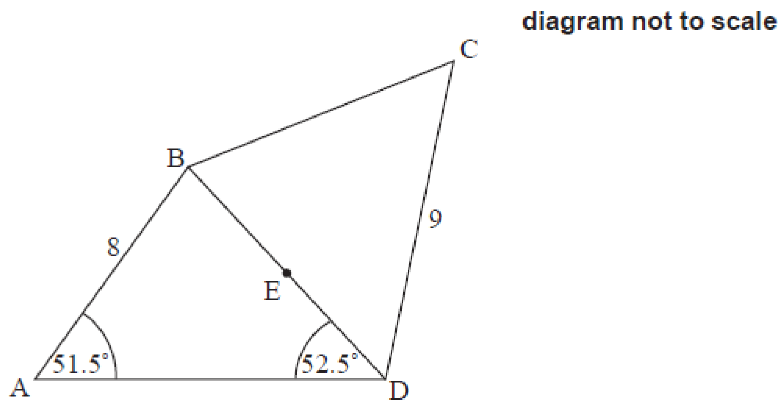
Trigonometry review *[100 marks]*

Using geometry software, Pedro draws a quadrilateral ABCD. $\mathrm{AB} = 8\ \mathrm{cm}$ and $\mathrm{CD} = 9\ \mathrm{cm}$. Angle $\mathrm{BAD} = 51.5^\circ$ and angle $\mathrm{ADB} = 52.5^\circ$. This information is shown in the diagram.



1a. Calculate the length of BD. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\frac{\mathrm{BD}}{\sin 51.5^\circ} = \frac{8}{\sin 52.5^\circ}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted sine rule, ***(A1)*** for correct substitution.

$(\mathrm{BD} =)\ 7.89\ (\mathrm{cm})\ \ (7.89164\ldots)$ ***(A1)(G2)***

Note: If radians are used the answer is $9.58723\ldots$ award at most ***(M1)(A1)(A0)***.

[3 marks]

$\mathrm{CE} = 7\ \mathrm{cm}$, where point E is the midpoint of BD.

1b. Show that angle $\mathrm{EDC} = 48.0^\circ$, correct to three significant figures. *[4 marks]*

Markscheme

$\cos \mathrm{EDC} = \frac{9^2+3.94582\ldots^2-7^2}{2\times 9\times 3.94582\ldots}$ ***(A1)*(ft)*(M1)(A1)*(ft)**

Note: Award ***(A1)*** for $3.94582\ldots$ or $\frac{7.89164\ldots}{2}$ seen, ***(M1)*** for substituted cosine rule, ***(A1)*(ft)** for correct substitutions.

$(\mathrm{EDC} =)\ 47.9515\ldots^\circ$ ***(A1)***

48.0° (3 sig figures) ***(AG)***

Note: Both an unrounded answer that rounds to the given answer and the rounded value must be seen for the final ***(M1)*** to be awarded.
Award at most ***(A1)*(ft)*(M1)(A1)*(ft)*(A0)*** if the known angle 48.0° is used to validate the result. Follow through from their BD in part (a).

[4 marks]

1c. Calculate the area of triangle BDC. *[3 marks]*

Markscheme

Units are required in this question.

$(\text{area} =)\ \frac{1}{2} \times 7.89164\ldots \times 9 \times \sin 48.0^\circ$ ***(M1)(A1)*(ft)**

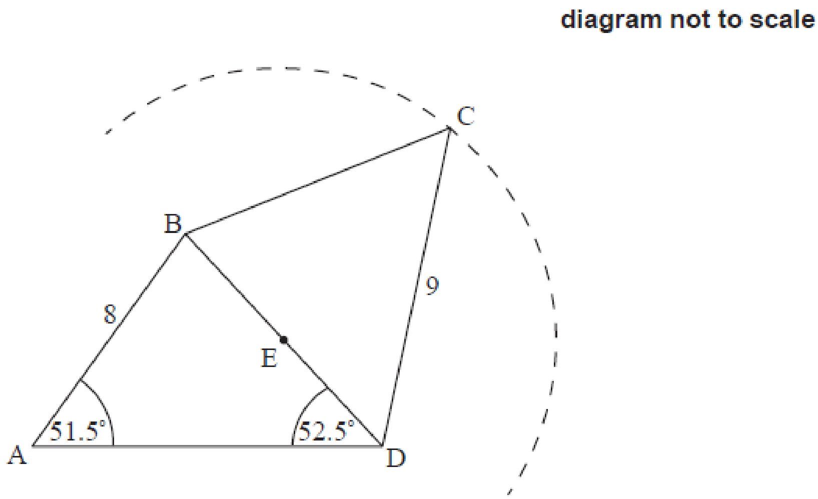
Note: Award ***(M1)*** for substituted area formula. Award ***(A1)*** for correct substitution.

$(\text{area} =)\ 26.4\ \text{cm}^2\ \ (26.3908\ldots)$ ***(A1)(ft)(G3)***

Note: Follow through from part (a).

[3 marks]

1d. Pedro draws a circle, with centre at point E, passing through point C. *[5 marks]*
Part of the circle is shown in the diagram.



Show that point A lies outside this circle. Justify your reasoning.

Markscheme

$\mathrm{AE}^2 = 8^2 + (3.94582\ldots)^2 - 2 \times 8 \times 3.94582\ldots \cos(76^\circ)$ ***(A1)(M1)(A1)*(ft)**

Note: Award ***(A1)*** for 76° seen. Award ***(M1)*** for substituted cosine rule to find AE, ***(A1)*(ft)** for correct substitutions.

$(\mathrm{AE} =)\ 8.02\,(\mathrm{cm})\ \ (8.01849\ldots)$ ***(A1)(ft)(G3)***

Note: Follow through from part (a).

OR

$\mathrm{AE}^2 = 9.78424\ldots^2 + (3.94582\ldots)^2 - 2 \times 9.78424\ldots \times 3.94582\ldots \cos(52.5^\circ$ ***(A1)(M1)(A1)*(ft)**

Note: Award ***(A1)*** for $\mathrm{AD}\ (9.78424\ldots)$ or 76° seen. Award ***(M1)*** for substituted cosine rule to find AE (do not award ***(M1)*** for cosine or sine rule to find AD), ***(A1)*(ft)** for correct substitutions.

$(\mathrm{AE} =)\ 8.02\,(\mathrm{cm})\ \ (8.01849\ldots)$ ***(A1)(ft)(G3)***

Note: Follow through from part (a).

$8.02 > 7.$ ***(A1)(ft)***

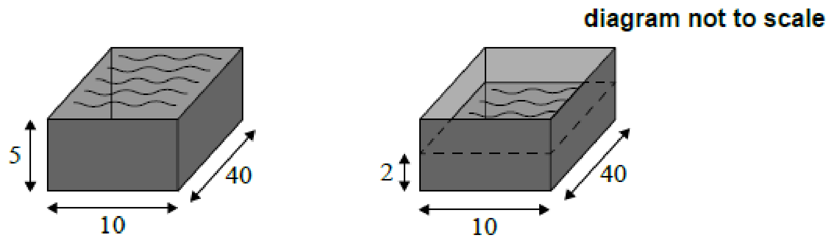
point A is outside the circle. ***(AG)***

Note: Award ***(A1)*** for a numerical comparison of AE and CE. Follow through for the final ***(A1)(ft)*** within the part for their 8.02. The final ***(A1)(ft)*** is contingent on a valid method to find the value of AE.
Do not award the final ***(A1)(ft)*** if the ***(AG)*** line is not stated.
Do not award the final ***(A1)(ft)*** if their point A is inside the circle.

[5 marks]

Yao drains the oil from his motorbike into two identical cuboids with rectangular bases of width 10 cm and length 40 cm. The height of each cuboid is 5 cm.

The oil from the motorbike fills the first cuboid completely and the second cuboid to a height of 2 cm. The information is shown in the following diagram.



2a. Calculate the volume of oil drained from Yao's motorbike. *[3 marks]*

Markscheme

units are required in both parts

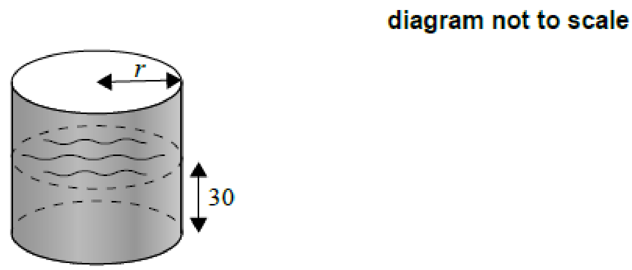
$(V =)\ 5 \times 10 \times 40 + 2 \times 10 \times 40$ ***(M1)(M1)***

Note: Award ***(M1)*** for correct substitutions in volume formula for both cuboids. Award ***(M1)*** for adding the volumes of both cuboids.

2800 cm^3 ***(A1) (C3)***

[3 marks]

2b. Yao then pours all the oil from the cuboids into an empty cylindrical container. The height of the oil in the container is 30 cm. *[3 marks]*



Find the internal radius, r, of the container.

Markscheme

units are required in both parts

$2800 = \pi \times r^2 \times 30$ ***(M1)(M1)***

Note: Award ***(M1)*** for correct substitution in volume of cylinder formula.
Award ***(M1)*** for equating *their* expression (must include π and r) to *their* 2800.

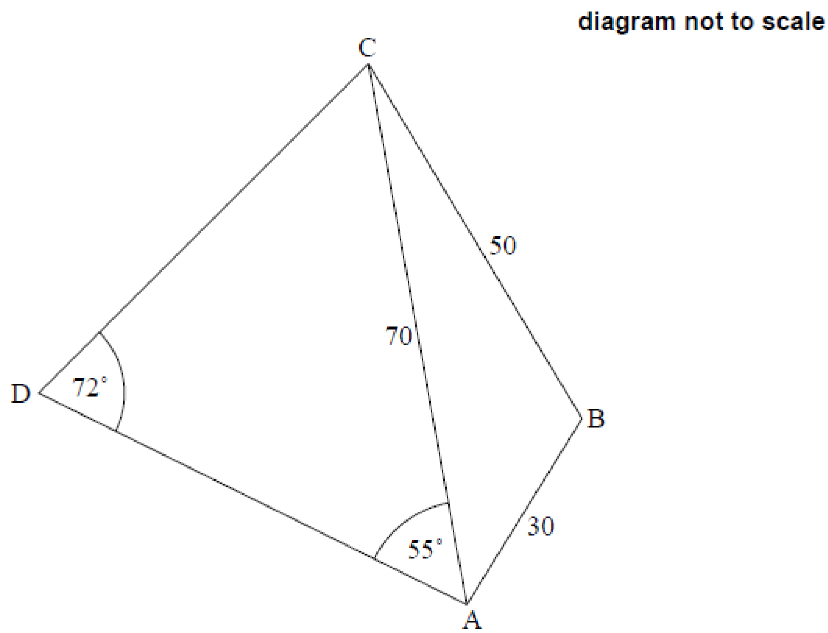
$(r =)\ 5.45$ cm $(5.45058...$ cm) ***(A1)(ft) (C3)***

Note: Follow through from *their* part (a).

[3 marks]

Haraya owns two triangular plots of land, ABC and ACD. The length of AB is $30\ \mathrm{m}$, BC is $50\ \mathrm{m}$ and AC is $70\ \mathrm{m}$. The size of $\mathrm{D\hat{A}C}$ is 55° and $\mathrm{A\hat{D}C}$ is 72°.

The following diagram shows this information.



3a. Find the length of AD. *[4 marks]*

Markscheme

$\text{A}\widehat{\text{C}}\text{D} = 53^\circ$ (or equivalent) ***(A1)***

Note: Award (***A1)*** for 53° (or equivalent) seen.

$\frac{\text{AD}}{\sin 53^\circ} = \frac{70}{\sin 72^\circ}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into sine rule formula, ***(A1)*** for correct substitution.

OR

$\left(\text{AD}^2 =\right) 60.2915\ldots^2 + 70^2 - 2 \times 70 \times 60.2915\ldots \times \cos 53$ ***(A1)(M1)(A1)***

Note: Award ***(A1)*** for 53 or $60.2915\ldots$ seen, ***(M1)*** for substitution into cosine rule formula, ***(A1)*** for correct substitution.

$(\text{AD} =)\ 58.8\ (\text{m})\ (58.7814\ldots)$ ***(A1)(G3)***

[4 marks]

3b. Find the size of $\text{A}\widehat{\text{B}}\text{C}$. *[3 marks]*

Markscheme

$\left(\cos \text{A}\widehat{\text{B}}\text{C}\right) = \frac{30^2 + 50^2 - 70^2}{2 \times 30 \times 50}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into cosine rule formula, ***(A1)*** for correct substitution.

$\left(\text{A}\widehat{\text{B}}\text{C} =\right) 120^\circ$ ***(A1)(G2)***

[3 marks]

3c. Calculate the area of the triangular plot of land ABC. *[3 marks]*

Markscheme

Units are required in part (c)

$A = \frac{1}{2} \times 50 \times 30 \times \sin 120^{\circ}$ ***(M1)(A1)*(ft)**

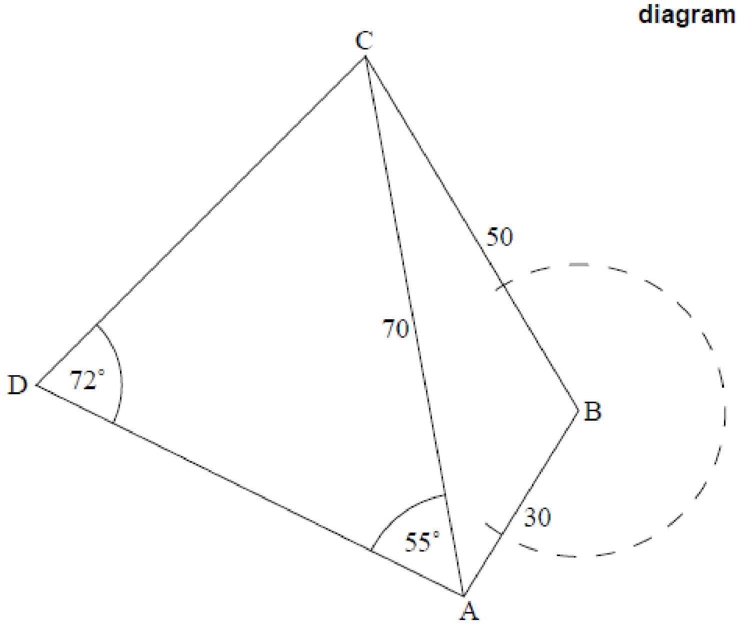
Note: Award ***(M1)*** for substitution into the area formula, ***(A1)*(ft)** for correct substitution. Award ***(M0)(A0)(A0)*** for $\frac{1}{2} \times 50 \times 30$.

$(A =)\ 650\ \mathrm{m}^2\ \ (649.519\ldots\ \mathrm{m}^2)$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (b).

[3 marks]

Haraya attaches a $20\ \mathrm{m}$ long rope to a vertical pole at point B.



3d. Determine whether the rope can extend into the triangular plot of land, ACD. Justify your answer. *[5 marks]*

Markscheme

METHOD 1 (equating part (c) to expression for area of triangle ABC)

$649.519\ldots = \frac{1}{2} \times 70 \times h$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for correctly substituted area of triangle formula. Award ***(A1)*(ft)** for equating the area formula to their area found in part (c).

$(h =)\ 18.6\,(\mathrm{m})\ (18.5576\ldots)$ ***(A1)*(ft)**

Note: Follow through from their part (c).

$20 > 18.5576\ldots$ ***(R1)*(ft)**

Note: Accept "the length of the rope is greater than the altitude of triangle ABC".

the rope passes inside the triangular plot of land ACD ***(A1)*(ft)**

Note: Follow through from their altitude. The final ***(A1)*** is contingent on ***(R1)*** being awarded.

METHOD 2 (finding $\mathrm{C\widehat{A}B}$ or $\mathrm{A\widehat{C}B}$ with sine rule and then trig ratio)

$\frac{\sin \mathrm{C\widehat{A}B}}{50} = \frac{\sin 120^\circ}{50}$ $\left(\mathrm{C\widehat{A}B} = 38.2132\ldots^\circ\right)$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into sine rule formula to find $\mathrm{C\widehat{A}B}$ or $\mathrm{A\widehat{C}B}$. Follow through from their part (b).

$(h =)\ 30 \times \sin(38.2132\ldots^\circ)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their $\mathrm{C\widehat{A}B}$ or $\mathrm{A\widehat{C}B}$ into trig formula.

$(h =)\ 18.6\,(\mathrm{m})\ (18.5576\ldots)$ ***(A1)*(ft)**

Note: Follow through from their part (b).

$20 > 18.5576\ldots$ ***(R1)*(ft)**

Note: Accept "the length of the rope is greater than the altitude of triangle ABC".

the rope passes inside the triangular plot of land ACD ***(A1)*(ft)**

Note: Follow through from their altitude. The final ***(A1)*** is contingent on ***(R1)*** being awarded.

METHOD 3 (finding $\mathrm{C\widehat{A}B}$ or $\mathrm{A\widehat{C}B}$ with with cosine rule and then trig ratio)

$\cos \mathrm{A\widehat{C}B} = \frac{50^2+70^2-30^2}{2(50)(70)}$ $\left(\mathrm{A\widehat{C}B} = 21.7867\ldots^\circ\right)$ ***(M1)***

Note: Award ***(M1)*** for for their correct substitution into cosine rule formula to find $\mathrm{C\widehat{A}B}$ or $\mathrm{A\widehat{C}B}$.

$(h =)\ 50 \times \sin(21.7867\ldots^\circ)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their $\mathrm{C\widehat{A}B}$ or $\mathrm{A\widehat{C}B}$ into trig formula.

$(h =)\ 18.6\,(\mathrm{m})\ (18.5576\ldots)$ ***(A1)*(ft)**

$20 > 18.5576\ldots$ ***(R1)*(ft)**

Note: Accept "the length of the rope is greater than the altitude of triangle ABC".

the rope passes inside the triangular plot of land ACD ***(A1)*****(ft)**

Note: Follow through from their altitude. The final ***(A1)*** is contingent on ***(R1)*** being awarded.

METHOD 4 (finding area of triangle with height 20, justifying the contradiction)

$A=\frac{1}{2}(70)(20)=700\left(\mathrm{m}^2\right)$ ***(M1)(A1)***

Note: Award ***(M1)*** for correct substitution into area of a triangle formula for a triangle with height 20 and base 70. Award ***(A1)*** for 700. Award ***(M0)(A0)*** for unsupported 700 unless subsequent reasoning explains how the 700 was found.

$700>649.519\ldots$ ***(R1)***

if rope exactly touches the AC then this triangle has an area greater than ABC and as the distance AC is fixed the altitude must be less than 20 ***(R1)***

OR

$\frac{1}{2}(70)(20)>\frac{1}{2}(70)$ (height perpendicular to AC) and therefore $20>$height perpendicular to AC ***(R1)*****(ft)**

Note: Award ***(R1)*** for an explanation that recognizes the actual triangle ABC and this new triangle have the same base (70) and hence the height of triangle ABC is less than 20.

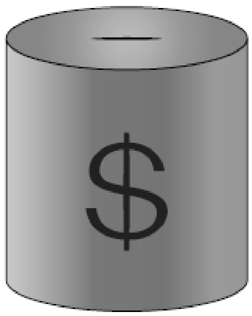
therefore, the rope passes inside the triangular plot of land ACD ***(A1)*****(ft)**

Note: Other methods, besides those listed here, may be possible. These methods can be summarized in two broad groups: the first is to find the altitude of the triangle, and compare it to 20, and the second is to create an artificial triangle with an altitude of 20 and explain why this triangle is not ABC by relating to area **and** the given lengths of the sides.

[5 marks]

Money boxes are coin containers used by children and come in a variety of shapes. The money box shown is in the shape of a cylinder. It has a radius of 4.43 cm and a height of 12.2 cm.

diagram not to scale



4a. Find the volume of the money box. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$(V =)\ \pi(4.43)^2 \times 12.2$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into volume of a cylinder formula, ***(A1)*** for correct substitution.

752 cm^3 (752.171…cm^3) ***(A1)(C3)***

[3 marks]

4b. A second money box is in the shape of a sphere and has the same volume as the cylindrical money box. *[3 marks]*

diagram not to scale

Find the diameter of the second money box.

Markscheme

$752.171\ldots = \frac{4}{3}\pi(r)^3$ ***(M1)***

Note: Award ***(M1)*** for equating their volume to the volume of a sphere formula.

$(r =)$ 5.64169...cm ***(A1)*****(ft)**

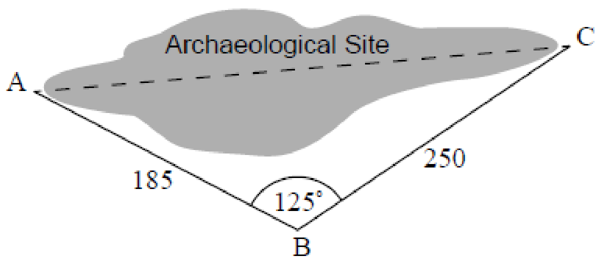
Note: Follow through from part (a).

$(d =)$ 11.3 cm (11.2833...cm) ***(A1)*****(ft)** ***(C3)***

[3 marks]

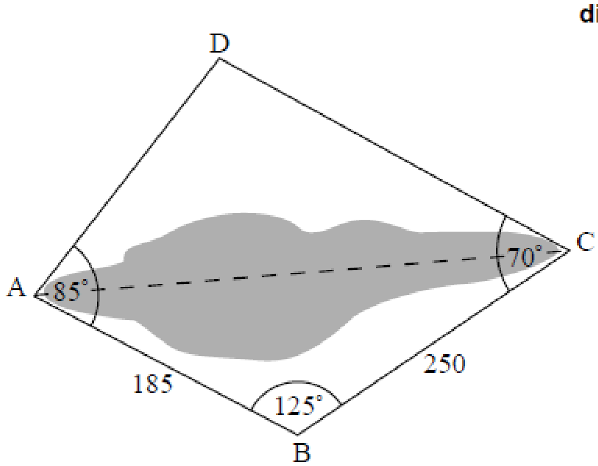
An archaeological site is to be made accessible for viewing by the public. To do this, archaeologists built two straight paths from point A to point B and from point B to point C as shown in the following diagram. The length of path AB is 185 m, the length of path BC is 250 m, and angle $A\hat{B}C$ is 125°.

diagram not to scale



The archaeologists plan to build two more straight paths, AD and DC. For the paths to go around the site, angle $B\hat{A}D$ is to be made equal to 85° and angle $B\hat{C}D$ is to be made equal to 70° as shown in the following diagram.

diagram not to scale



5a. Find the size of angle $C\hat{A}D$. *[1 mark]*

Markscheme

(CAD =) 53.1° (53.0521...°) ***(A1)*****(ft)**

Note: Follow through from their part (b)(i) only if working seen.

[1 mark]

5b. Find the size of angle $A\hat{C}D$. *[2 marks]*

Markscheme

(ACD =) $70° - (180° - 125° - 31.9478°...)$ ***(M1)***

Note: Award ***(M1)*** for subtracting their angle $\mathrm{A}\hat{\mathrm{C}}\mathrm{B}$ from 70°.

OR

(ADC =) $360 - (85 + 70 + 125) = 80$

(ACD =) $180 - 80 - 53.0521...$ ***(M1)***

46.9° (46.9478...°) ***(A1)*****(ft)*****(G2)***

Note: Follow through from part (b)(i).

[2 marks]

A factory packages coconut water in cone-shaped containers with a base radius of 5.2 cm and a height of 13 cm.

6a. Find the slant height of the cone-shaped container. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\frac{\pi(5.2)^2 \times 13}{3}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in the volume formula for cone.

368 (368.110...) cm^3 ***(A1)(G2)***

Note: Accept $117.173...\pi$ cm^3 or $\frac{8788}{75}\pi$ cm^3.

[2 marks]

6b. Find the slant height of the cone-shaped container. *[2 marks]*

Markscheme

(slant height^2) = $(5.2)^2 + 13^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into the formula.

14.0 (14.0014…) (cm) ***(A1)(G2)***

[2 marks]

6c. Show that the total surface area of the cone-shaped container is 314 cm^2, correct to three significant figures. *[3 marks]*

Markscheme

$14.0014\ldots \times (5.2) \times \pi + (5.2)^2 \times \pi$ ***(M1)(M1)***

Note: Award ***(M1)*** for their correct substitution in the curved surface area formula for cone; ***(M1)*** for adding the correct area of the base. The addition must be explicitly seen for the second ***(M1)*** to be awarded. Do not accept rounded values here as may come from working backwards.

313.679… (cm^2) ***(A1)***

Note: Use of 3 sf value 14.0 gives an unrounded answer of 313.656….

314 (cm^2) ***(AG)***

Note: Both the unrounded and rounded answers must be seen for the final ***(A1)*** to be awarded.

[3 marks]

The factory designers are currently investigating whether a cone-shaped container can be replaced with a cylinder-shaped container with the same radius and the same total surface area.

6d. Find the height, h, of this cylinder-shaped container. *[4 marks]*

Markscheme

$2 \times \pi \times (5.2) \times h + 2 \times \pi \times (5.2)^2 = 314$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for correct substitution in the curved surface area formula for cylinder; ***(M1)*** for adding two correct base areas of the cylinder; ***(M1)*** for equating their total cylinder surface area to 314 (313.679…). For this mark to be awarded the areas of the two bases must be added to the cylinder curved surface area and equated to 314. Award at most ***(M1)(M0)(M0)*** for cylinder curved surface area equated to 314.

(h =) 4.41 (4.41051…) (cm) ***(A1)(G3)***

[4 marks]

6e. The factory director wants to increase the volume of coconut water sold per container. *[4 marks]*

State whether or not they should replace the cone-shaped containers with cylinder-shaped containers. Justify your conclusion.

Markscheme

$\pi \times (5.2)^2 \times 4.41051\ldots$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in the volume formula for cylinder.

375 (374.666…) (cm^3) ***(A1)*****(ft)*****(G2)***

Note: Follow through from part (d).

375 (cm^3) > 368 (cm^3) ***(R1)*****(ft)**

OR

"volume of cylinder is larger than volume of cone" or similar ***(R1)*****(ft)**

Note: Follow through from their answer to part (a). The verbal statement should be consistent with their answers from parts (e) and (a) for the ***(R1)*** to be awarded.

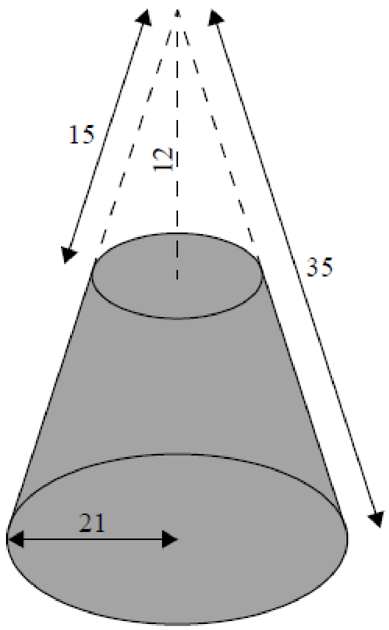
replace with the cylinder containers ***(A1)*****(ft)**

Note: Do not award ***(A1)*****(ft)*****(R0)***. **Follow through** from their **incorrect** volume for the cylinder **in this question part** but only if substitution in the volume formula shown.

[4 marks]

A solid right circular cone has a base radius of 21 cm and a slant height of 35 cm. A smaller right circular cone has a height of 12 cm and a slant height of 15 cm, and is removed from the top of the larger cone, as shown in the diagram.

diagram not to scale



7. Calculate the radius of the base of the cone which has been removed. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\sqrt{15^2 - 12^2}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras theorem.

OR

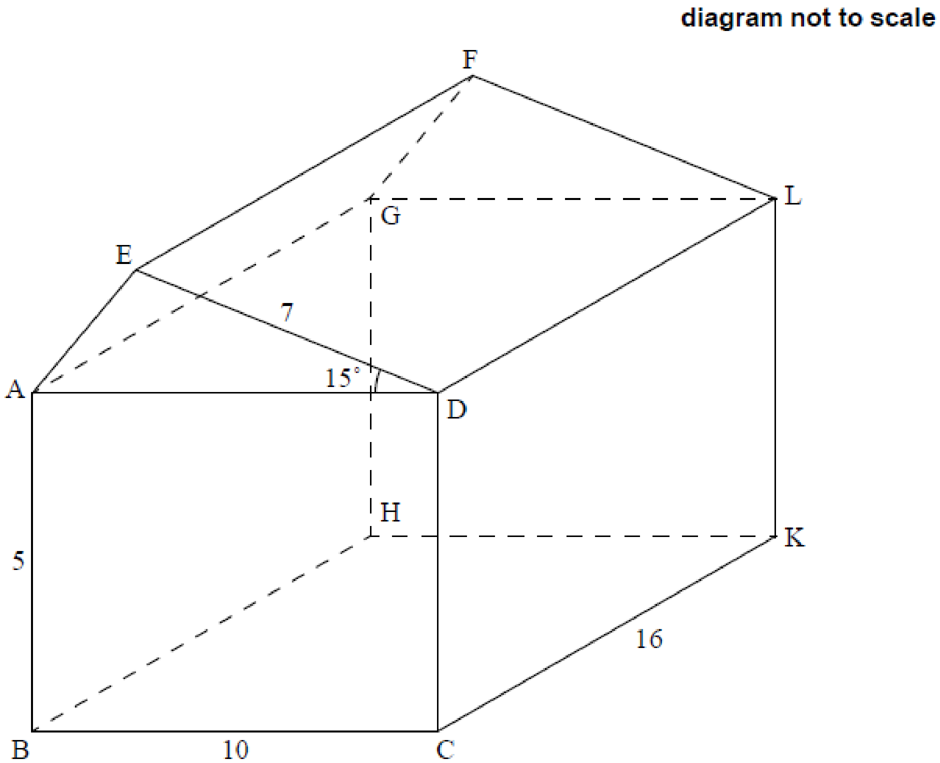
$\frac{\text{radius}}{21} = \frac{15}{35}$ ***(M1)***

Note: Award ***(M1)*** for a correct equation.

= 9 (cm) ***(A1) (C2)***

[2 marks]

Farmer Brown has built a new barn, on horizontal ground, on his farm. The barn has a cuboid base and a triangular prism roof, as shown in the diagram.



The cuboid has a width of 10 m, a length of 16 m and a height of 5 m.
The roof has two sloping faces and two vertical and identical sides, ADE and GLF.
The face DEFL slopes at an angle of 15° to the horizontal and ED = 7 m .

8a. Calculate the area of triangle EAD. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

(Area of EAD =) $\frac{1}{2} \times 10 \times 7 \times \sin 15$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into area of a triangle formula, **(A1)** for correct substitution. Award ***(M0)(A0)(A0)*** if EAD or AED is considered to be a right-angled triangle.

= 9.06 m^2 (9.05866... m^2) ***(A1) (G3)***

[3 marks]

8b. Calculate the **total** volume of the barn. *[3 marks]*

Markscheme

(10 × 5 × 16) + (9.05866... × 16)   ***(M1)(M1)***

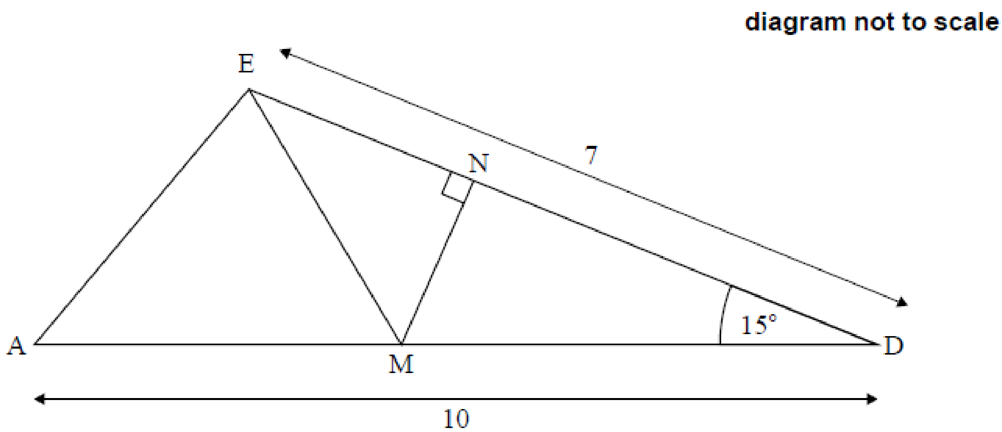
Note: Award ***(M1)*** for correct substitution into volume of a cuboid, ***(M1)*** for adding the correctly substituted volume of their triangular prism.

= 945 m^3 (944.938... m^3)   ***(A1)*****(ft)** ***(G3)***

Note: Follow through from part (a).

[3 marks]

The roof was built using metal supports. Each support is made from **five** lengths of metal AE, ED, AD, EM and MN, and the design is shown in the following diagram.



ED = 7 m , AD = 10 m and angle ADE = 15° .
M is the midpoint of AD.
N is the point on ED such that MN is at right angles to ED.

8c. Calculate the length of MN. *[2 marks]*

Markscheme

$\frac{\text{MN}}{5} = \sin 15$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into trigonometric equation.

(MN =) 1.29(m) (1.29409... (m))   ***(A1) (G2)***

[2 marks]

8d. Calculate the length of AE. *[3 marks]*

Markscheme

$(AE^2 =)\ 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos 15$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into cosine rule formula, and ***(A1)*** for correct substitution.

(AE =) 3.71(m) (3.71084… (m)) ***(A1) (G2)***

[3 marks]

Farmer Brown believes that N is the midpoint of ED.

8e. Show that Farmer Brown is incorrect. *[3 marks]*

Markscheme

$ND^2 = 5^2 - (1.29409...)^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras theorem.

(ND =) 4.83 (4.82962...) ***(A1)*(ft)**

Note: Follow through from part (c).

OR

$\frac{1.29409...}{\text{ND}} = \tan 15^\circ$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into tangent.

(ND =) 4.83 (4.82962...) ***(A1)*(ft)**

Note: Follow through from part (c).

OR

$\frac{\text{ND}}{5} = \cos 15^\circ$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into cosine.

(ND =) 4.83 (4.82962...) ***(A1)*(ft)**

Note: Follow through from part (c).

OR

$ND^2 = 1.29409...^2 + 5^2 - 2 \times 1.29409... \times 5 \times \cos 75^\circ$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into cosine rule.

(ND =) 4.83 (4.82962...) ***(A1)*(ft)**

Note: Follow through from part (c).

$4.82962... \neq 3.5$ (ND $\neq$ 3.5) ***(R1)*(ft)**

OR

$4.82962... \neq 2.17038...$ (ND $\neq$ NE) ***(R1)*(ft)**

(hence Farmer Brown is incorrect)

Note: Do not award ***(M0)(A0)(R1)*(ft)**. Award ***(M0)(A0)(R0)*** for a correct conclusion without any working seen.

[3 marks]

8f. Calculate the **total** length of metal required for one support. *[4 marks]*

Markscheme

(EM^2 =) 1.29409...^{2} + (7 − 4.82962...)^2 ***(M1)***

Note: Award ***(M1)*** for their correct substitution into Pythagoras theorem.

OR

(EM^2 =) $5^2 + 7^2 - 2 \times 5 \times 7 \times \cos 15$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into cosine rule formula.

(EM =) 2.53(m) (2.52689...(m)) ***(A1)*****(ft)** ***(G2)*****(ft)**

Note: Follow through from parts (c), (d) and (e).

(Total length =) 2.52689... + 3.71084... + 1.29409... +10 + 7 ***(M1)***

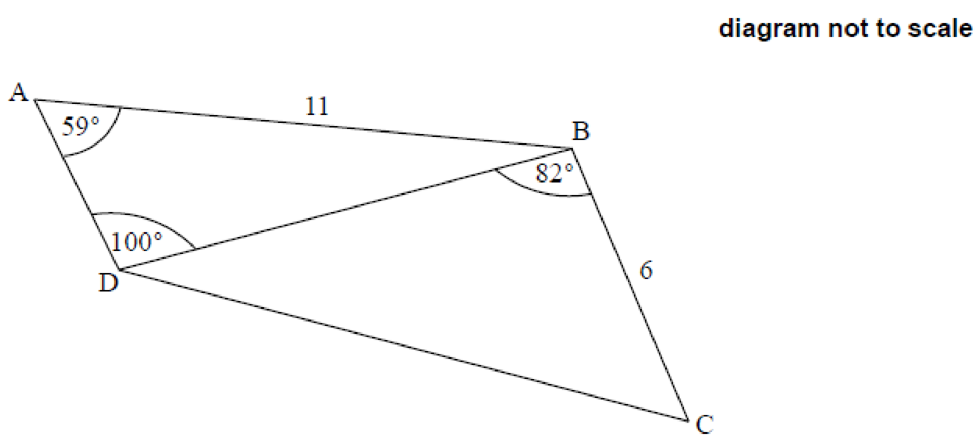
Note: Award ***(M1)*** for adding their EM, their parts (c) and (d), and 10 and 7.

= 24.5 (m) (24.5318... (m)) ***(A1)*****(ft)** ***(G4)***

Note: Follow through from parts (c) and (d).

[4 marks]

The following diagram shows quadrilateral ABCD.



$\mathrm{AB} = 11\,\mathrm{cm},\ \mathrm{BC} = 6\,\mathrm{cm},\ \mathrm{B}\hat{\mathrm{A}}\mathrm{D} = 100^\circ,\ \text{and}\ \mathrm{C}\hat{\mathrm{B}}\mathrm{D} = 82^\circ$

9a. Find DB. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

evidence of choosing sine rule ***(M1)***

eg $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

correct substitution ***(A1)***

eg $\frac{\text{DB}}{\sin 59^\circ} = \frac{11}{\sin 100^\circ}$

9.57429

DB = 9.57 (cm) ***A1 N2***

[3 marks]

9b. Find DC. *[3 marks]*

Markscheme

evidence of choosing cosine rule ***(M1)***

eg

$a^2 = b^2 + c^2 - 2bc\cos(A),\ \text{DC}^2 = \text{DB}^2 + \text{BC}^2 - 2\text{DB} \times \text{BC} \times \cos\left(\text{D}\hat{\text{B}}\text{C}\right.$

correct substitution into RHS ***(A1)***

eg $9.57^2 + 6^2 - 2 \times 9.57 \times 6 \times \cos 82^\circ,\ 111.677$

10.5677

DC = 10.6 (cm) ***A1 N2***

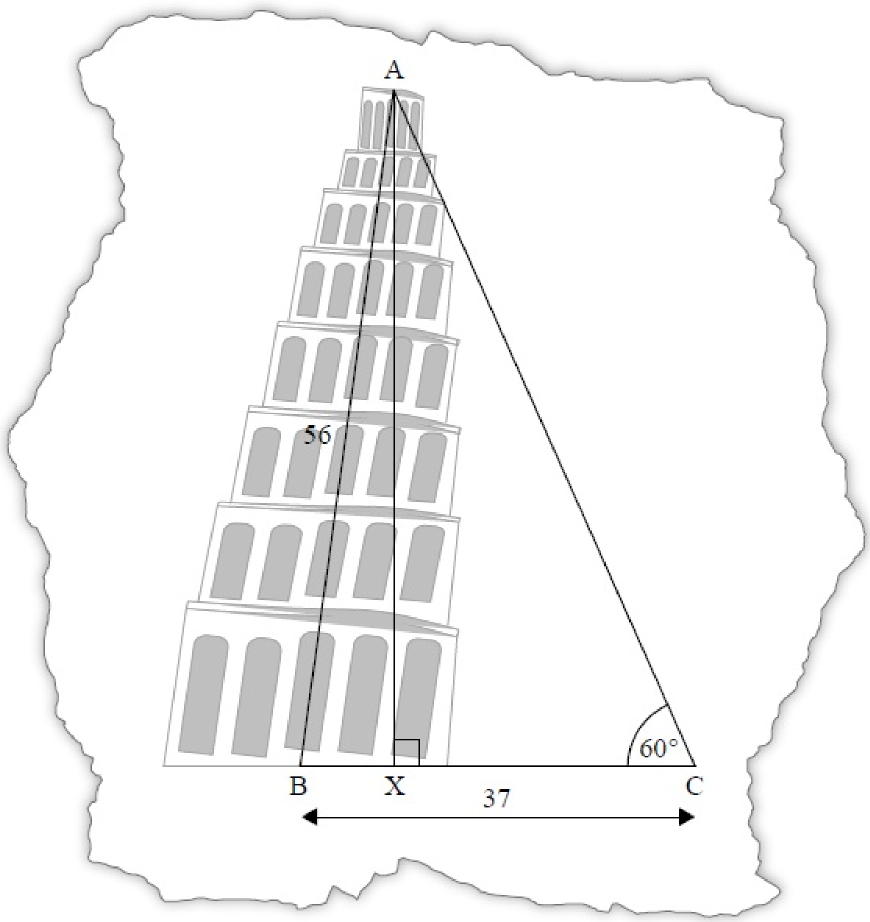
[3 marks]

The Tower of Pisa is well known worldwide for how it leans.

Giovanni visits the Tower and wants to investigate how much it is leaning. He draws a diagram showing a non-right triangle, ABC.

On Giovanni’s diagram the length of AB is 56 m, the length of BC is 37 m, and angle ACB is 60°. AX is the perpendicular height from A to BC.

diagram not to scale



10a. Use Giovanni’s diagram to show that angle ABC, the angle at which the Tower is leaning relative to the horizontal, is 85° to the nearest degree. *[5 marks]*

Markscheme

$\frac{\sin \mathrm{BAC}}{37} = \frac{\sin 60}{56}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituting the sine rule formula, ***(A1)*** for correct substitution.

angle $\mathrm{B}\hat{\mathrm{A}}\mathrm{C}$ = 34.9034...° ***(A1)***

Note: Award ***(A0)*** if unrounded answer does not round to 35. Award ***(G2)*** if 34.9034... seen without working.

angle $\mathrm{A}\hat{\mathrm{B}}\mathrm{C}$ = 180 − (34.9034... + 60) ***(M1)***

Note: Award ***(M1)*** for subtracting their angle BAC + 60 from 180.

85.0965...° ***(A1)***

85° ***(AG)***

Note: Both the unrounded and rounded value must be seen for the final ***(A1)*** to be awarded. If the candidate rounds 34.9034...° to 35° while substituting to find angle $\mathrm{A}\hat{\mathrm{B}}\mathrm{C}$, the final ***(A1)*** can be awarded but **only** if both 34.9034...° and 35° are seen.
If 85 is used as part of the workings, award at most ***(M1)(A0)(A0)(M0)(A0)(AG)***. This is the reverse process and not accepted.

10b. Use Giovanni's diagram to calculate the length of AX. *[2 marks]*

Markscheme

sin 85... × 56 ***(M1)***

= 55.8 (55.7869...) (m) ***(A1)(G2)***

Note: Award ***(M1)*** for correct substitution in trigonometric ratio.

10c. Use Giovanni's diagram to find the length of BX, the horizontal displacement of the Tower. *[2 marks]*

Markscheme

$\sqrt{56^2 - 55.7869\ldots^2}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in the Pythagoras theorem formula. Follow through from part (a)(ii).

OR

cos(85) × 56 ***(M1)***

Note: Award ***(M1)*** for correct substitution in trigonometric ratio.

= 4.88 (4.88072…) (m) ***(A1)*(ft)*(G2)***

Note: Accept 4.73 (4.72863…) (m) from using their 3 s.f answer. Accept equivalent methods.

[2 marks]

Giovanni’s tourist guidebook says that the actual horizontal displacement of the Tower, BX, is 3.9 metres.

10d. Find the percentage error on Giovanni’s diagram. *[2 marks]*

Markscheme

$\left|\frac{4.88-3.9}{3.9}\right| \times 100$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into the percentage error formula.

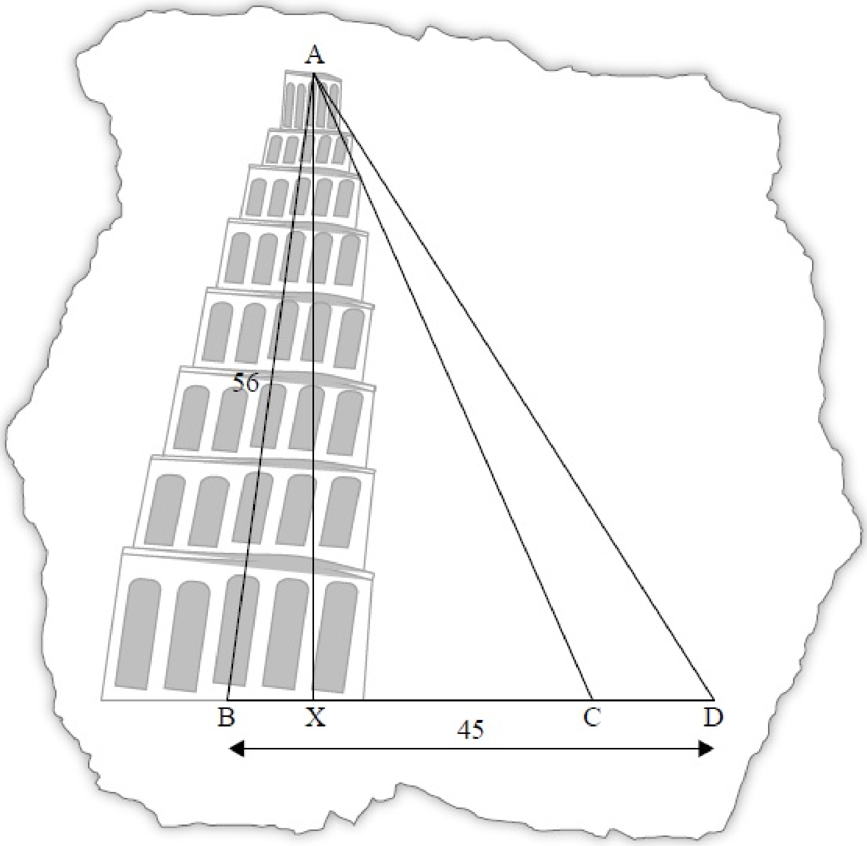
= 25.1 (25.1282) (%) ***(A1)*(ft)*(G2)***

Note: Follow through from part (a)(iii).

[2 marks]

10e. Giovanni adds a point D to his diagram, such that BD = 45 m, and another triangle is formed. *[3 marks]*

diagram not to scale



Find the angle of elevation of A from D.

Markscheme

$\tan^{-1}\left(\frac{55.7869...}{40.11927...}\right)$ ***(A1)*(ft)*(M1)***

Note: Award ***(A1)*(ft)** for their 40.11927... seen. Award ***(M1)*** for correct substitution into trigonometric ratio.

OR

$(37 - 4.88072...)^2 + 55.7869...^2$

(AC =) 64.3725...

$64.3726...^2 + 8^2 - 2 \times 8 \times 64.3726... \times \cos 120$

(AD =) 68.7226...

$\frac{\sin 120}{68.7226...} = \frac{\sin \mathrm{A\hat{D}C}}{64.3725...}$ ***(A1)*(ft)*(M1)***

Note: Award ***(A1)*(ft)** for their correct values seen, ***(M1)*** for correct substitution into the sine formula.

= 54.3° (54.2781...°) ***(A1)*(ft)*(G2)***

Note: Follow through from part (a). Accept equivalent methods.

[3 marks]