Homework 19.01 *[56 marks]*

The functions f and g are defined such that $f(x) = \frac{x+3}{4}$ and $g(x) = 8x + 5$.

1a. Show that $(g \circ f)(x) = 2x + 11$. *[2 marks]*

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1b. Given that $(g \circ f)^{-1}(a) = 4$, find the value of a. *[3 marks]*

2. The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = x - 2$ and $g(x) = ax + b$, where $a,\ b \in \mathbb{R}$. *[6 marks]*

Given that $(f \circ g)(2) = -3$ and $(g \circ f)(1) = 5$, find the value of a and the value of b.

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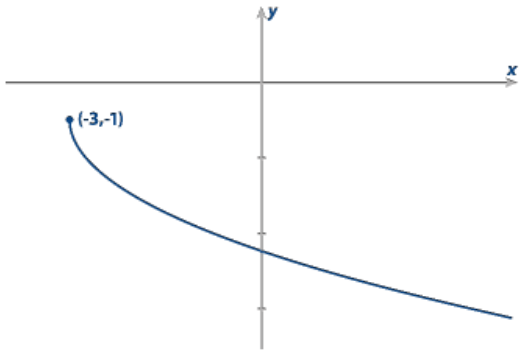
The following diagram shows the graph of $y = -1 - \sqrt{x+3}$ for $x \geq -3$.



3a. Describe a sequence of transformations that transforms the graph of $y = \sqrt{x}$ for $x \geq 0$ to the graph of $y = -1 - \sqrt{x+3}$ for $x \geq -3$. *[3 marks]*

A function f is defined by $f(x) = -1 - \sqrt{x+3}$ for $x \geq -3$.

3b. State the range of f. *[1 mark]*

3c. Find an expression for $f^{-1}(x)$, stating its domain. *[5 marks]*

3d. Find the coordinates of the point(s) where the graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect. *[5 marks]*

The following table shows values of $f(x)$ and $g(x)$ for different values of x.

Both f and g are one-to-one functions.

x	-2	0	3	4
$f(x)$	8	4	0	-3
$g(x)$	-5	-2	4	0

4a. Find $g(0)$. *[1 mark]*

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4b. Find $(f \circ g)(0)$. *[2 marks]*

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4c. Find the value of x such that $f(x) = 0$. *[2 marks]*

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A function f is defined by $f(x) = \frac{2x-1}{x+1}$, where $x \in \mathbb{R},\ x \neq -1$.

The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

5a. Write down the equation of the vertical asymptote. *[1 mark]*

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5b. Write down the equation of the horizontal asymptote. *[1 mark]*

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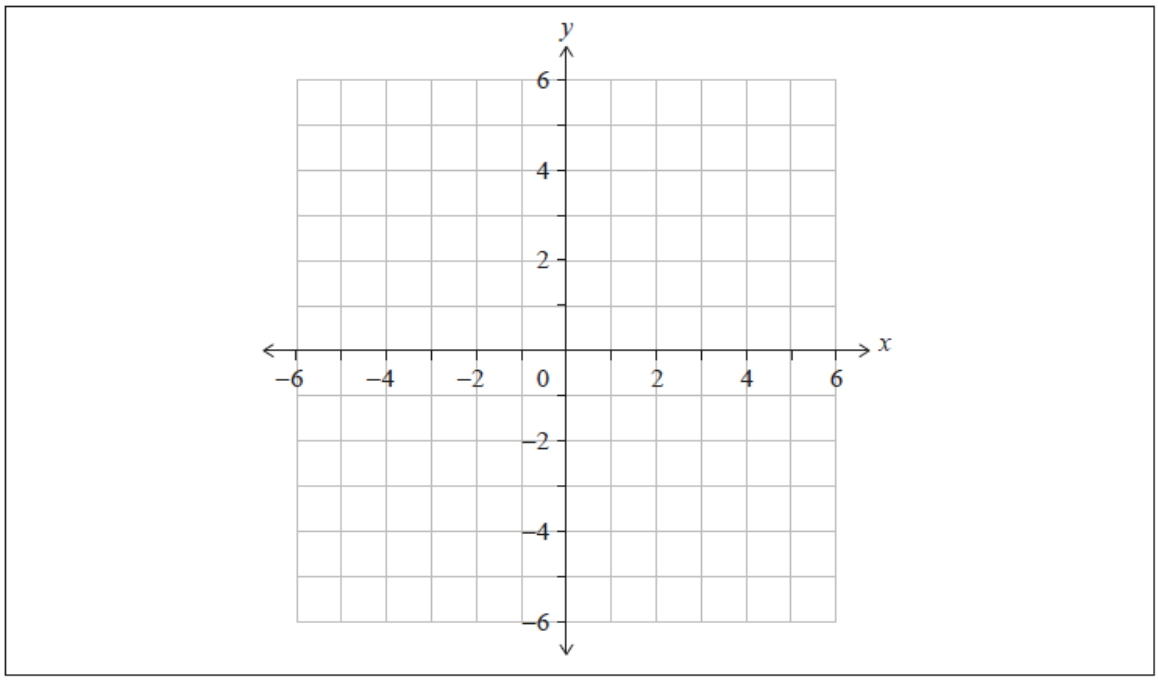
5c. On the set of axes below, sketch the graph of $y = f(x)$. *[3 marks]*

On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes.



5d. Hence, solve the inequality $0 < \frac{2x-1}{x+1} < 2$. *[1 mark]*

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The function f is defined by $f(x) = \frac{2x+4}{3-x}$, where $x \in \mathbb{R},\ x \neq 3$.

Write down the equation of

6a. the vertical asymptote of the graph of f. *[1 mark]*

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6b. the horizontal asymptote of the graph of f. *[1 mark]*

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Find the coordinates where the graph of f crosses

6c. the x-axis. *[1 mark]*

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6d. the y-axis. *[1 mark]*

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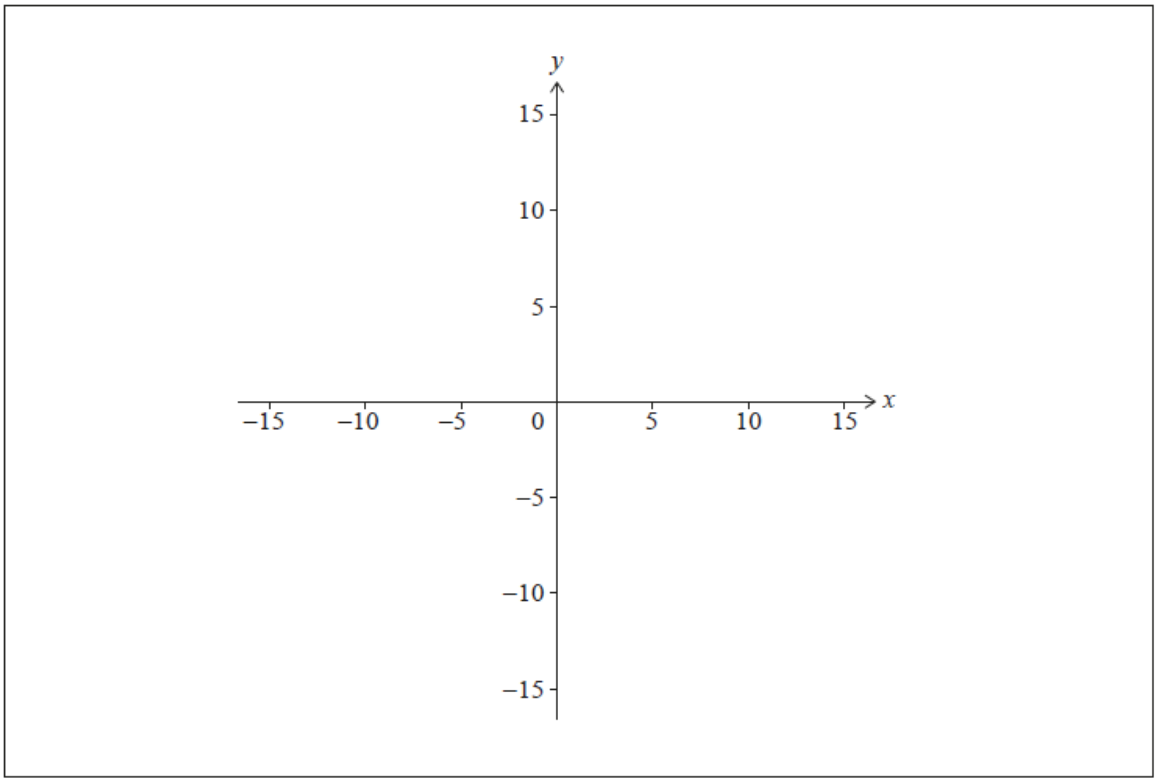
6e. Sketch the graph of f on the axes below. *[1 mark]*



The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = 6x^2 - 12x + 1$ and $g(x) = -x + c$, where $c \in \mathbb{R}$.

7a. Find the range of f. *[2 marks]*

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7b. Given that $(g \circ f)(x) \leq 0$ for all $x \in \mathbb{R}$, determine the set of possible values for c. *[4 marks]*

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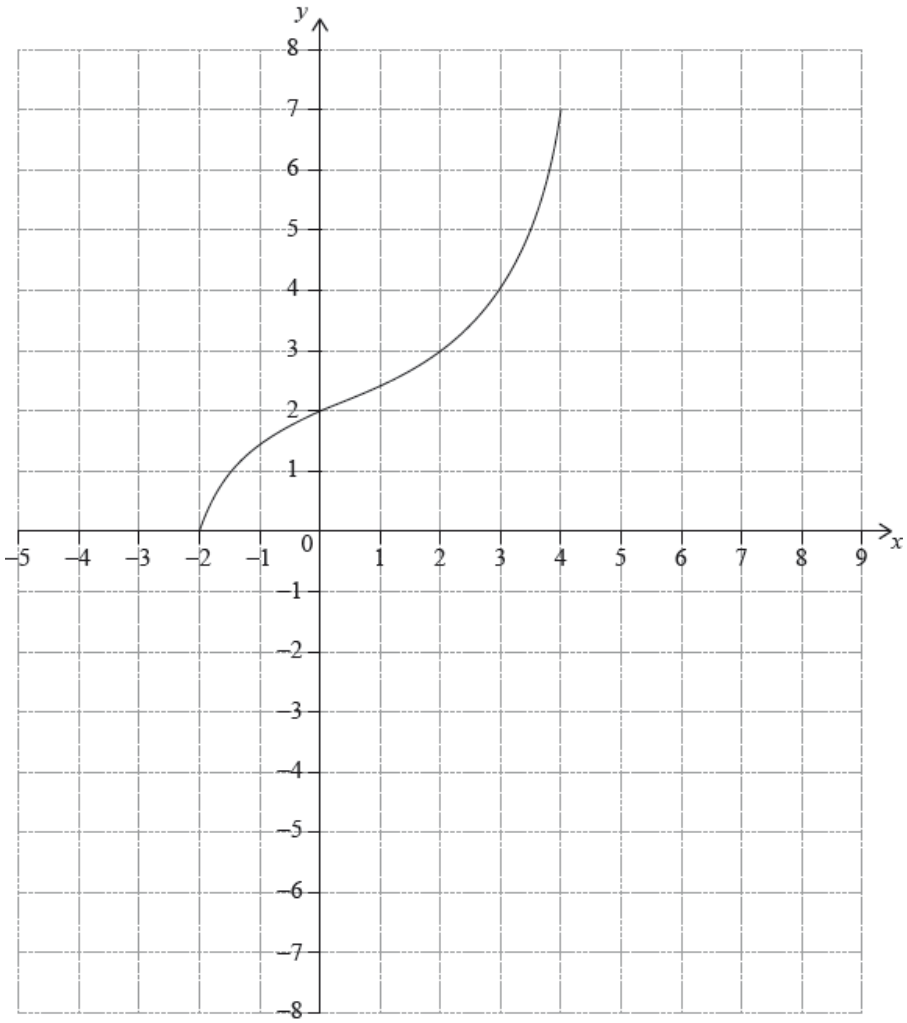
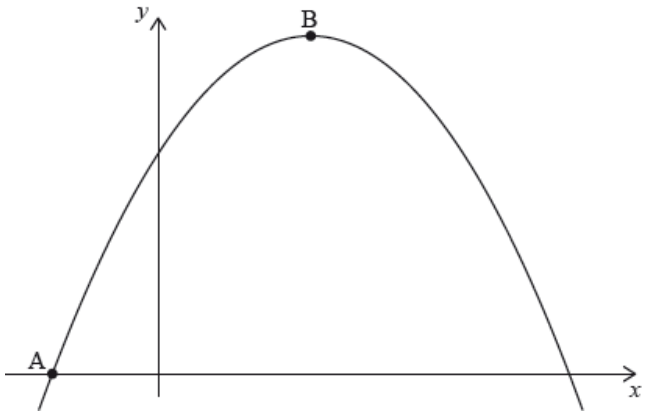
The following diagram shows the graph of a function f, with domain $-2 \leqslant x \leqslant 4$.



The points $(-2, 0)$ and $(4, 7)$ lie on the graph of f.

8. On the grid, sketch the graph of f^{-1}. *[3 marks]*

The graph of the quadratic function $f(x) = c + bx - x^2$ intersects the x-axis at the point $\mathrm{A}(-1, 0)$ and has its vertex at the point $\mathrm{B}(3, 16)$.



9a. Write down the equation of the axis of symmetry for this graph. *[2 marks]*

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9b. Find the value of b. *[2 marks]*

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9c. Write down the range of $f(x)$. *[2 marks]*

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