Probability review *[68 marks]*

All the children in a summer camp play at least one sport, from a choice of football (F) or basketball (B). 15 children play both sports.

The number of children who play only football is double the number of children who play only basketball.

Let x be the number of children who play only football.

1a. Write down an expression, in terms of x, for the number of children who play only basketball. *[1 mark]*

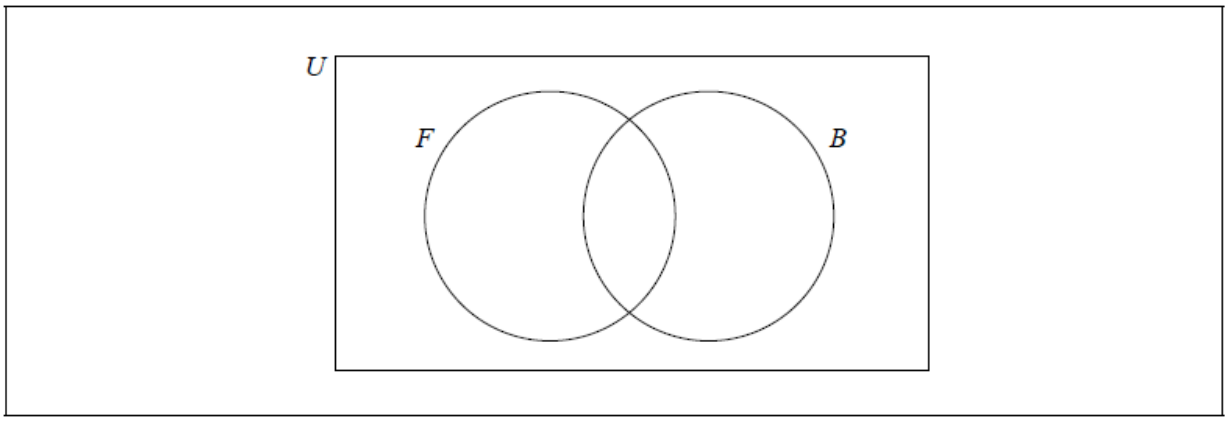
Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

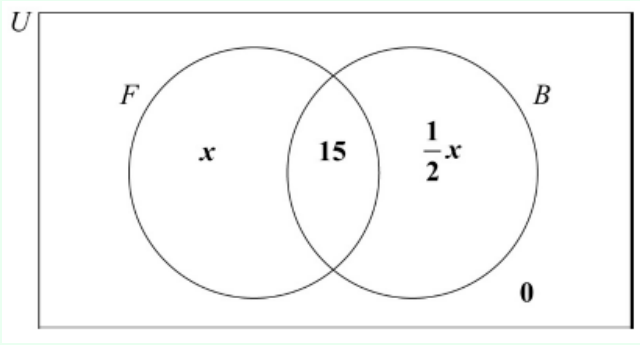
$\frac{1}{2}x$ ***(A1)*** ***(C1)***

[1 mark]

1b. Complete the Venn diagram using the above information. *[2 marks]*



Markscheme



(A1)(A1)*(ft)** ***(C2)

Notes: Award ***(A1)*** for 15 placed in the correct position, award ***(A1)*(ft)** for x and their $\frac{1}{2}x$ placed in the correct positions of diagram. Do not penalize the absence of 0 inside the rectangle and award at most ***(A1)(A0)*** if any value other than 0 is seen outside the circles. Award at most ***(A1)(A0)*** if 35 and 70 are seen instead of x and their $\frac{1}{2}x$.

[2 marks]

There are 120 children in the summer camp.

1c. Find the number of children who play only football. *[2 marks]*

Markscheme

$x + \frac{1}{2}x + 15 = 120$ or equivalent ***(M1)***

Note: Award ***(M1)*** for adding the values in their Venn and equating to 120 (or equivalent).

$(x =)\ 70$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from their Venn diagram, but only if the answer is a positive integer and x is seen in their Venn diagram.

[2 marks]

1d. Write down the value of $n(F)$. *[1 mark]*

Markscheme

85 ***(A1)*(ft)** ***(C1)***

Note: Follow through from their Venn diagram and their answer to part (c), but only if the answer is a positive integer and less than 120.

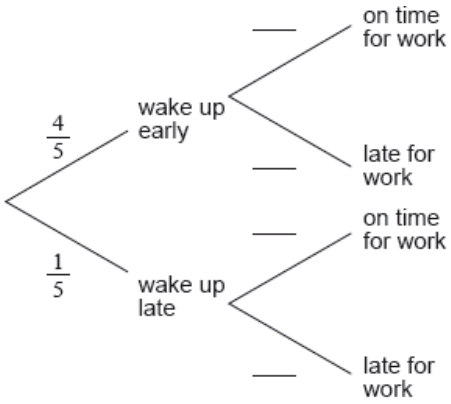
[1 mark]

On a work day, the probability that Mr Van Winkel wakes up early is $\frac{4}{5}$.

If he wakes up early, the probability that he is on time for work is p.

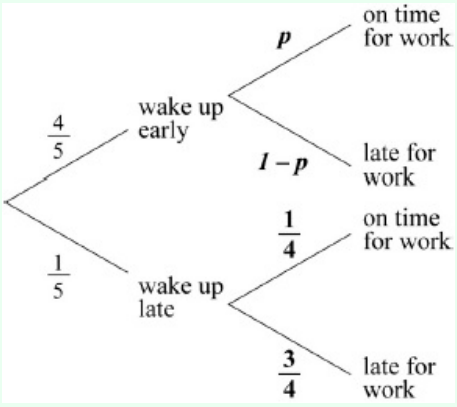
If he wakes up late, the probability that he is on time for work is $\frac{1}{4}$.

2a. Complete the tree diagram below. *[2 marks]*



Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.



(A1)(A1) ***(C2)***

Note: Award ***(A1)*** for each correct pair of probabilities.

[2 marks]

The probability that Mr Van Winkel arrives on time for work is $\frac{3}{5}$.

2b. Find the value of p. *[4 marks]*

Markscheme

$\frac{4}{5}p + \frac{1}{5} \times \frac{1}{4} = \frac{3}{5}$ ***(A1)*****(ft)*****(M1)*****(M1)***

Note: Award ***(A1)*****(ft)** for two correct products from part (a), ***(M1)*** for adding their products, ***(M1)*** for equating the sum of any two probabilities to $\frac{3}{5}$.

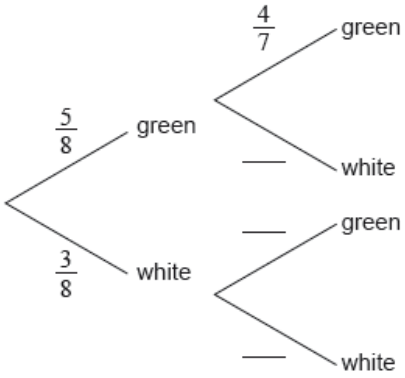
$(p =) \frac{11}{16}\ (0.688,\ 0.6875)$ ***(A1)*****(ft)** ***(C4)***

Note: Award the final ***(A1)*****(ft)** only if $0 \leqslant p \leqslant 1$. Follow through from part (a).

[4 marks]

A bag contains 5 green balls and 3 white balls. Two balls are selected at random without replacement.

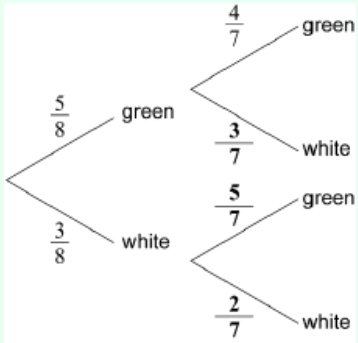
3a. Complete the following tree diagram. *[3 marks]*



Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

correct probabilities



A1A1A1 ***N3***

Note: Award ***A1*** for each correct **bold** answer.

[3 marks]

3b. Find the probability that exactly one of the selected balls is green. *[3 marks]*

Markscheme

multiplying along branches ***(M1)***

eg $\frac{5}{8} \times \frac{3}{7}$, $\frac{3}{8} \times \frac{5}{7}$, $\frac{15}{56}$

adding probabilities of correct mutually exclusive paths ***(A1)***

eg $\frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{5}{7}$, $\frac{15}{56} + \frac{15}{56}$

$\frac{30}{56}$ $\left(= \frac{15}{28}\right)$ ***A1*** ***N2***

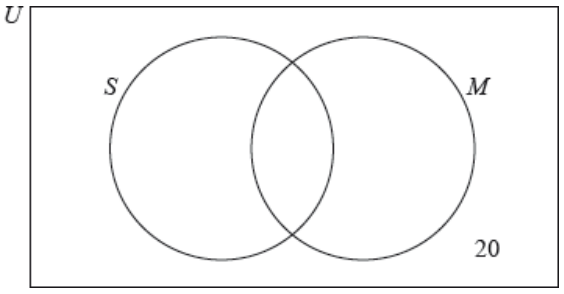
[3 marks]

Rosewood College has 120 students. The students can join the sports club (S) and the music club (M).

For a student chosen at random from these 120, the probability that they joined both clubs is $\frac{1}{4}$ and the probability that they joined the music club is $\frac{1}{3}$.

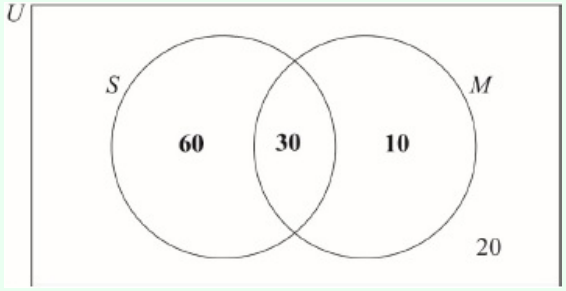
There are 20 students that did not join either club.

4a. Complete the Venn diagram for these students. *[2 marks]*



Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.



(A1)(A1) ***(C2)***

Note: Award ***(A1)*** for 30 in correct area, ***(A1)*** for 60 and 10 in the correct areas.

[2 marks]

4b. One of the students who joined the sports club is chosen at random. Find the probability that this student joined both clubs. *[2 marks]*

Markscheme

$\frac{30}{90}$ $\left(\frac{1}{3},\ 0.333333\ldots,\ 33.3333\ldots\%\right)$ ***(A1)*****(ft)*** ***(A1)*****(ft)*** ***(C2)***

Note: Award ***(A1)*****(ft)** for correct numerator of 30, ***(A1)*****(ft)** for correct denominator of 90. Follow through from their Venn diagram.

[2 marks]

4c. Determine whether the events S and M are independent. *[2 marks]*

Markscheme

$\mathrm{P}(S) \times \mathrm{P}(M) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$ ***(R1)***

Note: Award ***(R1)*** for multiplying their by $\frac{1}{3}$.

therefore the events are independent $\left(\text{as } \mathrm{P}(S \cap M) = \frac{1}{4}\right)$ ***(A1)*****(ft)** ***(C2)***

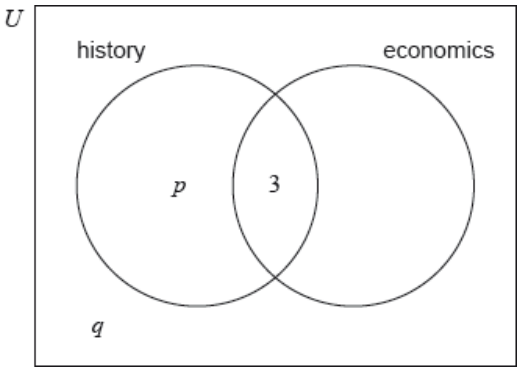
Note: Award ***(R1)(A1)*****(ft)** for an answer which is consistent with their Venn diagram.

Do not award ***(R0)(A1)*****(ft)**.

Do not award final ***(A1)*** if $\mathrm{P}(S) \times \mathrm{P}(M)$ is not calculated. Follow through from part (a).

[2 marks]

In a group of 20 girls, 13 take history and 8 take economics. Three girls take both history and economics, as shown in the following Venn diagram. The values p and q represent numbers of girls.



5a. Find the value of p; *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $p + 3 = 13,\ 13 - 3$

$p = 10$ ***A1*** ***N2***

[2 marks]

5b. Find the value of q. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $p + 3 + 5 + q = 20,\ 10 - 10 - 8$

$q = 2$ ***A1*** ***N2***

[2 marks]

5c. A girl is selected at random. Find the probability that she takes economics but not history. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $20 - p - q - 3,\ 1 - \frac{15}{20},\ n(E \cap H') = 5$

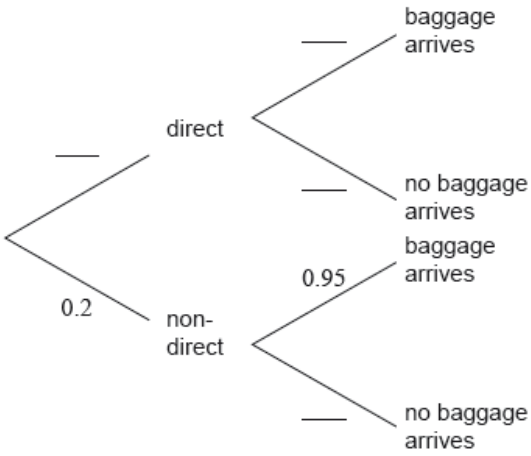
$\frac{5}{20}\ \left(\frac{1}{4}\right)$ ***A1*** ***N2***

[2 marks]

Sara regularly flies from Geneva to London. She takes either a direct flight or a non-directflight that goes via Amsterdam.

If she takes a direct flight, the probability that her baggage does not arrive in London is 0.01.
If she takes a non-direct flight the probability that her baggage arrives in London is 0.95.

The probability that she takes a non-direct flight is 0.2.



6a. Complete the tree diagram. *[3 marks]*

Markscheme

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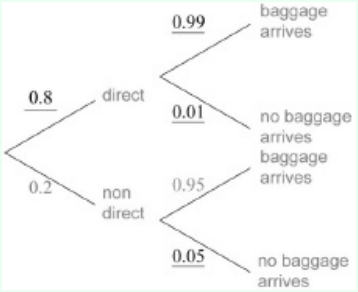


(A1)(A1)(A1) ***(C3)***

Note: Award ***(A1)*** for each correct pair of probabilities.

[3 marks]

6b. Find the probability that Sara's baggage arrives in London. *[3 marks]*

Markscheme

$0.8 \times 0.99 + 0.2 \times 0.95$ ***(A1)*****(ft)*****(M1)***

Note: Award ***(A1)*****(ft)** for two correct products of probabilities taken from their diagram, ***(M1)*** for the addition of their products.

$= 0.982 \ \left(98.2\%, \ \frac{491}{500}\right)$ ***(A1)*****(ft)** ***(C3)***

Note: Follow through from part (a).

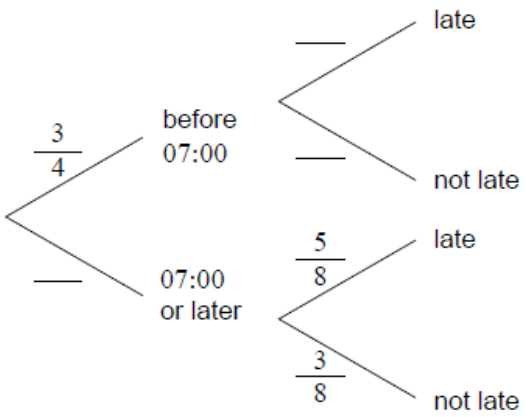
[3 marks]

Pablo drives to work. The probability that he leaves home before 07:00 is $\frac{3}{4}$.

If he leaves home before 07:00 the probability he will be late for work is $\frac{1}{8}$.

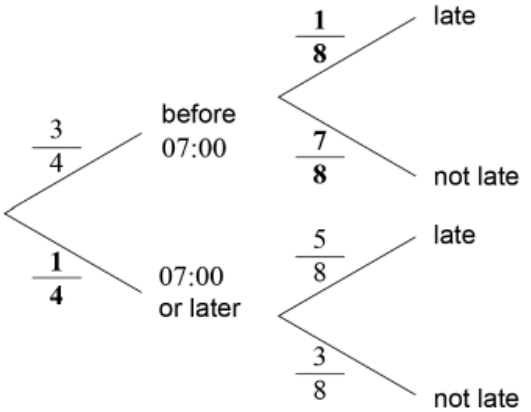
If he leaves home at 07:00 or later the probability he will be late for work is $\frac{5}{8}$.

7a. **Copy** and complete the following tree diagram. *[3 marks]*



Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.



A1A1A1 N3

Note: Award ***A1*** for each bold fraction.

[3 marks]

7b. Find the probability that Pablo leaves home before 07:00 and is late for work. *[2 marks]*

Markscheme

multiplying along correct branches ***(A1)***

eg $\frac{3}{4} \times \frac{1}{8}$

P(leaves before 07:00 ∩ late) = $\frac{3}{32}$ ***A1 N2***

[2 marks]

7c. Find the probability that Pablo is late for work. *[3 marks]*

Markscheme

multiplying along other “late” branch ***(M1)***
eg $\frac{1}{4} \times \frac{5}{8}$

adding probabilities of two mutually exclusive late paths ***(A1)***
eg $\left(\frac{3}{4} \times \frac{1}{8}\right) + \left(\frac{1}{4} \times \frac{5}{8}\right), \frac{3}{32} + \frac{5}{32}$

$\mathrm{P}(L) = \frac{8}{32} \left(= \frac{1}{4}\right)$ ***A1 N2***

[3 marks]

7d. Given that Pablo is late for work, find the probability that he left home before 07:00. *[3 marks]*

Markscheme

recognizing conditional probability (seen anywhere) ***(M1)***
eg $\mathrm{P}(A|B), \mathrm{P}(\text{before 7}|\text{late})$

correct substitution of **their** values into formula ***(A1)***
eg $\frac{\frac{3}{32}}{\frac{1}{4}}$

$\mathrm{P}(\text{left before 07:00}|\text{late}) = \frac{3}{8}$ ***A1 N2***

[3 marks]

7e. Two days next week Pablo will drive to work. Find the probability that he will be late at least once. *[3 marks]*

Markscheme

valid approach ***(M1)***
eg 1 − P(not late twice), P(late once) + P(late twice)

correct working ***(A1)***
eg $1 - \left(\frac{3}{4} \times \frac{3}{4}\right), 2 \times \frac{1}{4} \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{4}$

$\frac{7}{16}$ ***A1 N2***

[3 marks]

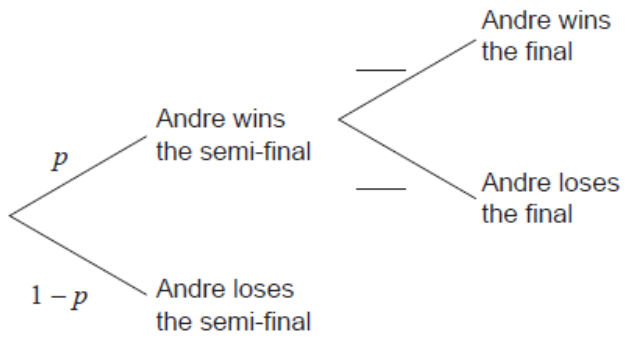
Andre will play in the semi-final of a tennis tournament.

If Andre wins the semi-final he will progress to the final. If Andre loses the semi-final, he will **not** progress to the final.

If Andre wins the final, he will be the champion.

The probability that Andre will win the semi-final is p. If Andre wins the semi-final, then the probability he will be the champion is 0.6.

8a. Complete the values in the tree diagram. *[1 mark]*



Markscheme

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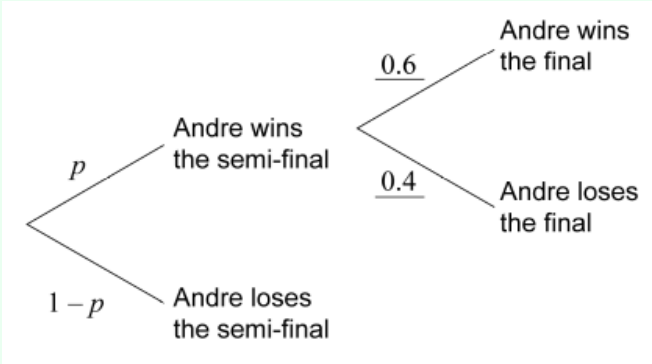


(A1) ***(C1)***

Note: Award ***(A1)*** for the correct pair of probabilities.

[1 mark]

The probability that Andre will not be the champion is 0.58.

8b. Find the value of p. *[2 marks]*

Markscheme

$p \times 0.4 + (1-p) = 0.58$ ***(M1)***

Note: Award ***(M1)*** for multiplying and adding correct probabilities for losing equated to 0.58.

OR

$p \times 0.6 = 1 - 0.58$ ***(M1)***

Note: Award ***(M1)*** for multiplying correct probabilities for winning equated to $1 - 0.58$ or 0.42.

$(p =)\ 0.7$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from their part (a). Award the final ***(A1)*****(ft)** only if their p is within the range $0 < p < 1$.

[2 marks]

8c. Given that Andre did not become the champion, find the probability that he lost in the semi-final. *[3 marks]*

Markscheme

$\frac{0.3}{0.58}\left(\frac{1-0.7}{0.58}\right)$ ***(A1)*****(ft)*****(A1)***

Note: Award ***(A1)*****(ft)** for their correct numerator. Follow through from part (b). Award ***(A1)*** for the correct denominator.

OR

$\frac{0.3}{0.3+0.7\times0.4}$ ***(A1)*****(ft)*****(A1)*****(ft)**

Note: Award ***(A1)*****(ft)** for their correct numerator. Follow through from part (b). Award ***(A1)*****(ft)** for their correct calculation of Andre losing the semi-final or winning the semi-final and then losing in the final. Follow through from their parts (a) and (b).

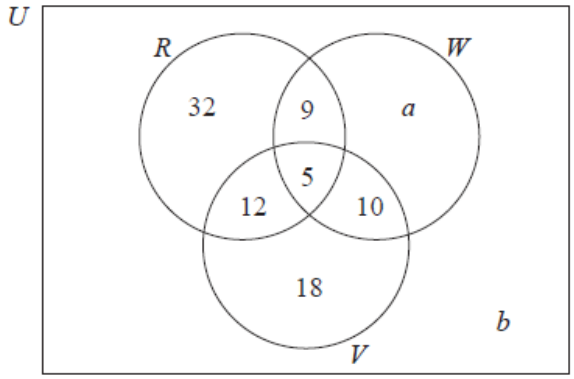
$\frac{15}{29}\ (0.517,\ 0.517241\ldots,\ 51.7\%)$ ***(A1)*****(ft)** ***(C3)***

Note: Follow through from parts (a) and (b).

[3 marks]

On a school excursion, 100 students visited an amusement park. The amusement park's main attractions are rollercoasters (R), water slides (W), and virtual reality rides (V).

The students were asked which main attractions they visited. The results are shown in the Venn diagram.



A total of 74 students visited the rollercoasters or the water slides.

9a. Find the value of a. *[2 marks]*

Markscheme

$74-(32+12+10+9+5)$ **OR** $74-68$ ***(M1)***

Note: Award ***(M1)*** for setting up a correct expression.

$(a=)\ 6$ ***(A1)(G2)***

[2 marks]

9b. Find the value of b. *[2 marks]*

Markscheme

$100-(74+18)$ ***(M1)***

OR

$100-92$ ***(M1)***

OR

$100-(32+9+5+12+10+18+6)$ ***(M1)***

Note: Award ***(M1)*** for setting up a correct expression. Follow through from part (a)(i) but only for $a \geq 0$.

$(b=)\ 8$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from part(a)(i). The value of b must be greater or equal to zero for the ***(A1)*****(ft)** to be awarded.

[2 marks]

9c. Find the number of students who visited at least two types of main attraction. *[2 marks]*

Markscheme

$9 + 5 + 12 + 10$ ***(M1)***

Note: Award ***(M1)*** for adding $9,\ 5,\ 12$ and 10.

36 ***(A1)(G2)***

[2 marks]

9d. Write down the value of $n(\ R \cap W)$. *[1 mark]*

Markscheme

14 ***(A1)***

[1 mark]

9e. Find the probability that a randomly selected student visited the rollercoasters. *[2 marks]*

Markscheme

$\frac{58}{100}\ \left(\frac{29}{50},\ 0.58,\ 58\%\right)$ ***(A1)(A1)(G2)***

Note: Award ***(A1)*** for correct numerator. Award ***(A1)*** for the correct denominator. Award ***(A0)*** for 58 only.

[2 marks]

9f. Find the probability that a randomly selected student visited the virtual reality rides. *[1 mark]*

Markscheme

$\frac{45}{100}$ $\left(\frac{9}{20}, 0.45, 45\%\right)$ ***(A1)*****(ft)**

Note: Follow through from their denominator from part (d)(i).

[1 mark]

9g. Hence determine whether the events in **parts (d)(i)** and **(d)(ii)** are independent. Justify your reasoning. *[2 marks]*

Markscheme

they are not independent ***(A1)*****(ft)**

$\frac{58}{100} \times \frac{45}{100} \neq \frac{17}{100}$ **OR** $0.261 \neq 0.17$ ***(R1)***

Note: Comparison of numerical values must be seen for ***(R1)*** to be awarded. Do not award ***(A1)(R0)***. Follow through from parts (d)(i) and (d)(ii).

[2 marks]