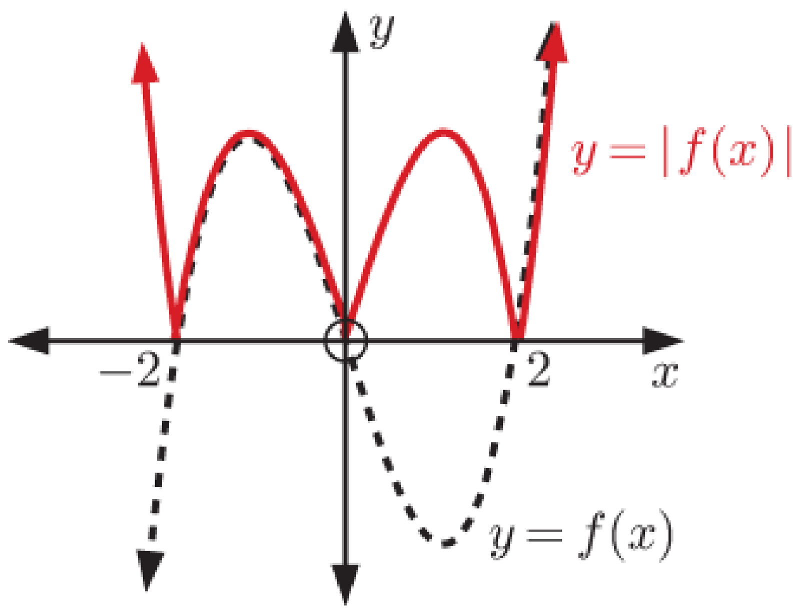
5 **a** **i**



ii



iii



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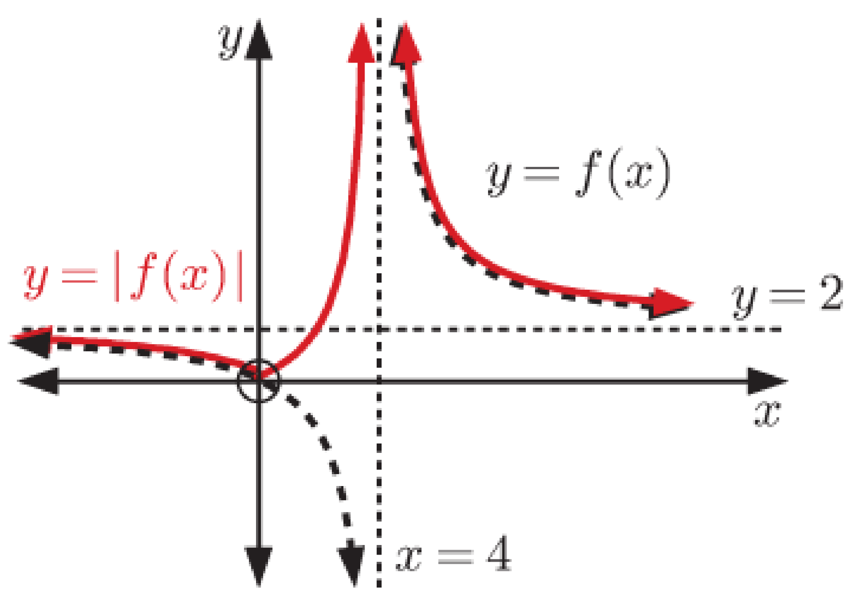
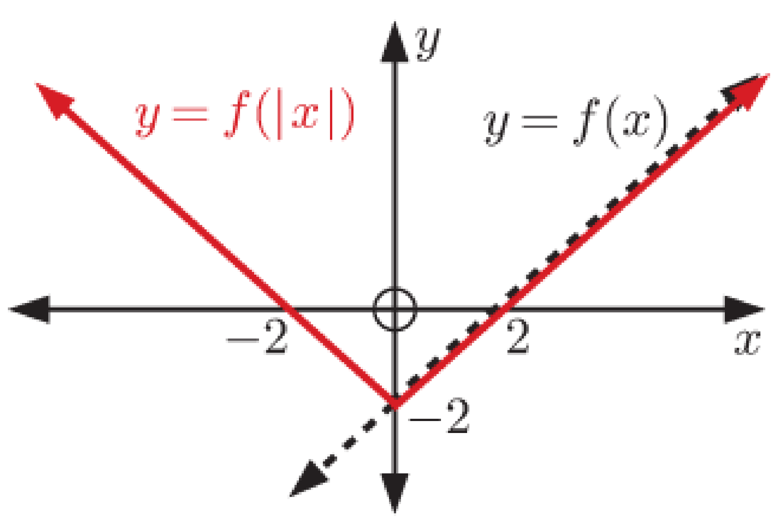
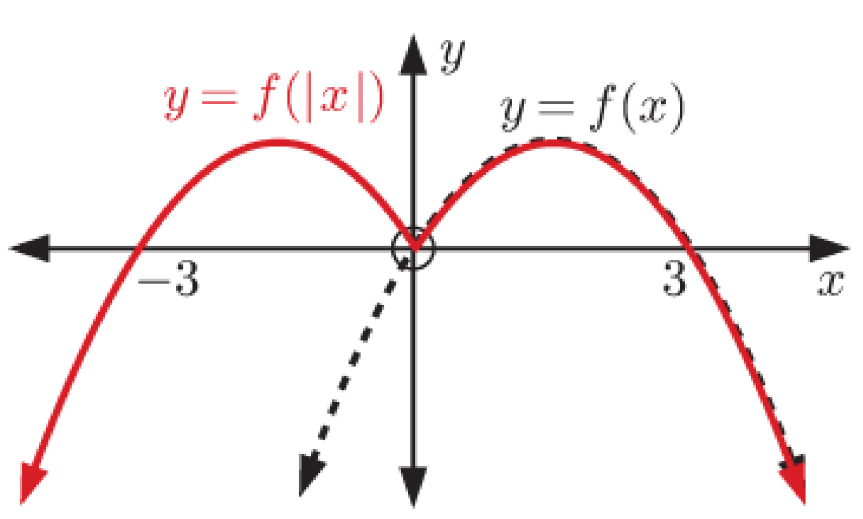


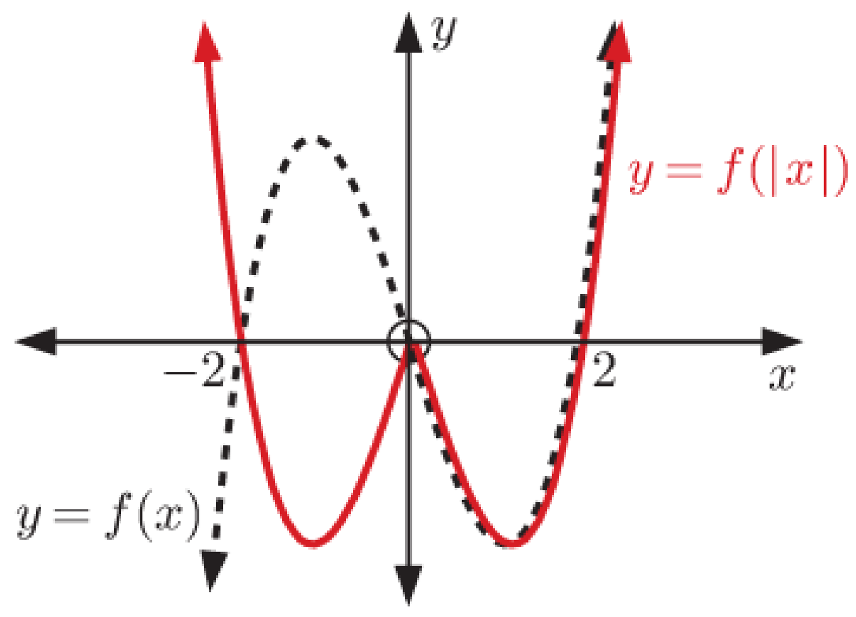
b **i**



ii



iii



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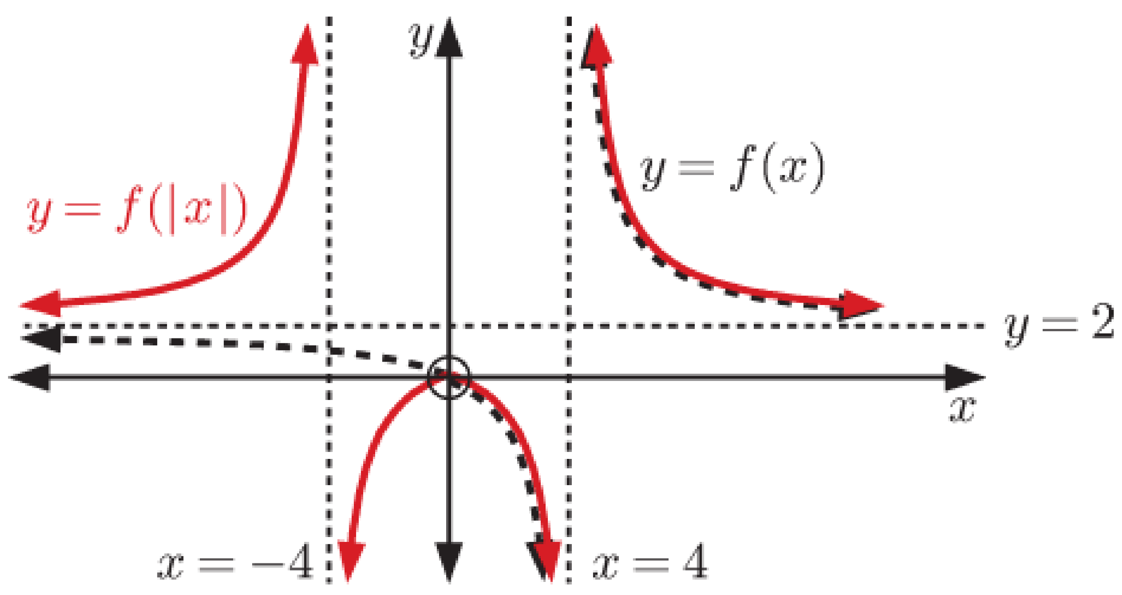
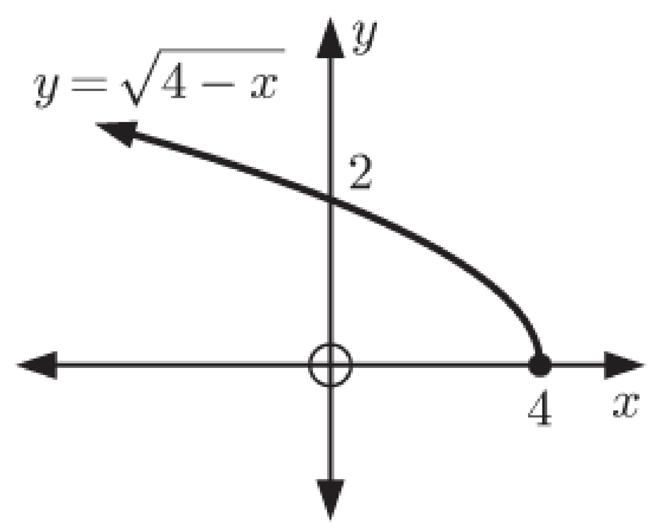
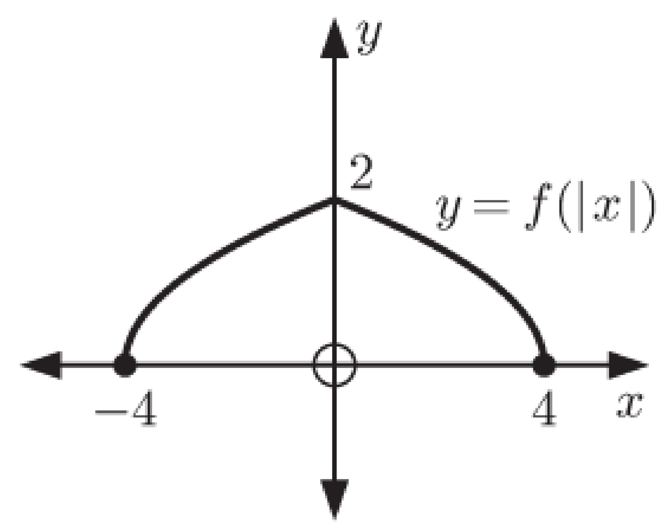


7 **a**



Domain is $\{x \mid x \leqslant 4\}$,
Range is $\{y \mid y \geqslant 0\}$

b



Domain is
$\{x \mid -4 \leqslant x \leqslant 4\}$,
Range is $\{y \mid 0 \leqslant y \leqslant 2\}$

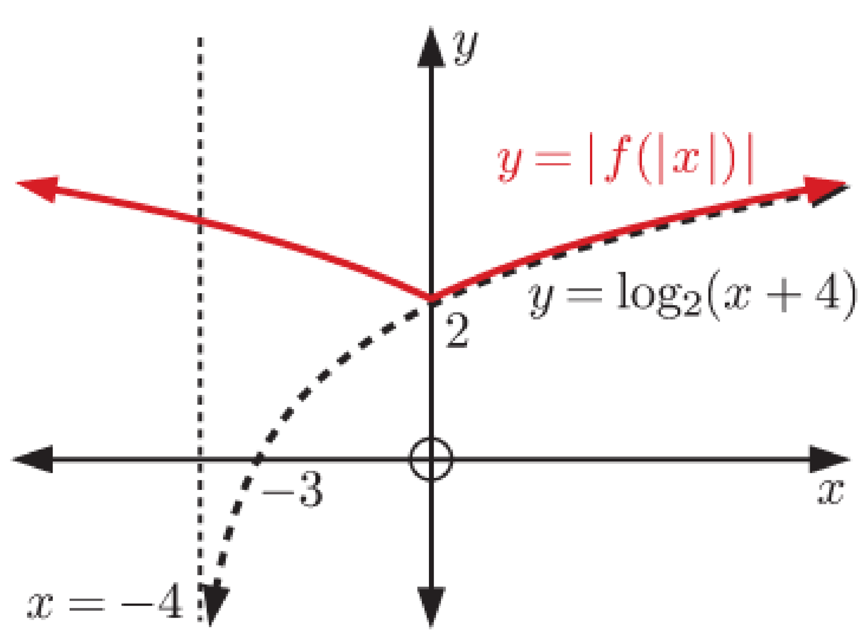
8 **a** **i** $\{x \mid -6 \leqslant x \leqslant 6\}$ **ii** $\{y \mid 0 \leqslant y \leqslant 7\}$

b No, we do not know the behaviour of $f(x)$ on $-2 \leqslant x < 0$, which is discarded when we find $f(|x|)$. This may or may not affect the range of $y = f(|x|)$.

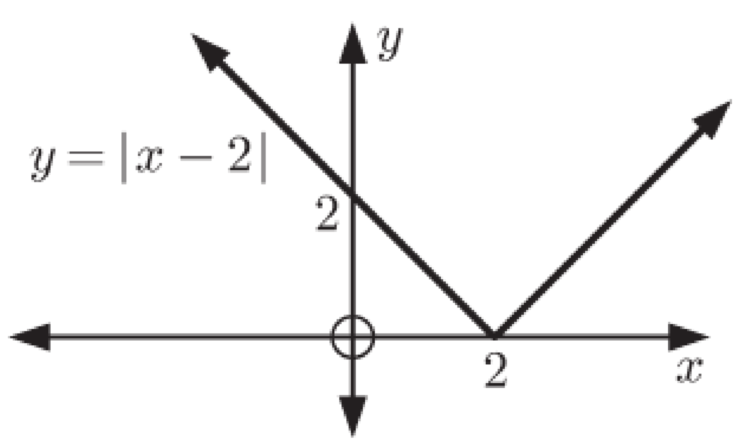
9 **a** x-intercepts -3 and 4, y-intercept 2

b x-intercepts 4 and -4, y-intercept -2

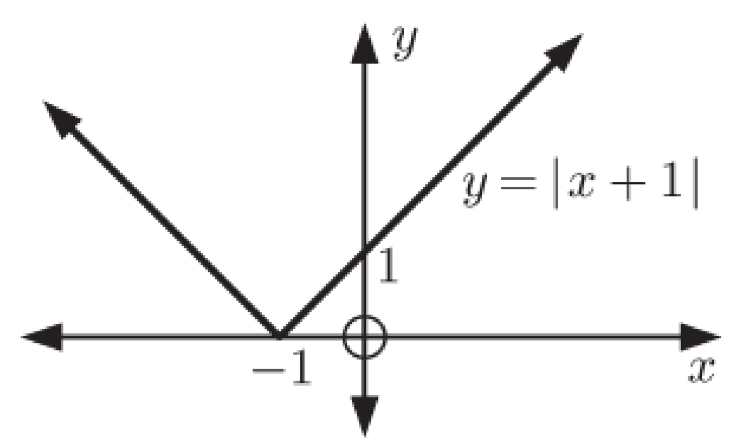
10



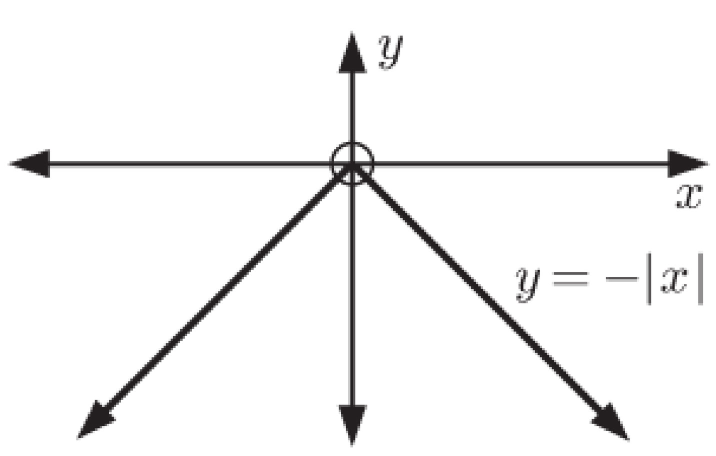
12 **a** $y = \begin{cases} x - 2, & x \geqslant 2 \\ 2 - x, & x < 2 \end{cases}$



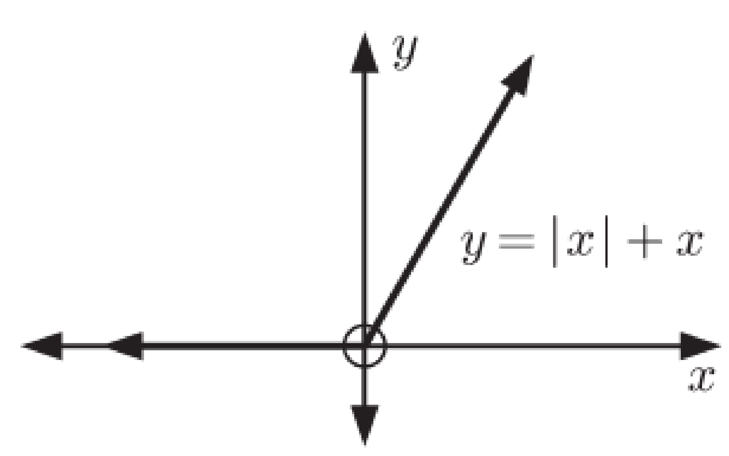
b $y = \begin{cases} x + 1, & x \geqslant -1 \\ -x - 1, & x < -1 \end{cases}$



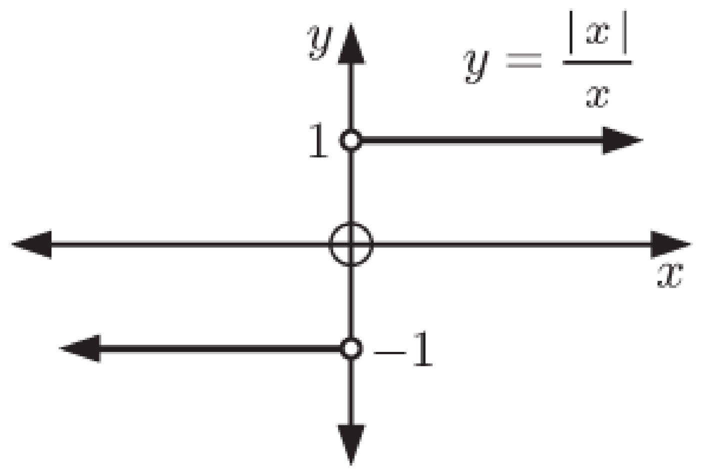
c $y = \begin{cases} -x, & x \geqslant 0 \\ x, & x < 0 \end{cases}$



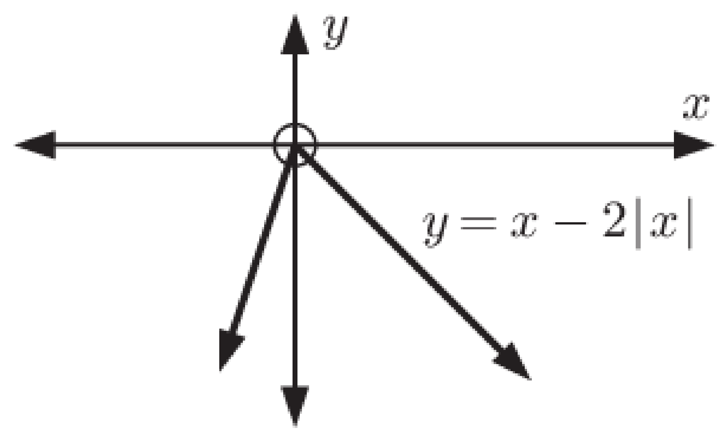
d $y = \begin{cases} 2x, & x \geqslant 0 \\ 0, & x < 0 \end{cases}$



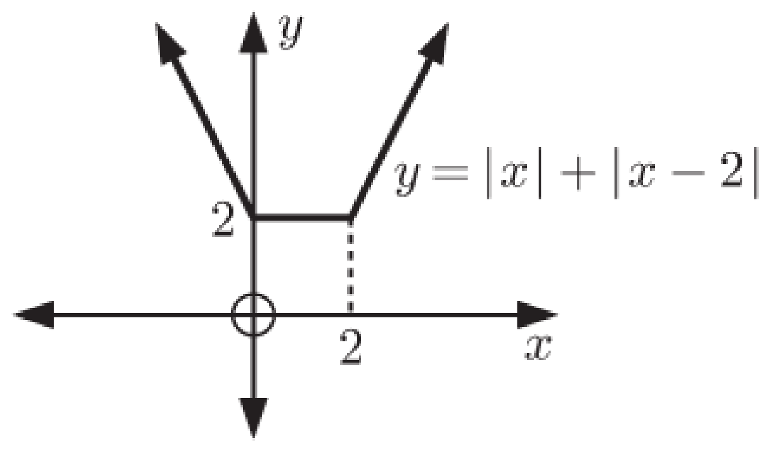
e $y = \begin{cases} 1, & x > 0 \\ \text{undefined}, & x = 0 \\ -1, & x < 0 \end{cases}$



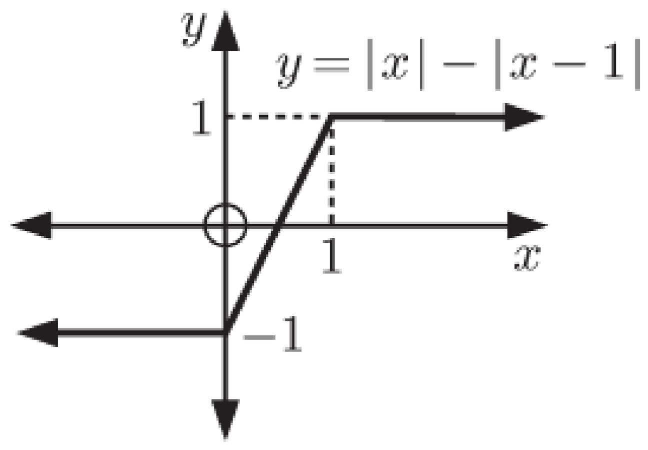
f $y = \begin{cases} -x, & x \geqslant 0 \\ 3x, & x < 0 \end{cases}$



g $y = \begin{cases} 2x-2, & x \geqslant 2 \\ 2, & 0 \leqslant x < 2 \\ 2-2x, & x < 0 \end{cases}$



h $y = \begin{cases} 1, & x \geqslant 1 \\ 2x-1, & 0 \leqslant x < 1 \\ -1, & x < 0 \end{cases}$



EXERCISE 6C.2

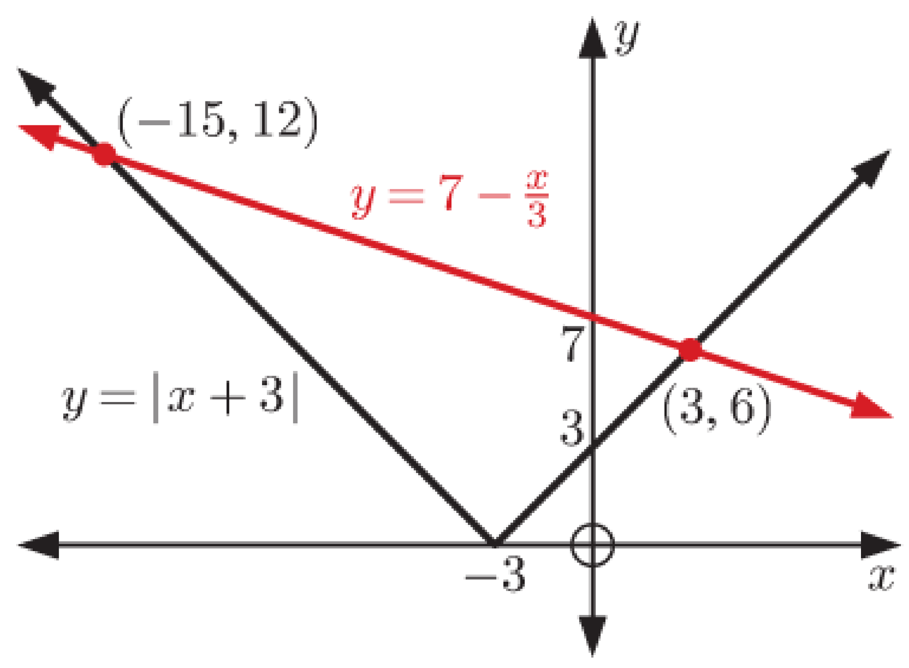
1 **a** $x = \pm 3$ **b** no solutions **c** $x = 0$
d $x = 4$ or -2 **e** $x = -1$ or 7 **f** no solutions
g $x = 1$ or $\frac{1}{3}$ **h** $x = 0$ or 3 **i** $x = -2$ or $\frac{14}{5}$

2 **a** $x = \frac{3}{2}$ or $\frac{3}{4}$ **b** $x = -2$ or $-\frac{4}{7}$ **c** $x = -1$ or 7

3 **a** In the case $\frac{3x+1}{x-1} = 3$ we get $3x + 1 = 3x - 3$ which has no solutions.
b $x = \frac{1}{3}$

5 **a** $x = \frac{3}{2}$ or $-\frac{1}{4}$ **b** $x = -\frac{4}{3}$ or -6 **c** $x = \frac{1}{2}$
d $x = \frac{5}{2}$ **e** $x = \frac{1}{2}$ or $-\frac{1}{2}$ **f** $x = \frac{2}{5}$ or -6

6 **a**



b $x = -15$ or 3

7 **a** $x = 1$ **b** $x = -\frac{4}{5}$ or 4
c $x \approx 0.714$ or $x = 5$ **d** $x \approx 2.69$
e $x \approx 1.28$ or 2.43 **f** $x \approx -1.91$, 0.304, or 2.09

EXERCISE 6C.3

1 **a** $-3 \leqslant x \leqslant 5$ **b** $x < -9$ or $x > 5$
c $1 < x < 2$ **d** $x \leqslant -\frac{7}{3}$ or $x \geqslant -1$
e $-\frac{3}{2} < x < \frac{5}{2}$ **f** $x < -\frac{9}{10}$ or $x > -\frac{7}{10}$

2 **a** $x < -2$ **b** $x < -1$ or $x > 1$
c all $x \in \mathbb{R}$ **d** $x \leqslant -\frac{7}{4}$ or $x \geqslant -\frac{1}{2}$
e $x < -10$ or $x > \frac{6}{5}$ **f** $\frac{7}{5} \leqslant x \leqslant 17$

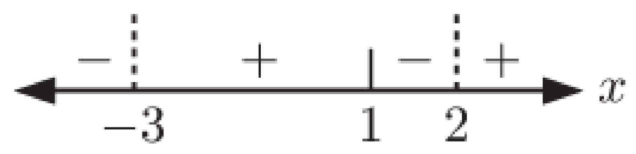
3 **a** $\frac{3}{2} \leqslant x \leqslant 3$, $x \neq 2$ **b** $x \geqslant -\frac{1}{4}$, $x \neq 1$
c $x < -10$ or $x > 2$

4 **a** $1 < x < 3$ **b** $x \geqslant 3$ **c** $x < -2.79$ or $x > 2.30$
d $-\frac{5}{2} \leqslant x \leqslant \frac{7}{2}$ **e** $-2.24 \leqslant x \leqslant 2.11$
f $-2.80 \leqslant x < -1$ or $2 < x \leqslant 2.92$
g $-1 \leqslant x < -0.189$ or $0.254 < x < 0.937$
h $-3.87 \leqslant x < -2.97$ or $-1.72 < x < 2.19$ or $2.59 < x < 3.5$

EXERCISE 6D.1

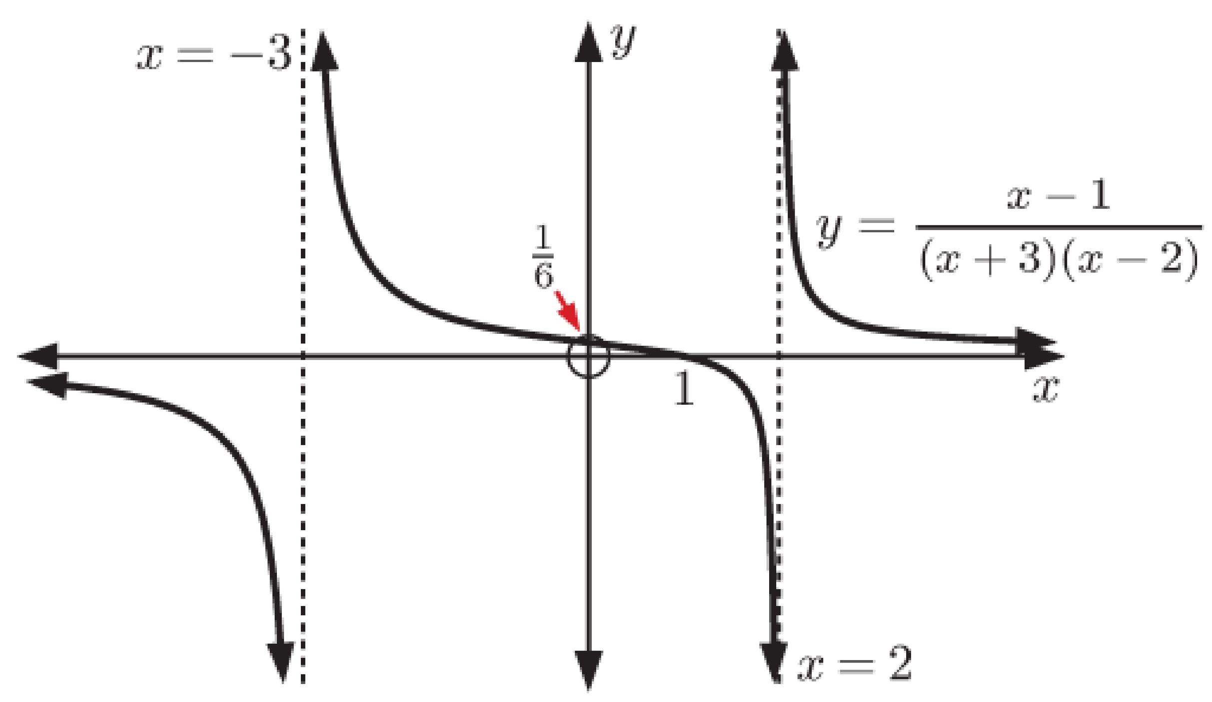
1 **a** vertical asymptotes $x = 2$ and $x = 6$, horizontal asymptote $y = 0$
b x-intercept $\frac{5}{4}$, y-intercept $-\frac{5}{12}$

2 **a** **i** vertical asymptotes $x = -3$ and $x = 2$, horizontal asymptote $y = 0$
ii x-intercept 1, y-intercept $\frac{1}{6}$
iii

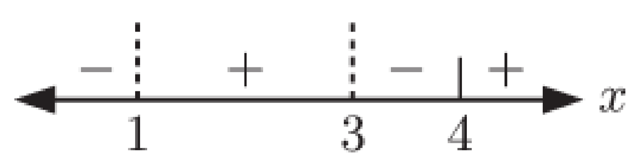


iv As $x \to -3^-$, $y \to -\infty$
As $x \to -3^+$, $y \to \infty$
As $x \to 2^-$, $y \to -\infty$
As $x \to 2^+$, $y \to \infty$
As $x \to -\infty$, $y \to 0^-$
As $x \to \infty$, $y \to 0^+$

v

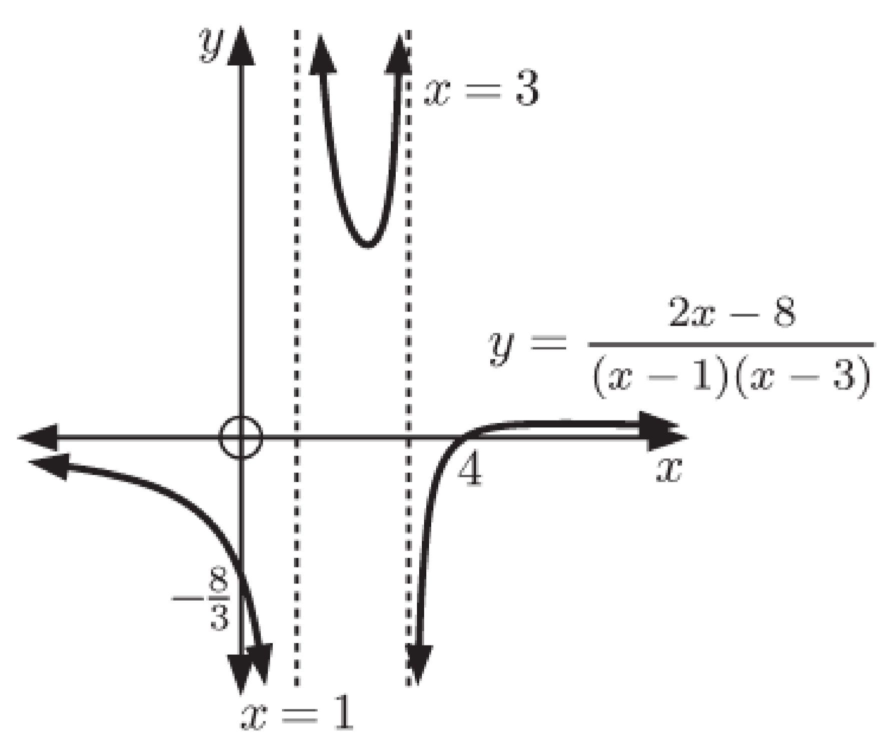


b **i** vertical asymptotes $x = 1$ and $x = 3$, horizontal asymptote $y = 0$
ii x-intercept 4, y-intercept $-\frac{8}{3}$
iii



iv As $x \to 1^-$, $y \to -\infty$
As $x \to 1^+$, $y \to \infty$
As $x \to 3^-$, $y \to \infty$
As $x \to 3^+$, $y \to -\infty$
As $x \to -\infty$, $y \to 0^-$
As $x \to \infty$, $y \to 0^+$

v



c **i** vertical asymptotes $x = -2$ and $x = 4$, horizontal asymptote $y = 0$
ii x-intercept $\frac{5}{3}$, y-intercept $-\frac{5}{8}$
iii