Statistics 01.02 [106 marks]

A large company surveyed 160 of its employees to find out how much time they spend traveling to work on a given day. The results of the survey are shown in the following cumulative frequency diagram.



1a. Find the median number of minutes spent traveling to work.



1b. Find the number of employees whose travelling time is within 15 [3 marks] minutes of the median.

Only 10% of the employees spent more than k minutes traveling to work.

1c. Find the value of k.

[3 marks]



The results of the survey can also be displayed on the following box-and-whisker diagram.



1d. Write down the value of b.

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Markscheme

b = 70 A1

[1 mark]
```

1e. Find the value of a.

Marksch	eme	
recognizing a is first a	quartile value	(M1)
40 employees		
<i>a</i> = 33 <i>A1</i>		
[2 marks]		

1f. Hence, find the interquartile range.

 Markscheme

 47 - 33
 (M1)

 IQR = 14
 A1

 [2 marks]

1g. Travelling times of less than p minutes are considered outliers.[2 marks]Find the value of p.

[2 marks]

[1 mark]



2. A data set consisting of 16 test scores has mean 14.5. One test score of [4 marks] 9 requires a second marking and is removed from the data set.

Find the mean of the remaining 15 test scores.

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

 $\frac{\frac{16}{\sum}}{16} = 14.5 \text{ (M1)}$ Note: Award M1 for use of $\bar{x} = \frac{\frac{n}{\sum}}{\frac{16}{n}}$. $\frac{\frac{16}{\sum}}{\frac{1}{10}} = 232 \text{ (A1)}$ new $\bar{x} = \frac{232-9}{15}$ (A1) $= 14.9 (= 14.8\bar{6}, = \frac{223}{15}) \text{ A1}$ Note: Do not accept 15. [4 marks]

A set of data comprises of five numbers x_1 , x_2 , x_3 , x_4 , x_5 which have been placed in ascending order.

^{3a.} Recalling definitions, such as the Lower Quartile is the $\frac{n+1}{4}th$ piece of [2 marks] data with the data placed in order, find an expression for the Interquartile Range.



3b. Hence, show that a data set with only 5 numbers in it cannot have any *[5 marks]* outliers.

```
MarkschemeUQ + 1.5IQR = 1.25x_4 + 1.25x_5 - 0.75x_1 - 0.75x_2 \ge x_5M1A1Since 1.25x_4 + 0.25x_5 \ge 0.75x_1 + 0.75x_2 due to the ascending order.R1Similarly LQ - 1.5IQR = 1.25x_1 + 1.25x_2 - 0.75x_4 - 0.75x_5 \le x_1M1A1Since 0.25x_1 + 1.25x_2 \le 0.75x_3 + 0.75x_4 due to the ascending order.So there are no outliers for a data set of 5 numbers.GrantsAG
```

3c. Give an example of a set of data with 7 numbers in it that does have an *[2 marks]* outlier, justify this fact by stating the Interquartile Range.

Markscheme For example 1, 2, 3, 4, 5, 6, 100 where *IQR* = 4 **A1A1** [2 marks]

The number of hours spent exercising each week by a group of students is shown in the following table.

Exercising time (in hours)	Number of students
2	5
3	1
4	4
5	3
6	x

The median is 4.5 hours.

4a. Find the value of x.

[2 marks]

Markscheme

EITHER

recognising that half the total frequency is 10 (may be seen in an ordered list or indicated on the frequency table) (A1)

OR

5 + 1 + 4 = 3 + x (A1) OR $\sum f = 20$ (A1) THEN x = 7 A1 [2 marks]

4b. Find the standard deviation.

METHOD 1

- $1.58429\ldots$
- 1.58 **A2**

METHOD 2

EITHER

$$\sigma^{2} = \frac{5 \times (2-4.3)^{2} + 1 \times (3-4.3)^{2} + 4 \times (4-4.3)^{2} + 3 \times (5-4.3)^{2} + 7 \times (6-4.3)^{2}}{20} \quad (=2.51)$$
(A1)

OR

$$\sigma^2 = rac{5 imes 2^2 + 1 imes 3^2 + 4 imes 4^2 + 3 imes 5^2 + 7 imes 6^2}{20} - 4.3^2 \ (= 2.51)$$
 (A1)

THEN

 $\sigma = \sqrt{2.51} = 1.58429...$ = 1.58 **A1**

A random sample of nine adults were selected to see whether sleeping well affected their reaction times to a visual stimulus. Each adult's reaction time was measured twice.

The first measurement for reaction time was taken on a morning after the adult had slept well. The second measurement was taken on a morning after the same adult had not slept well.

The box and whisker diagrams for the reaction times, measured in seconds, are shown below.



Consider the box and whisker diagram representing the reaction times after sleeping well.

5a. State the median reaction time after sleeping well.



5b. Verify that the measurement of $0.\,46$ seconds is not an outlier.

[3 marks]

[1 mark]

Markscheme
${ m IQR} = 0.35 - 0.27\;(=0.08)$ (s) (A1)
substituting their IQR into correct expression for upper fence (A1)
0.35+1.5 imes 0.08~(=0.47) (s)
0.46 < 0.47 R1
so 0.46 (s) is not an outlier \textbf{AG}
[3 marks]

5c. State why it appears that the mean reaction time is greater than the *[1 mark]* median reaction time.

Markscheme

EITHER

the median is closer to the lower quartile (positively skewed) **R1**

OR

The distribution is positively skewed **R1**

OR

the range of reaction times below the median is smaller than the range of reaction times above the median **R1**

Note: These are sample answers from a range of acceptable correct answers. Award **R1** for any correct statement that explains this. Do not award **R1** if there is also an incorrect statement, even if another statement in the answer is correct. Accept a correctly and clearly labelled diagram.

[1 mark]

5d. Now consider the two box and whisker diagrams.

[1 mark]

Comment on whether these box and whisker diagrams provide any evidence that might suggest that not sleeping well causes an increase in reaction time.

EITHER

the distribution for 'not sleeping well' is centred at a higher reaction time *R1*

OR

The median reaction time after not sleeping well is equal to the upper quartile reaction time after sleeping well *R1*

OR

75% of reaction times are $<0.\,35$ seconds after sleeping well, compared with 50% after not sleeping well $$\it R1$$

OR

the sample size of 9 is too small to draw any conclusions \qquad **R1**

Note: These are sample answers from a range of acceptable correct answers. Accept any relevant correct statement **that relates to the median and/or quartiles shown in the box plots**. **Do not accept** a comparison of means. Do not award **R1** if there is also an incorrect statement, even if another statement in the answer is correct.

Award **RO** to "correlation does not imply causation".

[1 mark]

A research student weighed lizard eggs in grams and recorded the results. The following box and whisker diagram shows a summary of the results where L and U are the lower and upper quartiles respectively.

diagram not to scale



The interquartile range is 20 grams and there are no outliers in the results.

6a. Find the minimum possible value of U.

[3 marks]

attempt to use definition of outlier 1. $5 \times 20 + Q_3$ (M1) 1. $5 \times 20 + U \ge 75$ ($\Rightarrow U \ge 45$, accept U > 45) OR $1.5 \times 20 + Q_3 = 75$ A1 minimum value of U = 45 A1 [3 marks]

6b. Hence, find the minimum possible value of L.

[2 marks]

Markscheme

attempt to use interquartile range **(M1)** U-L=20 (may be seen in part (a)) OR $L\geq 25$ (accept L>25) minimum value of L=25 **A1 [2 marks]**

Each athlete on a running team recorded the distance (M miles) they ran in $30\,$ minutes.

The median distance is $4\ {\rm miles}$ and the interquartile range is $1.1\ {\rm miles}.$

This information is shown in the following box-and-whisker plot.



7a. Find the value of a.

Markscheme * This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure. valid approach (*M1*) $eg \quad Q_3 - Q_1 , \ Q_3 - 1.1 , \ 4.5 - a = 1.1$ a = 3.4 A1 N2 [2 marks]

The distance in miles, M, can be converted to the distance in kilometres, K, using the formula $K = \frac{8}{5}M$.

7b. Write down the value of the median distance in kilometres (km). [1 mark]

22	
$rac{32}{5}(=6.4)$ (km) A1 N1	= 6.4) (km) A1 N1
[1 mark]	ark]

The variance of the distances run by the athletes is $\frac{16}{9}$ km².

The standard deviation of the distances is b miles.

7c. Find the value of b.

[4 marks]

METHOD 1 (standard deviation first)

valid approach (M1) eg standard deviation = $\sqrt{\text{variance}}$, $\sqrt{\frac{16}{9}}$ standard deviation = $\frac{4}{3}$ (km) (A1) valid approach to convert **their** standard deviation (M1) eg $\frac{4}{3} \times \frac{5}{8}$, $\sqrt{\frac{16}{9}} = \frac{8}{5}M$ $\frac{20}{24}$ (miles) $\left(=\frac{5}{6}\right)$ A1 N3

Note: If no working shown, award *M1A1M0A0* for the value $\frac{4}{3}$. If working shown, and candidate's final answer is $\frac{4}{3}$, award *M1A1M0A0*.

METHOD 2 (variance first) valid approach to convert variance **(M1)** $eg \left(\frac{5}{8}\right)^2, \frac{64}{25}, \frac{16}{9} \times \left(\frac{5}{8}\right)^2$ variance $= \frac{25}{36}$ **(A1)** valid approach **(M1)** eg standard deviation $= \sqrt{\text{variance}}, \sqrt{\frac{25}{36}}, \sqrt{\frac{16}{9} \times \left(\frac{5}{8}\right)^2}$

 $rac{20}{24}$ (miles) $\left(=rac{5}{6}
ight)$ A1 N3 [4 marks] A total of 600 athletes from different teams compete in a 5 km race. The times the 600 athletes took to run the 5 km race are shown in the following cumulative frequency graph.



There were 400 athletes who took between 22 and m minutes to complete the $5 \ \rm km$ race.

(M1)

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7d. Find m.
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[3 marks]

Markschemecorrect frequency for 22 minutes (A1)eg 20adding their frequency (do not accept 22 + 400)eg 20 + 400 , 420 athletesm = 30 (minutes)A1 N3[3 marks]

7e. The first 150 athletes that completed the race won a prize.

Given that an athlete took between 22 and m minutes to complete the 5 km race, calculate the probability that they won a prize.

Markscheme 27 (minutes) (A1) correct working (A1) 130 athletes between 22 and 27 minutes, $\mathrm{P}(22 < t < 27) {=} rac{150 {-} 20}{600} \;, \; rac{13}{60}$ eg evidence of conditional probability or reduced sample space (M1) eg $P(A \mid B)$, $P(t < 27 \mid 22 < t < 30)$, $\frac{P(22 < t < 27)}{P(22 < t < m)}$, $\frac{150}{400}$ correct working (A1) $\frac{600}{400}$, $\frac{150-20}{400}$ eg $\frac{130}{400}$ $\left(\frac{13}{40} = \frac{78000}{240000} = \frac{390}{1200} = 0.325\right)$ **A1** N5 **Note:** If no other working is shown, award **AOAOM1AOAO** for answer of $\frac{150}{400}$. Award **NO** for answer of $\frac{3}{8}$ with no other working shown. [5 marks]

Hafizah harvested 49 mangoes from her farm. The weights of the mangoes, w, in grams, are shown in the following grouped frequency table.

Weight (g)	$100 \le w < 200$	$200 \le w < 300$	$300 \le w < 400$	$400 \le w < 500$	$500 \le w < 600$
Frequency	4	7	14	16	8

8a. Write down the modal group for these data.

[1 mark]

[5 marks]

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure. It appeared in a paper that permitted the use of a calculator, and so might not be suitable for all forms of practice.

 $400 \le w < 500$ (A1) (C1)

Note: Accept alternative notation [400, 500) or [400, 500[. Do not accept "400-500".

[1 mark]

8b. Use your graphic display calculator to find an estimate of the standard *[2 marks]* deviation of the weights of mangoes from this harvest.

Markscheme						
115 $(115.265(g))$ (A2) (C2)						
Note: Award <i>(A1)(A0)</i> for an answer of $116 (116.459)$.						
[2 marks]						

[3 marks]





Note: Award **(A2)** for all correct heights of bars or **(A1)** for three or four correct heights of bars.

Award **(A1)** for rectangular bars all with correct left and right end points (100, 200, 300, 400, 500 and 600) and for no gaps; the bars do **not** have to be shaded.

Award at most (A2)(A0) if a ruler is not used for all lines.

[3 marks]

The fastest recorded speeds of eight animals are shown in the following table.

Animal	Speed ($\mathrm{km}\mathrm{h}^{-1}$)
Golden eagle	300
Swordfish	97
Hare	80
Lion	80
Horse	71
Zebra	64
Komodo dragon	21
Tiger beetle	6

9a. State whether **speed** is a continuous or discrete variable.

[1 mark]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

continuous (A1) (C1)

[1 mark]

9b. Write down the median speed for these animals.

 Markscheme

 75.5 (km h⁻¹)

 (A1) (C1)

 Note: Answer must be exact.

 [1 mark]

9c. Write down the range of the animal speeds.

[1 mark]

Markscheme

294 (km h⁻¹) *(A1) (C1)* [1 mark] [1 mark]



9e. For these eight animals write down the standard deviation.

[1 mark]

Markscheme

84.6 (84.5597...)(km h⁻¹) (A1) (C1)

Note: If the response to part (d)(i) is awarded zero marks, a correct response to part (d)(ii) is awarded *(C2)*.

[1 mark]

A florist sells bouquets of roses. The florist recorded, in **Table 1**, the number of roses in each bouquet sold to customers.

Table 1

Number of roses in a bouquet (<i>n</i>)	2	3	4	5	6	7	8	9	10	11	12
Number of customers (<i>f</i>)	9	2	4	5	7	3	10	2	3	1	4

The roses can be arranged into bouquets of size small, medium or large. The data from **Table 1** has been organized into a cumulative frequency table, **Table 2**.

Table 2

Bouquet size	Number of roses (n)	Frequency (f)	Cumulative frequency
small	$2 \le n \le 4$	15	
medium	$5 \le n \le 8$	25	
large	$9 \le n \le 12$		

10a. Complete the cumulative frequency table.

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

Bouquet size	Number of roses (n)	Frequency (f)	Cumulative frequency	
small	$2 \le n \le 4$	15	15	
medium	$5 \le n \le 8$	25	40	
large	$9 \le n \le 12$	10	50	

(A1)(ft) (C2)

Note: Award **(A1)** for 10; **(A1)(ft)** for the last column all correct. Follow through from *their* 10 for *their* 50 in the last column.

[2 marks]

10b. Write down the probability that a bouquet of roses sold is **not** small. [2 marks]



10c. A customer buys a large bouquet.

[2 marks]

)

Find the probability that there are 12 roses in this bouquet.

Markscheme

 $\frac{4}{10} \left(0.4, \frac{2}{5}, 40\% \right)$ (A1)(A1)(ft) (C2)

Note: Award **(A1)** for a numerator of 4 and **(A1)(ft)** for *their* 10 as denominator. Follow through from part (a).

A group of 10 girls recorded the number of hours they spent watching television during a particular week. Their results are summarized in the box-and-whisker plot below.



11a. The range of the data is 16. Find the value of a.

Markscheme
* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.
valid approach (M1)
eg 16 + 8, a - 8
24 (hours) A1 N2
[2 marks]

11b. Find the value of the interquartile range.

 Markscheme

 valid approach
 (M1)

 eg 20 - 15, $Q_3 - Q_1$, 15 - 20

 IQR = 5
 A1 N2

 [2 marks]

The group of girls watched a total of 180 hours of television.

11c. Find the mean number of hours that the girls in this group spent [2 marks] watching television that week.

[2 marks]

	Markscheme	
	correct working (A1)	
	$eg = \frac{180}{10}$, $\frac{180}{n}$, $\frac{\sum x}{10}$	
	mean = 18 (hours) A1 N2	
	[2 marks]	
	A group of 20 boys also recorded the number of hours they spent watching television that same week. Their results are summarized in the table below. $\overline{x} = 21$ $\sigma = 3$	
Lld	. Find the total number of hours the group of boys spent watching <i>[2 marks]</i> television that week.	
	Markscheme	

11e. Find the mean number of hours that all 30 girls and boys spent[3 marks]watching television that week.

attempt to find total hours for group B (M1)

group B total hours = 420 (seen anywhere) **A1 N2**

eg $ar{x} imes n$

Markscheme
attempt to find sum for combined group (may be seen in working) (M1)
eg 180 + 420, 600
correct working (A1)
eg $rac{180+420}{30}$, $rac{600}{30}$
mean = 20 (hours) A1 N2
[3 marks]

The following week, the group of boys had exams. During this exam week, the boys spent half as much time watching television compared to the previous week.

For this exam week, find

11f. the mean number of hours that the group of boys spent watching [2 marks] television.

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Markschemevalid approach to find the new mean(M1)eg \quad \frac{1}{2}\mu, \quad \frac{1}{2} \times 21mean = \frac{21}{2} (= 10.5) hoursA1 N2[2 marks]
```

University students were surveyed and asked how many hours, h , they worked each month. The results are shown in the following table.

Hours per month, <i>h</i>	Frequency	Cumulative frequency
$0 < h \le 10$	3	3
$10 < h \le 20$	7	10
$20 < h \le 30$	10	20
$30 < h \le 40$	14	34
$40 < h \le 50$	р	44
$50 < h \le 60$	6	50
$60 < h \le 70$	4	54
$70 < h \le 80$	2	q

Use the table to find the following values.

12a.*p*.

[1 mark]

Markscheme p = 10 (A1) (C1)

Note: Award **(A1)** for each correct value. [1 mark]

Markscheme
q = 56 (A1) (C1)
Note: Award (A1) for each correct value
[1 mark]

The first five class intervals, indicated in the table, have been used to draw part of a cumulative frequency curve as shown.



12c. On the same grid, complete the cumulative frequency curve for these [2 marks] data.



[2 marks]

12d. Use the cumulative frequency curve to find an estimate for the number [2 marks] of students who worked at most 35 hours per month.



A health inspector analysed the amount of sugar in 500 different **snacks** prepared in various school cafeterias. The collected data are shown in the following boxand-whisker diagram.



13a. State what 13 represents in the given diagram.



13b. Write down the interquartile range for this data.

[2 marks]

[1 mark]

Markscheme18 - 12 (A1)Note: Award (M1) for correct quartiles seen.6 (g) (A1) (C2)[2 marks]

13c. Write down the approximate number of snacks whose amount of sugar [1 mark] ranges from 18 to 20 grams.



125 (A1) (C1) [1 mark] 13d. The health inspector visits two school cafeterias. She inspects the same [2 marks] number of **meals** at each cafeteria. The data is shown in the following box-and-whisker diagrams.



Meals prepared in the school cafeterias are required to have less than 10 grams of sugar.

State, giving a reason, which school cafeteria has more meals that **do not** meet the requirement.



Ten students were asked for the distance, in km, from their home to school. Their responses are recorded below.

0.3 0.4 3 3 3.5 5 7 8 8 10

14a. For these data, find the mean distance from a student's home to school.[2 marks]

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

evidence of finding $\frac{\sum x}{n}$ (M1) $eg \quad \frac{0.3+0.4+3+\ldots+10}{10}$, $\frac{48.2}{10}$ $\bar{x} = 4.82$ (exact) A1 N2 [2 marks]

The following box-and-whisker plot represents this data.



14b. Find the value of p.

Markschemep = 4.25 (exact)A1 N1[1 mark]

14c. Find the interquartile range.

Markscheme valid approach (M1) $eg Q_3 - Q_1 3 - 8, 3 to 8$ IQR = 5 A1 N2 [2 marks]

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[1 mark]

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