Sequences review *[112 marks]*

The Osaka Tigers basketball team play in a multilevel stadium.

The most expensive tickets are in the first row. The ticket price, in Yen (¥), for each row forms an arithmetic sequence. Prices for the first three rows are shown in the following table.

Ticket pricing per game	
1st row	6800 Yen
2nd row	6550 Yen
3rd row	6300 Yen

1a. Write down the value of the common difference, d *[1 mark]*

Markscheme

$(d =) -250$ ***A1***

[1 mark]

1b. Calculate the price of a ticket in the 16th row. *[2 marks]*

Markscheme

$(u_{16} =)\, 6800 + (16 - 1)\,(-250)$ ***M1***

(¥)3050 ***A1***

[2 marks]

1c. Find the total cost of buying 2 tickets in each of the first 16 rows. *[3 marks]*

Markscheme

$(S_{16} =) \left(\frac{16}{2}\right)(2 \times 6800 + (16 - 1)(-250)) \times 2$ ***M1M1***

Note: Award ***M1*** for correct substitution into arithmetic series formula. Award ***M1*** for multiplication by 2 seen.

OR

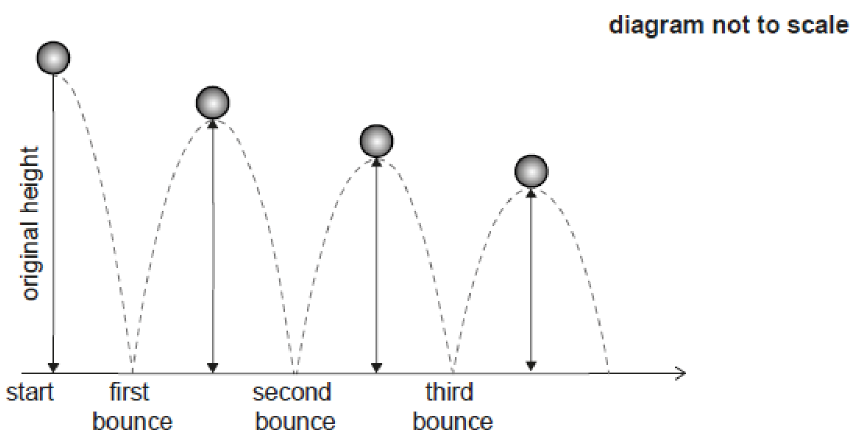
$(S_{16} =) \left(\frac{16}{2}\right)(6800 + 3050) \times 2$ ***M1M1***

Note: Award ***M1*** for correct substitution into arithmetic series formula. Award ***M1*** for multiplication by 2 seen.

(¥)158 000 (157 600) ***A1***

[3 marks]

A ball is dropped from a height of 1.8 metres and bounces on the ground. The maximum height reached by the ball, after each bounce, is 85% of the previous maximum height.



2a. Show that the maximum height reached by the ball after it has bounced for the sixth time is 68 cm, to the nearest cm. *[2 marks]*

Markscheme

use of geometric sequence with $r = 0.85$ ***M1***

EITHER

$(0.85)^6(1.8)$ **OR** $0.678869\ldots$ **OR** $(0.85)^5(1.53)$ ***A1***

$= 0.68 \text{ m}$

$= 68 \text{ cm}$ ***AG***

OR

$(0.85)^6(180)$ **OR** $(0.85)^5(153)$ ***A1***

$= 68 \text{ cm}$ ***AG***

[2 marks]

2b. Find the number of times, after the first bounce, that the maximum height reached is greater than 10 cm. *[2 marks]*

Markscheme

EITHER

$(0.85)^n(1.8) > 0.1$ **OR** $(0.85)^{n-1}(1.53) > 0.1$ ***(M1)***

Note: If $1.8\ \text{m}$ (or $180\ \text{cm}$) is used then ***(M1)*** only awarded for use of n in $(0.85)^n(1.8) > 0.1$.
If $1.53\ \text{m}$ (or $153\ \text{cm}$) is used then ***(M1)*** only awarded for use of $n-1$ in $(0.85)^{n-1}(1.53) > 0.1$.

17 ***A1***

OR

$(0.85)^{17}(1.8) = 0.114\ \text{m}$ and $(0.85)^{18}(1.8) = 0.0966\ \text{m}$ ***(M1)***

17 ***A1***

OR

solving $(0.85)^n(1.8) = 0.1$ to find $n = 17.8$ ***(M1)***

17 ***A1***

Note: Evidence of solving may be a graph **OR** the “solver” function **OR** use of logs to solve the equation. Working may use cm.

[2 marks]

2c. Find the total **vertical** distance travelled by the ball from the point at which it is dropped until the fourth bounce. *[3 marks]*

Markscheme

EITHER

distance (in one direction) travelled between first and fourth bounce

$= \frac{(1.8\times0.85)(1-0.85^3)}{1-0.85} \ (= 3.935925\ldots)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$18 + 2(3.935925)$

$= 9.67 \text{ m} \ (9.67185\ldots \text{ m})$ ***A1***

OR

distance (in one direction) travelled between drop and fourth bounce

$= \frac{(1.8)(1-0.85^4)}{1-0.85} \ (= 5.735925\ldots)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$2(5.735925)-1.8$

$= 9.67 \text{ m} \ (9.67185\ldots \text{ m})$ ***A1***

OR

distance (in one direction) travelled between first and fourth bounce

$(0.85)(1.8)+(0.85)^2(1.8)+(0.85)^3(1.8) \ (= 3.935925\ldots)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$1.8 + 2(0.85)(1.8)+2(0.85)^2(1.8)+2(0.85)^3(1.8)$

$= 9.67 \text{ m} \ (9.67185\ldots \text{ m})$ ***A1***

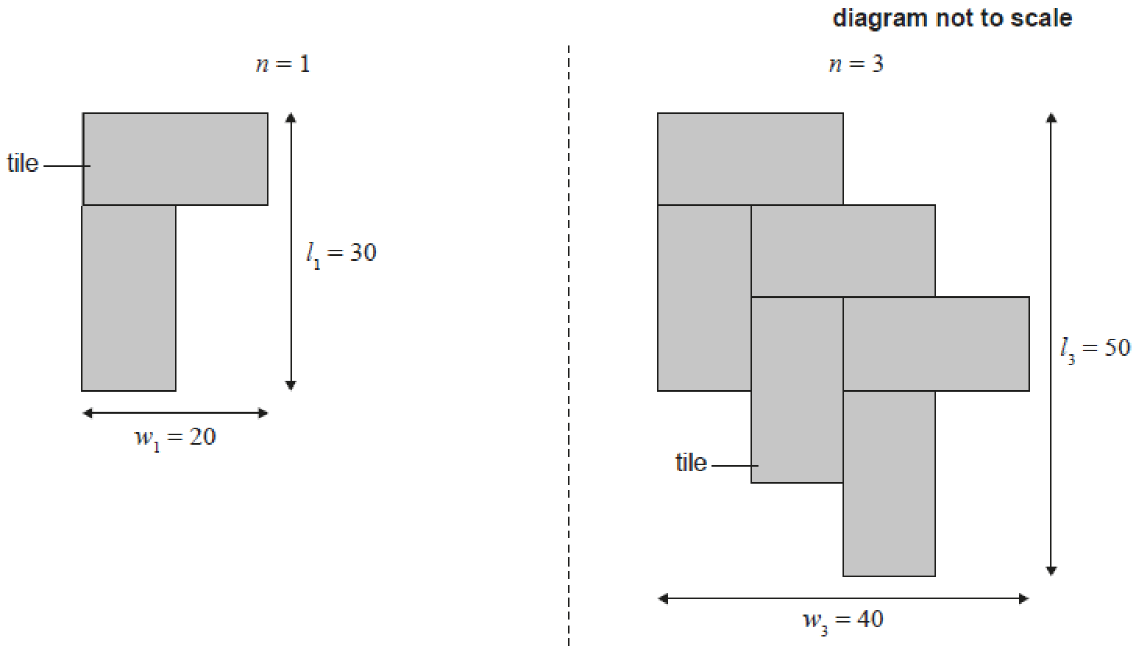
Note: Answers may be given in cm.

[3 marks]

Eddie decides to construct a path across his rectangular grass lawn using pairs of tiles.

Each tile is 10 cm wide and 20 cm long. The following diagrams show the path after Eddie has laid one pair and three pairs of tiles. This pattern continues until Eddie reaches the other side of his lawn. When n pairs of tiles are laid, the path has a width of w_n centimetres and a length l_n centimetres.

The following diagrams show this pattern for one pair of tiles and for three pairs of tiles, where the white space around each diagram represents Eddie's lawn.



The following table shows the values of w_n and l_n for the first three values of n.

Number of pairs of tiles, n	Width of lawn crossed by path, w_n (cm)	Length of lawn crossed by path, l_n (cm)
1	20	30
2	a	b
3	40	50

Find the value of

3a. a. *[1 mark]*

Markscheme

30 ***A1***

[1 mark]

3b. b. *[1 mark]*

Markscheme

40 ***A1***

[1 mark]

Write down an expression in terms of n for

3c. w_n. *[2 marks]*

Markscheme

arithmetic formula chosen ***(M1)***

$w_n = 20 + (n-1)10 \quad (= 10 + 10n)$ ***A1***

[2 marks]

3d. l_n. *[1 mark]*

Markscheme

arithmetic formula chosen

$l_n = 30 + (n-1)10 \quad (= 20 + 10n)$ ***A1***

[1 mark]

Eddie's lawn has a length $740\ \mathrm{cm}$.

3e. Show that Eddie needs 144 tiles. *[2 marks]*

Markscheme

$740 = 30 + (n - 1)10$ **OR** $740 = 20 - 10n$ ***M1***

$n = 72$ ***A1***

144 tiles ***AG***

Note: The ***AG*** line must be stated for the final ***A1*** to be awarded.

[2 marks]

3f. Find the value of w_n for this path. *[1 mark]*

Markscheme

$w_{72} = 730$ ***A1***

[1 mark]

3g. Find the total area of the tiles in Eddie's path. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer. *[3 marks]*

Markscheme

$(10 \times 20) \times 144$ ***(M1)***

$= 28800$ ***(A1)***

$2.88 \times 10^4 \text{ cm}^2$ ***A1***

Note: Follow through within the question for correctly converting *their* intermediate value into standard form (but only if the pre-conversion value is seen).

[3 marks]

The tiles cost $\$24.50$ per square metre and are sold in packs of five tiles.

3h. Find the cost of a single pack of five tiles. *[3 marks]*

Markscheme

EITHER

1 square metre $= 100 \text{ cm} \times 100 \text{ cm}$ ***(M1)***

(so, 50 tiles) and hence 10 packs of tiles in a square metre ***(A1)***

(so each pack is $\frac{\$24.50}{10 \text{ packs}}$)

OR

area covered by one pack of tiles is $(0.2 \text{ m} \times 0.1 \text{ m} \times 5 =)\ 0.1 \text{ m}^2$ ***(A1)***

24.5×0.1 ***(M1)***

THEN

$\$2.45$ per pack (of 5 tiles) ***A1***

[3 marks]

To allow for breakages Eddie wants to have at least 8% more tiles than he needs.

3i. Find the minimum number of packs of tiles Eddie will need to order. *[3 marks]*

Markscheme

$\frac{1.08 \times 144}{5}$ $(= 31.104)$ ***(M1)(M1)***

Note: Award ***M1*** for correct numerator, ***M1*** for correct denominator.

32 (packs of tiles) ***A1***

[3 marks]

There is a fixed delivery cost of $\$35$.

3j. Find the total cost for Eddie’s order. *[2 marks]*

Markscheme

$35 + (32 \times 2.45)$ ***(M1)***

$\$113\ \ (113.4)$ ***A1***

[2 marks]

Scott purchases food for his dog in large bags and feeds the dog the same amount of dog food each day. The amount of dog food left in the bag at the end of each day can be modelled by an arithmetic sequence.

On a particular day, Scott opened a new bag of dog food and fed his dog. By the end of the third day there were 115.5 cups of dog food remaining in the bag and at the end of the eighth day there were 108 cups of dog food remaining in the bag.

Find the number of cups of dog food

4a. fed to the dog per day. *[3 marks]*

Markscheme

EITHER

$115.5 = u_1 + (3-1) \times d \quad (115.5 = u_1 + 2d)$

$108 = u_1 + (8-1) \times d \quad (108 = u_1 + 7d)$ ***(M1)(A1)***

Note: Award ***M1*** for attempting to use the arithmetic sequence term formula, ***A1*** for both equations correct. Working for ***M1*** and ***A1*** can be found in parts (i) or (ii).

$(d = -1.5)$

1.5 (cups/day) ***A1***

Note: Answer must be written as a positive value to award ***A1***.

OR

$(d =) \frac{115.5 - 108}{5}$ ***(M1)(A1)***

Note: Award ***M1*** for attempting a calculation using the difference between term 3 and term 8; ***A1*** for a correct substitution.

$(d =)\ 1.5$ (cups/day) ***A1***

[3 marks]

4b. remaining in the bag at the end of the first day. *[1 mark]*

Markscheme

$(u_1 =)\ 118.5$ (cups) ***A1***

[1 mark]

4c. Calculate the number of days that Scott can feed his dog with one bag of food. *[2 marks]*

Markscheme

attempting to substitute their values into the term formula for arithmetic sequence equated to zero ***(M1)***

$0 = 118.5 + (n - 1) \times (-1.5)$

$(n =)\ 80$ days ***A1***

Note: Follow through from part (a) only if their answer is positive.

[2 marks]

In 2021, Scott spent $\$625$ on dog food. Scott expects that the amount he spends on dog food will increase at an annual rate of 6.4%.

4d. Determine the amount that Scott expects to spend on dog food in 2025. Round your answer to the nearest dollar. *[3 marks]*

Markscheme

$(t_5 =)\ 625 \times 1.064^{(5-1)}$ ***(M1)(A1)***

Note: Award ***M1*** for attempting to use the geometric sequence term formula; ***A1*** for a correct substitution

$\$801$ ***A1***

Note: The answer must be rounded to a whole number to award the final ***A1***.

[3 marks]

4e. Calculate the value of $\sum_{n=1}^{10}\left(625 \times 1.064^{(n-1)}\right)$. *[1 mark]*

Markscheme

$(S_{10} =)$ $(\$)\ 8390$ $(8394.39\ldots)$ ***A1***

[1 mark]

4f. Describe what the value in part (d)(i) represents in this context. *[2 marks]*

Markscheme

EITHER

the total cost (of dog food) ***R1***

for 10 years beginning in 2021 **OR** 10 years before 2031 ***R1***

OR

the total cost (of dog food) ***R1***

from 2021 to 2030 (inclusive) **OR** from 2021 to (the start of) 2031 ***R1***

[2 marks]

4g. Comment on the appropriateness of modelling this scenario with a geometric sequence. *[1 mark]*

Markscheme

EITHER
According to the model, the cost of dog food per year will eventually be too high to keep a dog.

OR
The model does not necessarily consider changes in inflation rate.

OR
The model is appropriate as long as inflation increases at a similar rate.

OR
The model does not account for changes in the amount of food the dog eats as it ages/becomes ill/stops growing.

OR
The model is appropriate since dog food bags can only be bought in discrete quantities. ***R1***

Note: Accept reasonable answers commenting on the appropriateness of the model for the specific scenario. There should be a reference to the given context. A reference to the geometric model must be clear: either “model” is mentioned specifically, or other mathematical terms such as “increasing” or “discrete quantities” are seen. Do not accept a contextual argument in isolation, e.g. “The dog will eventually die”.

[1 mark]

The admissions team at a new university are trying to predict the number of student applications they will receive each year.

Let n be the number of years that the university has been open. The admissions team collect the following data for the first two years.

Year, n	Number of applications received in year n
1	12 300
2	12 669

5a. Calculate the percentage increase in applications from the first year to the second year. *[2 marks]*

Markscheme

$\frac{12669-12300}{12300} \times 100$ ***(M1)***

3% ***A1***

[2 marks]

It is assumed that the number of students that apply to the university each year will follow a geometric sequence, u_n.

5b. Write down the common ratio of the sequence. *[1 mark]*

Markscheme

1.03 ***A1***

Note: Follow through from part (a).

[1 mark]

5c. Find an expression for u_n. *[1 mark]*

Markscheme

$(u_n =)\ 12\,300 \times 1.03^{n-1}$ ***A1***

[1 mark]

5d. Find the number of student applications the university expects to receive when $n = 11$. Express your answer to the nearest integer. *[2 marks]*

Markscheme

$(u_{11} =)\ 12\,300 \times 1.03^{10}$ ***(M1)***

16530 ***A1***

Note: Answer must be to the nearest integer. Do not accept 16500.

[2 marks]

In the first year there were $10\,380$ places at the university available for applicants. The admissions team announce that the number of places available will increase by 600 every year.

Let v_n represent the number of places available at the university in year n.

5e. Write down an expression for v_n . *[2 marks]*

Markscheme

$(v_n =)\ 10380 + 600(n-1)$ **OR** $600n + 9780$ ***M1A1***

Note: Award ***M1*** for substituting into arithmetic sequence formula, ***A1*** for correct substitution.

[2 marks]

For the first 10 years that the university is open, all places are filled. Students who receive a place each pay an $\$80$ acceptance fee.

5f. Calculate the total amount of acceptance fees paid to the university in the first 10 years. *[3 marks]*

Markscheme

$80 \times \frac{10}{2}(2(10380)+9(600))$ ***(M1)(M1)***

Note: Award ***(M1)*** for multiplying by 80 and ***(M1)*** for substitution into sum of arithmetic sequence formula.

$\$10\ 500\ 000 \quad (\$10\ 464\ 000)$ ***A1***

[3 marks]

When $n = k$, the number of places available will, for the first time, exceed the number of students applying.

5g. Find k. *[3 marks]*

Markscheme

$12\,300 \times 1.03^{n-1} < 10\,380 + 600(n-1)$ or equivalent ***(M1)***

Note: Award ***(M1)*** for equating their expressions from parts (b) and (c).

EITHER

graph showing $y = 12\,300 \times 1.03^{n-1}$ and $y = 10\,380 + 600(n-1)$ ***(M1)***

OR

graph showing $y = 12\,300 \times 1.03^{n-1} - (10\,380 + 600(n-1))$ ***(M1)***

OR

list of values including, $(u_{n=})\ 17537$ and $(v_{n=})\ 17580$ ***(M1)***

OR

$12.4953\ldots$ from graphical method or solving numerical equality ***(M1)***

Note: Award ***(M1)*** for a valid attempt to solve.

THEN

$(k =) 13$ ***A1***

[3 marks]

5h. State whether, for all $n > k$, the university will have places available for all applicants. Justify your answer. *[2 marks]*

Markscheme

this will not guarantee enough places. ***A1***

EITHER

A written statement that $u_n > v_n$, with range of n. ***R1***

Example: "when $n = 24$ (or greater), the number of applications will exceed the number of places again" ("$u_n > v_n,\ n \geq 24$").

OR

exponential growth will always exceed linear growth ***R1***

Note: Accept an equivalent sketch. Do not award ***A1R0***.

[2 marks]

Charlie and Daniella each began a fitness programme. On day one, they both ran $500\ \mathrm{m}$. On each subsequent day, Charlie ran $100\ \mathrm{m}$ more than the previous day whereas Daniella increased her distance by 2% of the distance ran on the previous day.

Calculate how far

6a. Charlie ran on day 20 of his fitness programme. *[2 marks]*

Markscheme

attempt to find u_{20} using an arithmetic sequence ***(M1)***

e.g. $u_1 = 500$ and $d = 100$ **OR** $u_{20} = 500 + 1900$ **OR** $500, 600, 700,\ \ldots$

(Charlie ran) $2400\ \mathrm{m}$ ***A1***

[2 marks]

6b. Daniella ran on day 20 of her fitness programme. *[3 marks]*

Markscheme

$(r =) 1.02$ ***(A1)***

attempt to find u_{20} using a geometric sequence ***(M1)***

e.g. $u_1 = 500$ and a value for r **OR** $500 \times r^{19}$ **OR** $500, 510, 520.2, \ldots$

(Daniella ran) $728 \text{ m} (728.405\ldots)$ ***A1***

[3 marks]

6c. On day n of the fitness programmes Daniella runs more than Charlie for the first time. *[3 marks]*

Find the value of n.

Markscheme

$500 \times 1.02^{n-1} > 500 + (n-1) \times 100$ ***(M1)***

attempt to solve inequality ***(M1)***

$n > 184.215\ldots$

$n = 185$ ***A1***

[3 marks]

A new concert hall was built with 14 seats in the first row. Each subsequent row of the hall has two more seats than the previous row. The hall has a total of 20 rows.

Find:

7a. the number of seats in the last row. *[3 marks]*

Markscheme

recognition of arithmetic sequence with common difference 2 ***(M1)***

use of arithmetic sequence formula ***(M1)***

$14+2(20-1)$

52 ***A1***

[3 marks]

7b. the total number of seats in the concert hall. *[2 marks]*

Markscheme

use of arithmetic series formula ***(M1)***

$\frac{14+52}{2}\times 20$

660 ***A1***

[2 marks]

The concert hall opened in 2019. The average number of visitors per concert during that year was 584. In 2020, the average number of visitors per concert increased by 1.2%.

7c. Find the average number of visitors per concert in 2020. *[2 marks]*

Markscheme

$584+(584\times 0.012)$ **OR** $584\times(1.012)^1$ ***(M1)***

$591(591.008)$ ***A1***

Note: Award ***M0A0*** if incorrect r used in part (b), and ***FT*** with their r in parts (c) and (d).

[2 marks]

The concert organizers use this data to model future numbers of visitors. It is assumed that the average number of visitors per concert will continue to increase each year by 1.2%.

7d. Determine the first year in which this model predicts the average number of visitors per concert will exceed the total seating capacity of the concert hall. *[5 marks]*

Markscheme

recognition of geometric sequence ***(M1)***

equating their nth geometric sequence term to their 660 ***(M1)***

Note: Accept inequality.

METHOD 1

EITHER

$600 = 584 \times (1.012)^{x-1}$ ***A1***

$(x - 1 =)10.3(10.2559\ldots)$

$x = 11.3(11.2559\ldots)$ ***A1***

2030 ***A1***

OR

$600 = 584 \times (1.012)^{x}$ ***A1***

$x = 10.3(10.2559\ldots)$ ***A1***

2030 ***A1***

METHOD 2

11^{th} term $658(657.987\ldots)$ ***(M1)A1***

12^{th} term $666(666.883\ldots)$ ***(M1)A1***

2030 ***A1***

Note: The last mark can be awarded if both their 11^{th} and 12^{th} correct terms are seen.

[5 marks]

7e. It is assumed that the concert hall will host 50 concerts each year. *[4 marks]*

Use the average number of visitors per concert per year to predict the **total** number of people expected to attend the concert hall from when it opens until the end of 2025.

Markscheme

7 seen ***(A1)***

EITHER

$584\left(\frac{1.012^7-1}{1.012-1}\right)$ ***(M1)***

multiplying their sum by 50 ***(M1)***

OR

sum of the number of visitors for their r and their seven years ***(M1)***

multiplying their sum by 50 ***(M1)***

OR

$29\ 200\left(\frac{1.012^7-1}{1.012-1}\right)$ ***(M1)(M1)***

THEN

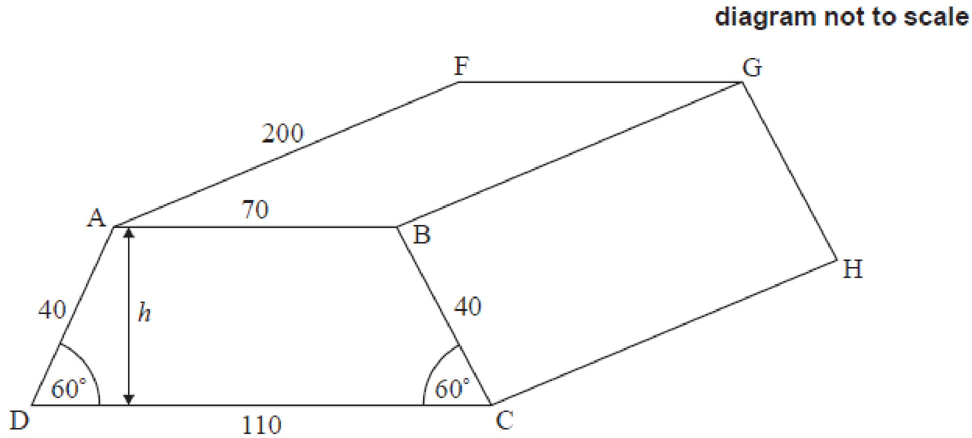
$212000(211907.3\ldots)$ ***A1***

Note: Follow though from their r from part (b).

[4 marks]

A large underground tank is constructed at Mills Airport to store fuel. The tank is in the shape of an isosceles trapezoidal prism, $\mathrm{ABCDEFGH}$.

$\mathrm{AB} = 70\ \mathrm{m}$, $\mathrm{AF} = 200\ \mathrm{m}$, $\mathrm{AD} = 40\ \mathrm{m}$, $\mathrm{BC} = 40\ \mathrm{m}$ and $\mathrm{CD} = 110\ \mathrm{m}$. Angle $\mathrm{ADC} = 60^\circ$ and angle $\mathrm{BCD} = 60^\circ$. The tank is illustrated below.



8a. Find h, the height of the tank. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\sin 60^\circ = \frac{h}{40}$ **OR** $\tan 60^\circ = \frac{h}{20}$ ***(M1)***

Note: Award ***(M1)*** for correct substitutions in trig ratio.

OR

$20^2 + h^2 = 40^2 \quad \left(\sqrt{40^2 - 20^2}\right)$ ***(M1)***

Note: Award ***(M1)*** for correct substitutions in Pythagoras' theorem.

$(h =)\ 34.6\ (\mathrm{m}) \quad \left(\sqrt{1200},\ 20\sqrt{3},\ 34.6410\ldots\right)$ ***(A1)(G2)***

[2 marks]

8b. Show that the volume of the tank is $624\ 000\ \mathrm{m}^3$, correct to three significant figures. *[3 marks]*

Markscheme

$\frac{1}{2}(70+110)(34.6410\ldots)\times 200$ ***(M1)(M1)***

Note: Award ***(M1)*** for their correctly substituted area of trapezium formula, provided all substitutions are positive. Award ***(M1)*** for multiplying by 200. Follow through from part (a).

OR

$\left(2\times\frac{1}{2}\times 20\times 34.6410\ldots+70\times 34.6410\ldots\right)\times 200$ ***(M1)(M1)***

Note: Award ***(M1)*** for the addition of correct areas for two triangles and one rectangle. Award ***(M1)*** for multiplying by 200. Follow through from part (a).

OR

$70\times 34.6410\ldots\times 200+2\times\frac{1}{2}\times 34.6410\ldots\times 20\times 200$ ***(M1)(M1)***

Note: Award ***(M1)*** for their correct substitution in volume of cuboid formula. Award ***(M1)*** for correctly substituted volume of triangular prism(s). Follow through from part (a).

$623538\ldots$ ***(A1)***

$624000\left(\mathrm{m}^3\right)$ ***(AG)***

Note: Both an unrounded answer that rounds to the given answer and the rounded value must be seen for the ***(A1)*** to be awarded.

[3 marks]

Once construction was complete, a fuel pump was used to pump fuel **into** the empty tank. The amount of fuel pumped into the tank by this pump **each hour** decreases as an arithmetic sequence with terms $u_1, u_2, u_3, \ldots, u_n$.

Part of this sequence is shown in the table.

Hour (n)	1st	2nd	3rd	...
Amount of fuel pumped into the tank in each hour, in m^3 (u_n)	45 000	43 200	41 400	...

8c. Write down the common difference, d. *[1 mark]*

Markscheme

$(d =)\ -1800$ ***(A1)***

[1 mark]

8d. Find the amount of fuel pumped into the tank in the 13th hour. *[2 marks]*

Markscheme

$(u_{13} =)\ 45000 + (13 - 1)(-1800)$ ***(M1)***

Note: Award ***(M1)*** for correct substitutions in arithmetic sequence formula.
OR
Award ***(M1)*** for a correct 4^{th} term seen as part of list.

$23400\ \left(\text{m}^3\right)$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from part (c) for their value of d.

[2 marks]

8e. Find the value of n such that $u_n = 0$. *[2 marks]*

Markscheme

$0 = 45000 + (n - 1)(-1800)$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into arithmetic sequence formula, equated to zero.

$(n =)\ \ 26$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (c). Award at most ***(M1)(A0)*** if their n is not a positive integer.

[2 marks]

8f. Write down the number of hours that the pump was pumping fuel into the tank. *[1 mark]*

Markscheme

25 ***(A1)*(ft)**

Note: Follow through from part (e)(i), but only if their final answer in (e)(i) is positive. If their n in part (e)(i) is not an integer, award ***(A1)*(ft)** for the nearest lower integer.

[1 mark]

At the end of the 2nd hour, the total volume of fuel in the tank was $88\ 200\ \text{m}^3$.

8g. Find the total amount of fuel pumped into the tank in the first 8 hours. *[2 marks]*

Markscheme

$(S_8 =)\ \frac{8}{2}(2 \times 45000 + (8-1) \times (-1800))$ ***(M1)***

Note: Award ***(M1)*** for their correct substitutions in arithmetic series formula. If a list method is used, award ***(M1)*** for the addition of their 8 correct terms.

$310\ 000\ \left(\mathrm{m}^3\right)\ (309\ 600)$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from part (c). Award at most ***(M1)(A0)*** if their final answer is greater than $624\ 000$.

[2 marks]

8h. Show that the tank will never be completely filled using this pump. *[3 marks]*

Markscheme

$(S_{25} =)\ \frac{25}{2}(2 \times 45000 + (25-1) \times (-1800))$, $(S_{25} =)\ \frac{25}{2}(45000 + 1800)$ ***(M1)***

Note: Award ***(M1)*** for their correct substitutions into arithmetic series formula.

$S_{25} = 585000\ \left(\mathrm{m}^3\right)$ ***(A1)*****(ft)*****(G1)***

Note: Award ***(M1)(A1)*** for correctly finding $S_{26} = 585000\ \left(\mathrm{m}^3\right)$, provided working is shown e.g. $(S_{26} =)\ \frac{26}{2}(2 \times 45000 + (26-1) \times (-1800))$, $(S_{26} =)\ \frac{26}{2}(45000 + 0)$. Follow through from part (c) and either their (e)(i) or (e)(ii). If $d < 0$ and their final answer is greater than $624\ 000$, award at most ***(M1)(A1)*****(ft)*****(R0)***. If $d > 0$, there is no maximum, award at most ***(M1)(A0)(R0)***. Award no marks if their number of terms is not a positive integer.

$585000\ \left(\mathrm{m}^3\right) < 624000\ \left(\mathrm{m}^3\right)$ ***(R1)***

Hence it will never be filled ***(AG)***

Note: The ***(AG)*** line must be seen. If it is omitted do not award the final ***(R1)***. Do not follow through within the part.
For unsupported $(S_{25}) = 585000$ seen, award at most ***(G1)(R1)(AG)***. Working

must be seen to follow through from parts (c) and (e)(i) or (e)(ii).

OR

$(S_n =)\ \frac{n}{2}(2 \times 45000 + (n-1) \times (-1800))$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into arithmetic series formula, with n.

Maximum of this function $585225\ \left(\mathrm{m}^3\right)$ ***(A1)***

Note: Follow through from part (c). Award at most ***(M1)(A1)*****(ft)*****(R0)*** if their final answer is greater than $624\ 000$. Award at most ***(M1)(A0)(R0)*** if their common difference is not -1800. Award at most ***(M1)(A0)(R0)*** if $585\ 225$ is not explicitly identified as the maximum of the function.

$585225\ \left(\mathrm{m}^3\right) < 624000\ \left(\mathrm{m}^3\right)$ ***(R1)***

Hence it will never be filled ***(AG)***

Note: The ***(AG)*** line must be seen. If it is omitted do not award the final ***(R1)***. Do not follow through within the part.

OR

sketch with concave down curve **and** labelled 624000 horizontal line ***(M1)***

Note: Accept a label of “tank volume” instead of a numerical value. Award ***(M0)*** if the line and the curve intersect.

curve explicitly labelled as $(S_n =)\ \frac{n}{2}(2 \times 45000 + (n-1) \times (-1800))$ or equivalent ***(A1)***

Note: Award ***(A1)*** for a written explanation interpreting the sketch. Accept a comparison of values, e.g $585225\ \left(\mathrm{m}^3\right) < 624000\ \left(\mathrm{m}^3\right)$, where 585225 is the graphical maximum. Award at most ***(M1)(A0)(R0)*** if their common difference is not -1800.

the line and the curve do not intersect ***(R1)***

hence it will never be filled ***(AG)***

Note: The ***(AG)*** line must be seen. If it is omitted do not award the final ***(R1)***. Do not follow through within the part.

OR

$624000 = \frac{n}{2}(2 \times 45000 + (n-1) \times (-1800))$ ***(M1)***

Note: Award ***(M1)*** for their correctly substituted arithmetic series formula equated to 624000 (623538).

Demonstrates there is no solution ***(A1)***

Note: Award ***(A1)*** for a correct working that the discriminant is less than zero **OR** correct working indicating there is no real solution in the quadratic formula.

There is no (real) solution (to this equation) ***(R1)***

hence it will never be filled ***(AG)***

Note: At most ***(M1)(A0)(R0)*** for their correctly substituted arithmetic series formula $= 624000,\ 623538$ or 622800 with a statement "no solution". Follow through from their part (b).

[3 marks]

John purchases a new bicycle for 880 US dollars (USD) and pays for it with a Canadian credit card. There is a transaction fee of 4.2 % charged to John by the credit card company to convert this purchase into Canadian dollars (CAD).

The exchange rate is 1 USD = 1.25 CAD.

9a. Calculate, in CAD, the total amount John pays for the bicycle. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

1.042 × 880 × 1.25 **OR** (880 + 0.042 × 880) × 1.25 ***(M1)(M1)***

Note: Award ***(M1)*** for multiplying 880 by 1.042 and ***(M1)*** for multiplying 880 by 1.25.

1150 (CAD) (1146.20 (CAD)) ***(A1)(G2)***

Note: Accept 1146.2 (CAD)

[3 marks]

John insures his bicycle with a US company. The insurance company produces the following table for the bicycle’s value during each year.

Year	Value of the bicycle (USD)
1st	880
2nd	704
3rd	563.20
…	…

The values of the bicycle form a geometric sequence.

9b. Find the value of the bicycle during the 5th year. **Give your answer to two decimal places**. *[3 marks]*

Markscheme

$\frac{704}{880}$ **OR** $\frac{563.20}{704}$ ***(M1)***

Note: Award ***(M1)*** for correctly dividing sequential terms to find the common ratio, or 0.8 seen.

$880(0.8)^{5-1}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into geometric sequence formula.

360.45 (USD) ***(A1)(G3)***

Note: Do not award the final ***(A1)*** if the answer is not correct to 2 decimal places. Award at most ***(M0)(M1)(A0)*** if $r = 1.25$.

[3 marks]

9c. Calculate, in years, when the bicycle value will be less than 50 USD. *[2 marks]*

Markscheme

$880(0.8)^{n-1} < 50$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into geometric sequence formula and (in)equating to 50. Accept weak or strict inequalities. Accept an equation. Follow through from their common ratio in part (b). Accept a sketch of their GP with $y = 50$ as a valid method.

OR

$u_{13} = 60.473$ **AND** $u_{14} = 48.379$ ***(M1)***

Note: Award ***(M1)*** for their u_{13} and u_{14} **both** seen. If the student states $u_{14} = 48.379 < 50$, without $u_{13} = 60.473$ seen, this is not sufficient to award ***(M1)***.

14 or "14th year" or "after the 13th year" ***(A1)*(ft)*(G2)***

Note: The context of the question requires the final answer to be an integer. Award at most ***(M1)(A0)*** for a final answer of 13.9 years. Follow through from their 0.8 in part (b).

[2 marks]

During the 1st year John pays 120 USD to insure his bicycle. Each year the amount he pays to insure his bicycle is reduced by 3.50 USD.

9d. Find the total amount John has paid to insure his bicycle for the first 5 years. *[3 marks]*

Markscheme

$\frac{5}{2}((2 \times 120) + (-3.5\,(5-1)))$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into arithmetic series formula, ***(A1)*** for correct substitution.

565 (USD) ***(A1)(G2)***

[3 marks]