Sequences review *[112 marks]*

The Osaka Tigers basketball team play in a multilevel stadium.

The most expensive tickets are in the first row. The ticket price, in Yen (¥), for each row forms an arithmetic sequence. Prices for the first three rows are shown in the following table.

Ticket pricing per game	
1st row	6800 Yen
2nd row	6550 Yen
3rd row	6300 Yen

1a. Write down the value of the common difference, d *[1 mark]*

. . . .

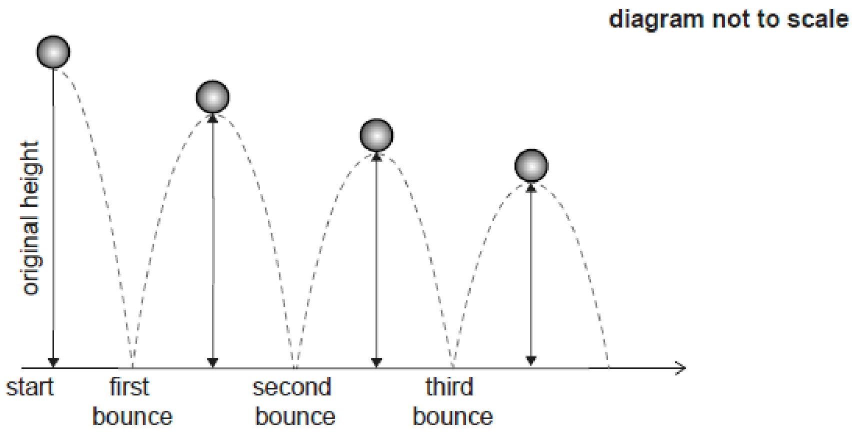
. . . .

. . . .

1b. Calculate the price of a ticket in the 16th row. *[2 marks]*

1c. Find the total cost of buying 2 tickets in each of the first 16 rows. *[3 marks]*

A ball is dropped from a height of 1.8 metres and bounces on the ground. The maximum height reached by the ball, after each bounce, is 85% of the previous maximum height.



2a. Show that the maximum height reached by the ball after it has bounced for the sixth time is $68\ \mathrm{cm}$, to the nearest cm. *[2 marks]*

....

....

....

....

....

....

2b. Find the number of times, after the first bounce, that the maximum height reached is greater than $10\ \mathrm{cm}$. *[2 marks]*

....

....

....

....

....

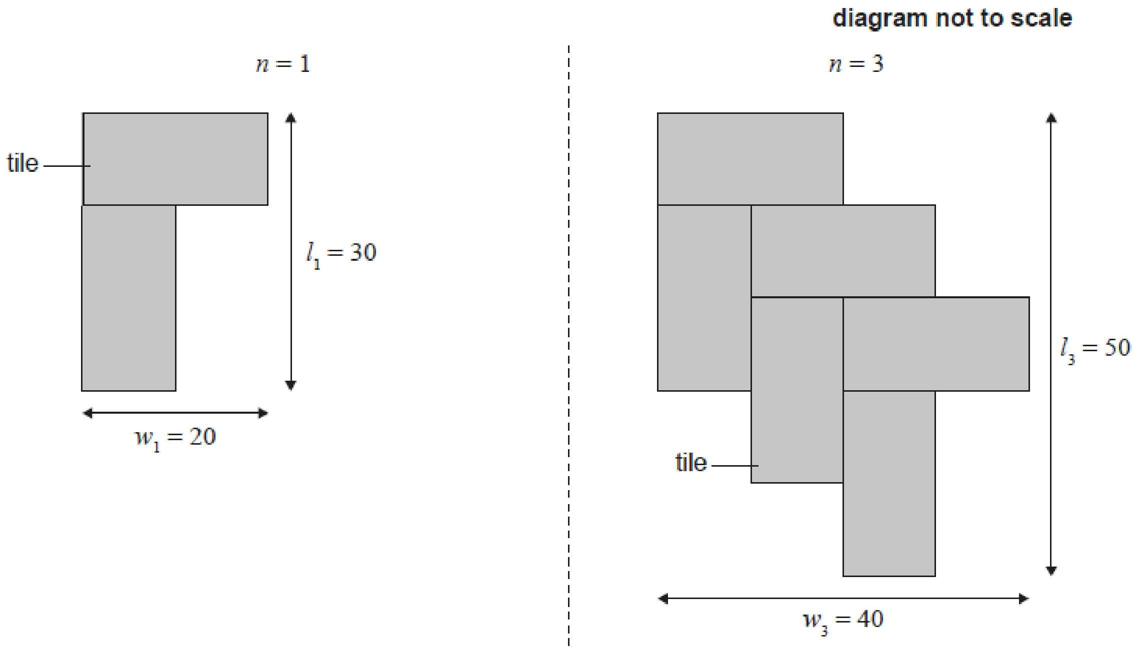
....

2c. Find the total **vertical** distance travelled by the ball from the point at which it is dropped until the fourth bounce. *[3 marks]*

Eddie decides to construct a path across his rectangular grass lawn using pairs of tiles.

Each tile is 10 cm wide and 20 cm long. The following diagrams show the path after Eddie has laid one pair and three pairs of tiles. This pattern continues until Eddie reaches the other side of his lawn. When n pairs of tiles are laid, the path has a width of w_n centimetres and a length l_n centimetres.

The following diagrams show this pattern for one pair of tiles and for three pairs of tiles, where the white space around each diagram represents Eddie's lawn.



The following table shows the values of w_n and l_n for the first three values of n.

Number of pairs of tiles, n	Width of lawn crossed by path, w_n (cm)	Length of lawn crossed by path, l_n (cm)
1	20	30
2	a	b
3	40	50

Find the value of

3a. a. *[1 mark]*

. . . .

. . . .

. . . .

3b. b. *[1 mark]*

Write down an expression in terms of n for

3c. w_n. *[2 marks]*

3d. l_n. *[1 mark]*

Eddie’s lawn has a length $740 \mathrm{~cm}$.

3e. Show that Eddie needs 144 tiles. *[2 marks]*

3f. Find the value of w_n for this path. *[1 mark]*

3g. Find the total area of the tiles in Eddie’s path. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer. *[3 marks]*

The tiles cost $\$24.50$ per square metre and are sold in packs of five tiles.

3h. Find the cost of a single pack of five tiles. *[3 marks]*

....

....

....

....

....

....

....

....

....

To allow for breakages Eddie wants to have at least 8% more tiles than he needs.

3i. Find the minimum number of packs of tiles Eddie will need to order. *[3 marks]*

....

....

....

....

....

....

....

....

....

There is a fixed delivery cost of $\$35$.

3j. Find the total cost for Eddie's order. *[2 marks]*

Scott purchases food for his dog in large bags and feeds the dog the same amount of dog food each day. The amount of dog food left in the bag at the end of each day can be modelled by an arithmetic sequence.

On a particular day, Scott opened a new bag of dog food and fed his dog. By the end of the third day there were 115.5 cups of dog food remaining in the bag and at the end of the eighth day there were 108 cups of dog food remaining in the bag.

Find the number of cups of dog food

4a. fed to the dog per day. *[3 marks]*

4b. remaining in the bag at the end of the first day. *[1 mark]*

....

....

....

4c. Calculate the number of days that Scott can feed his dog with one bag of food. *[2 marks]*

....

....

....

....

....

....

In 2021, Scott spent $\$625$ on dog food. Scott expects that the amount he spends on dog food will increase at an annual rate of 6.4%.

4d. Determine the amount that Scott expects to spend on dog food in 2025. *[3 marks]*
Round your answer to the nearest dollar.

4e. *[1 mark]*

Calculate the value of $\sum_{n=1}^{10}\left(625 \times 1.064^{(n-1)}\right)$.

4f. Describe what the value in part (d)(i) represents in this context. *[2 marks]*

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

4g. Comment on the appropriateness of modelling this scenario with a geometric sequence. *[1 mark]*

. . . .

. . . .

. . . .

The admissions team at a new university are trying to predict the number of student applications they will receive each year.

Let n be the number of years that the university has been open. The admissions team collect the following data for the first two years.

Year, n	Number of applications received in year n
1	12300
2	12669

5a. Calculate the percentage increase in applications from the first year to the second year. *[2 marks]*

....

....

....

....

....

....

It is assumed that the number of students that apply to the university each year will follow a geometric sequence, u_n.

5b. Write down the common ratio of the sequence. *[1 mark]*

....

....

....

5c. Find an expression for u_n. *[1 mark]*

....

....

....

5d. Find the number of student applications the university expects to receive when $n = 11$. Express your answer to the nearest integer. *[2 marks]*

In the first year there were $10\ 380$ places at the university available for applicants. The admissions team announce that the number of places available will increase by 600 every year.

Let v_n represent the number of places available at the university in year n.

5e. Write down an expression for v_n . *[2 marks]*

For the first 10 years that the university is open, all places are filled. Students who receive a place each pay an $\$80$ acceptance fee.

5f. Calculate the total amount of acceptance fees paid to the university in the first 10 years. *[3 marks]*

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

When $n = k$, the number of places available will, for the first time, exceed the number of students applying.

5g. Find k. *[3 marks]*

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

5h. State whether, for all $n > k$, the university will have places available for all applicants. Justify your answer. *[2 marks]*

....

....

....

....

....

....

Charlie and Daniella each began a fitness programme. On day one, they both ran $500 \mathrm{~m}$. On each subsequent day, Charlie ran $100 \mathrm{~m}$ more than the previous day whereas Daniella increased her distance by 2% of the distance ran on the previous day.

Calculate how far

6a. Charlie ran on day 20 of his fitness programme. *[2 marks]*

6b. Daniella ran on day 20 of her fitness programme. *[3 marks]*

6c. On day n of the fitness programmes Daniella runs more than Charlie for the first time. *[3 marks]*

Find the value of n.

....

....

....

....

....

....

....

....

....

A new concert hall was built with 14 seats in the first row. Each subsequent row of the hall has two more seats than the previous row. The hall has a total of 20 rows.

Find:

7a. the number of seats in the last row. *[3 marks]*

7b. the total number of seats in the concert hall. *[2 marks]*

The concert hall opened in 2019. The average number of visitors per concert during that year was 584. In 2020, the average number of visitors per concert increased by 1.2%.

7c. Find the average number of visitors per concert in 2020. *[2 marks]*

....

....

....

....

....

....

The concert organizers use this data to model future numbers of visitors. It is assumed that the average number of visitors per concert will continue to increase each year by 1.2%.

7d. Determine the first year in which this model predicts the average number of visitors per concert will exceed the total seating capacity of the concert hall. *[5 marks]*

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

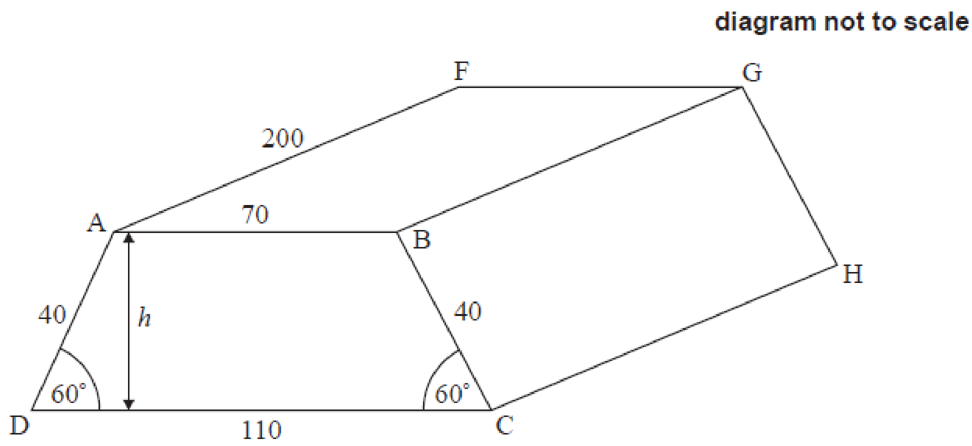
. . . .

7e. It is assumed that the concert hall will host 50 concerts each year. *[4 marks]*

Use the average number of visitors per concert per year to predict the **total** number of people expected to attend the concert hall from when it opens until the end of 2025.

A large underground tank is constructed at Mills Airport to store fuel. The tank is in the shape of an isosceles trapezoidal prism, $\mathrm{ABCDEFGH}$.

$\mathrm{AB} = 70\ \mathrm{m}$, $\mathrm{AF} = 200\ \mathrm{m}$, $\mathrm{AD} = 40\ \mathrm{m}$, $\mathrm{BC} = 40\ \mathrm{m}$ and $\mathrm{CD} = 110\ \mathrm{m}$. Angle $\mathrm{ADC} = 60^{\circ}$ and angle $\mathrm{BCD} = 60^{\circ}$. The tank is illustrated below.



8a. Find h, the height of the tank. *[2 marks]*

. . . .

8b. Show that the volume of the tank is $624\ 000\ \mathrm{m}^3$, correct to three significant figures. *[3 marks]*

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

Once construction was complete, a fuel pump was used to pump fuel **into** the empty tank. The amount of fuel pumped into the tank by this pump **each hour** decreases as an arithmetic sequence with terms $u_1, u_2, u_3, \ldots, u_n$.

Part of this sequence is shown in the table.

Hour (n)	1st	2nd	3rd	…
Amount of fuel pumped into the tank in each hour, in $\mathbf{m}^3$ (u_n)	45 000	43 200	41 400	…

8c. Write down the common difference, d. *[1 mark]*

. . . .

. . . .

. . . .

8d. Find the amount of fuel pumped into the tank in the 13th hour. *[2 marks]*

8e. Find the value of n such that $u_n = 0$. *[2 marks]*

8f. Write down the number of hours that the pump was pumping fuel into the tank. *[1 mark]*

At the end of the 2nd hour, the total volume of fuel in the tank was $88\ 200\ \text{m}^3$.

8g. Find the total amount of fuel pumped into the tank in the first 8 hours. *[2 marks]*

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

8h. Show that the tank will never be completely filled using this pump. *[3 marks]*

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

John purchases a new bicycle for 880 US dollars (USD) and pays for it with a Canadian credit card. There is a transaction fee of 4.2 % charged to John by the credit card company to convert this purchase into Canadian dollars (CAD).

The exchange rate is 1 USD = 1.25 CAD.

9a. Calculate, in CAD, the total amount John pays for the bicycle. *[3 marks]*

John insures his bicycle with a US company. The insurance company produces the following table for the bicycle's value during each year.

Year	Value of the bicycle (USD)
1st	880
2nd	704
3rd	563.20
…	…

The values of the bicycle form a geometric sequence.

9b. Find the value of the bicycle during the 5th year. **Give your answer to two decimal places**. *[3 marks]*

....

....

....

....

....

....

....

....

....

9c. Calculate, in years, when the bicycle value will be less than 50 USD. *[2 marks]*

....

....

....

....

....

....

During the 1st year John pays 120 USD to insure his bicycle. Each year the amount he pays to insure his bicycle is reduced by 3.50 USD.

9d. Find the total amount John has paid to insure his bicycle for the first 5 years. *[3 marks]*

....

....

....

....

....

....