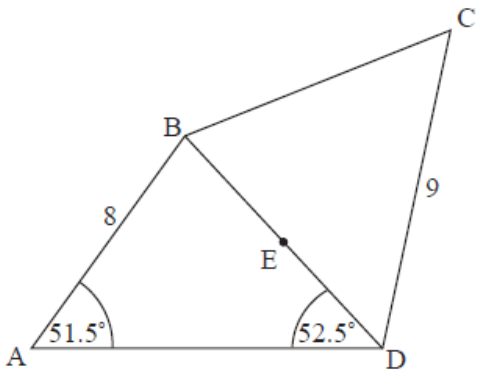
Trigonometry review *[100 marks]*

Using geometry software, Pedro draws a quadrilateral ABCD. $\mathrm{AB} = 8\ \mathrm{cm}$ and $\mathrm{CD} = 9\ \mathrm{cm}$. Angle $\mathrm{BAD} = 51.5^\circ$ and angle $\mathrm{ADB} = 52.5^\circ$. This information is shown in the diagram.

diagram not to scale



1a. Calculate the length of BD. *[3 marks]*

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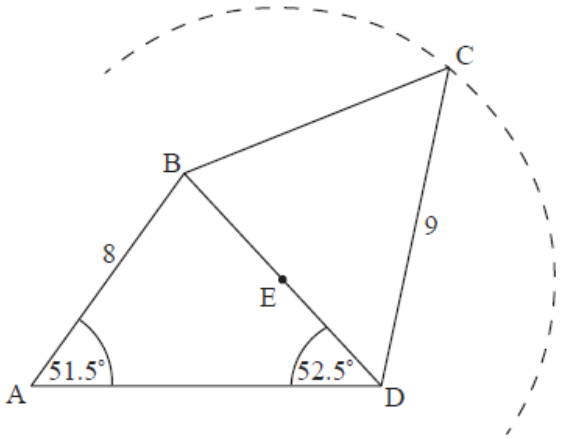
$\mathrm{CE} = 7\ \mathrm{cm}$, where point E is the midpoint of BD.

1b. Show that angle $\mathrm{EDC} = 48.0^{\circ}$, correct to three significant figures. *[4 marks]*

1c. Calculate the area of triangle BDC. *[3 marks]*

1d. Pedro draws a circle, with centre at point E, passing through point C. *[5 marks]*
Part of the circle is shown in the diagram.

diagram not to scale



Show that point A lies outside this circle. Justify your reasoning.

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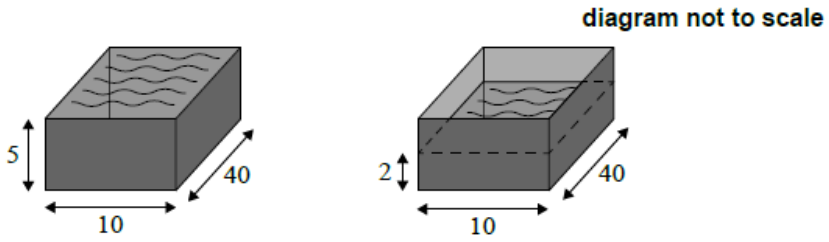
Yao drains the oil from his motorbike into two identical cuboids with rectangular bases of width 10 cm and length 40 cm. The height of each cuboid is 5 cm.

The oil from the motorbike fills the first cuboid completely and the second cuboid to a height of 2 cm. The information is shown in the following diagram.



2a. Calculate the volume of oil drained from Yao's motorbike. *[3 marks]*

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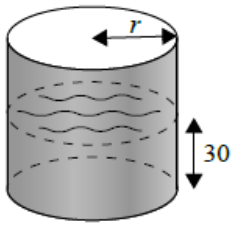
2b. Yao then pours all the oil from the cuboids into an empty cylindrical container. The height of the oil in the container is 30 cm. *[3 marks]*

diagram not to scale



Find the internal radius, r, of the container.

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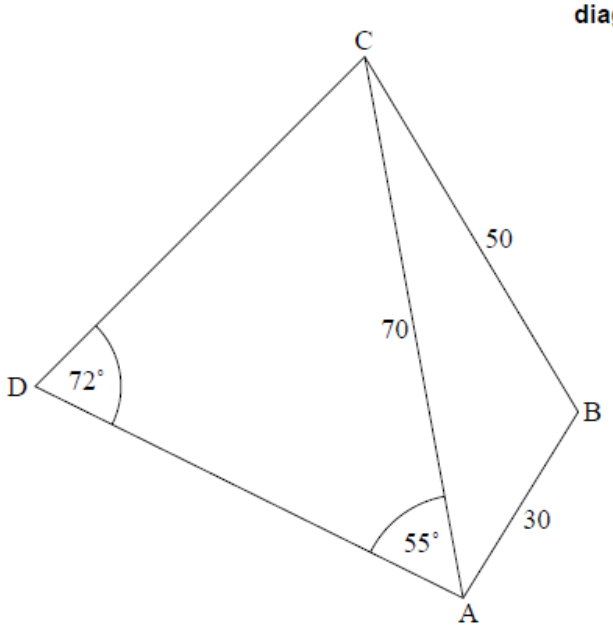
Haraya owns two triangular plots of land, ABC and ACD. The length of AB is $30\ \mathrm{m}$, BC is $50\ \mathrm{m}$ and AC is $70\ \mathrm{m}$. The size of $\mathrm{D\hat{A}C}$ is 55° and $\mathrm{A\hat{D}C}$ is 72°.

The following diagram shows this information.

diagram not to scale



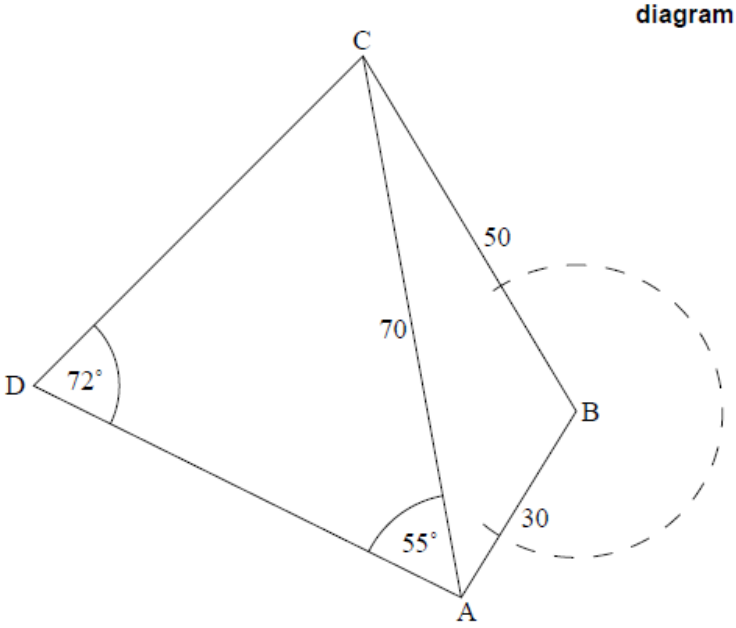
3a. Find the length of AD. *[4 marks]*

3b. Find the size of $\mathrm{A\hat{B}C}$. *[3 marks]*

3c. Calculate the area of the triangular plot of land ABC. *[3 marks]*

Haraya attaches a $20\ \mathrm{m}$ long rope to a vertical pole at point B.

diagram not to scale



3d. Determine whether the rope can extend into the triangular plot of land, *[5 marks]* ACD. Justify your answer.

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Money boxes are coin containers used by children and come in a variety of shapes. The money box shown is in the shape of a cylinder. It has a radius of 4.43 cm and a height of 12.2 cm.

diagram not to scale

4a. Find the volume of the money box. *[3 marks]*

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4b. A second money box is in the shape of a sphere and has the same volume as the cylindrical money box. *[3 marks]*

diagram not to scale

Find the diameter of the second money box.

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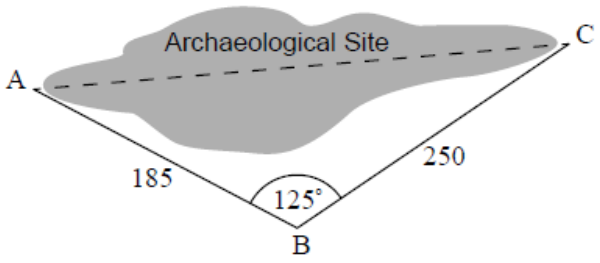
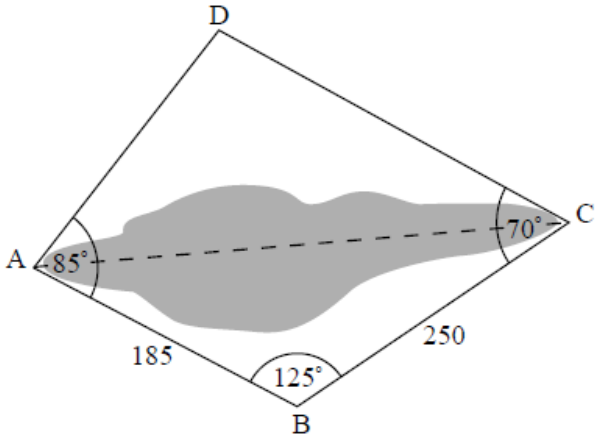
An archaeological site is to be made accessible for viewing by the public. To do this, archaeologists built two straight paths from point A to point B and from point B to point C as shown in the following diagram. The length of path AB is 185 m, the length of path BC is 250 m, and angle $\mathrm{A\hat{B}C}$ is 125°.

diagram not to scale



The archaeologists plan to build two more straight paths, AD and DC. For the paths to go around the site, angle $\mathrm{B\hat{A}D}$ is to be made equal to 85° and angle $\mathrm{B\hat{C}D}$ is to be made equal to 70° as shown in the following diagram.

diagram not to scale



5a. Find the size of angle $\mathrm{C\hat{A}D}$. *[1 mark]*

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5b. Find the size of angle $\hat{A}CD$. *[2 marks]*

A factory packages coconut water in cone-shaped containers with a base radius of 5.2 cm and a height of 13 cm.

6a. Find the slant height of the cone-shaped container. *[2 marks]*

6b. Find the slant height of the cone-shaped container. *[2 marks]*

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6c. Show that the total surface area of the cone-shaped container is 314 cm^2, correct to three significant figures. *[3 marks]*

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The factory designers are currently investigating whether a cone-shaped container can be replaced with a cylinder-shaped container with the same radius and the same total surface area.

6d. Find the height, h, of this cylinder-shaped container. *[4 marks]*

6e. The factory director wants to increase the volume of coconut water sold per container. *[4 marks]*

State whether or not they should replace the cone-shaped containers with cylinder-shaped containers. Justify your conclusion.

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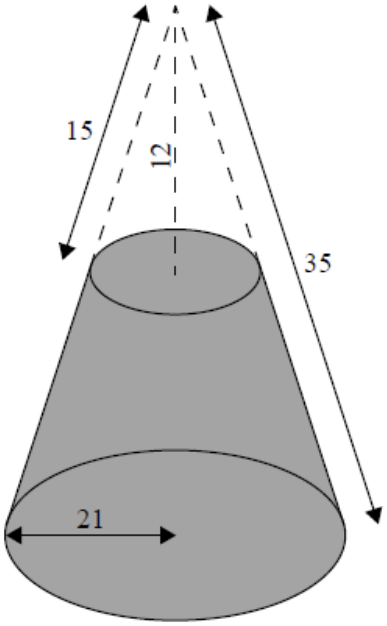
A solid right circular cone has a base radius of 21 cm and a slant height of 35 cm. A smaller right circular cone has a height of 12 cm and a slant height of 15 cm, and is removed from the top of the larger cone, as shown in the diagram.

diagram not to scale



7. Calculate the radius of the base of the cone which has been removed. *[2 marks]*

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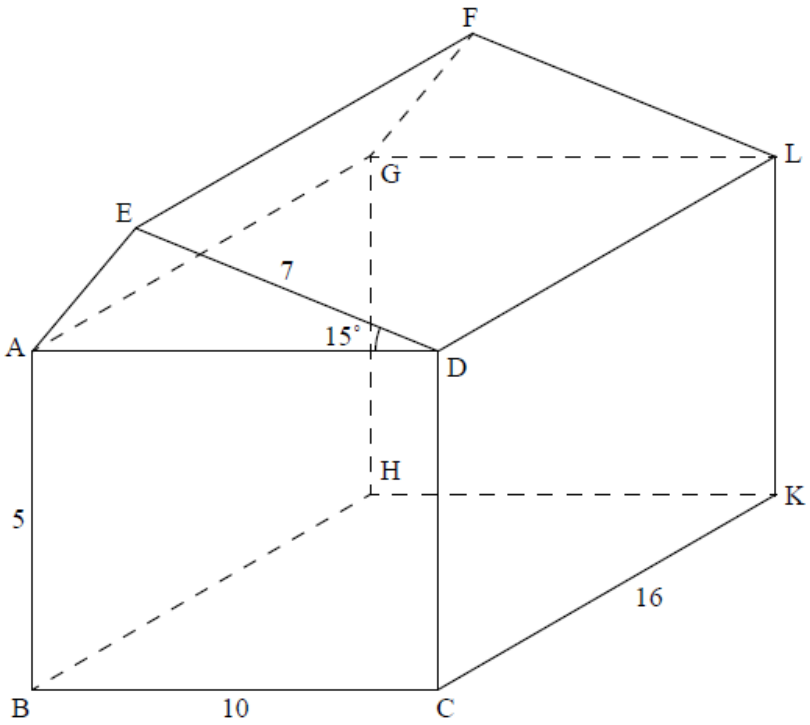
Farmer Brown has built a new barn, on horizontal ground, on his farm. The barn has a cuboid base and a triangular prism roof, as shown in the diagram.

diagram not to scale



The cuboid has a width of 10 m, a length of 16 m and a height of 5 m.
The roof has two sloping faces and two vertical and identical sides, ADE and GLF.
The face DEFL slopes at an angle of 15° to the horizontal and ED = 7 m .

8a. Calculate the area of triangle EAD. *[3 marks]*

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8b. Calculate the **total** volume of the barn. *[3 marks]*

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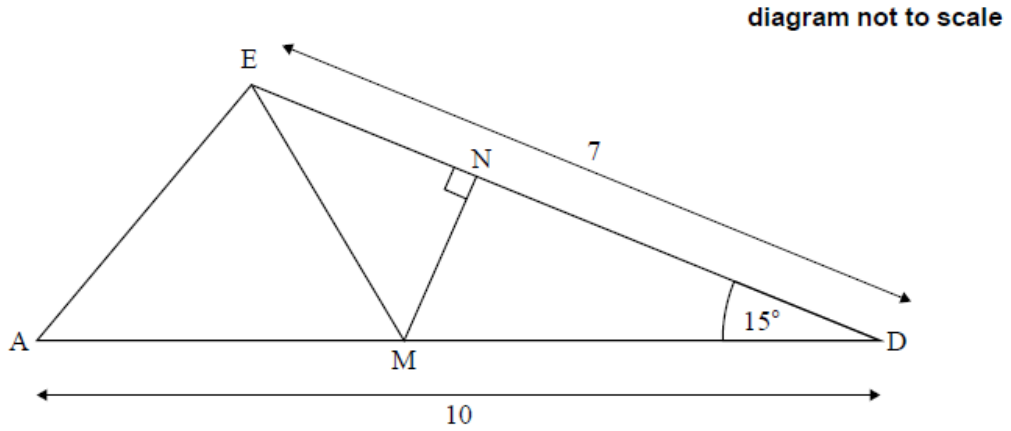
The roof was built using metal supports. Each support is made from **five** lengths of metal AE, ED, AD, EM and MN, and the design is shown in the following diagram.



ED = 7 m , AD = 10 m and angle ADE = 15° .
M is the midpoint of AD.
N is the point on ED such that MN is at right angles to ED.

8c. Calculate the length of MN. *[2 marks]*

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8d. Calculate the length of AE. *[3 marks]*

Farmer Brown believes that N is the midpoint of ED.

8e. Show that Farmer Brown is incorrect. *[3 marks]*

8f. Calculate the **total** length of metal required for one support. *[4 marks]*

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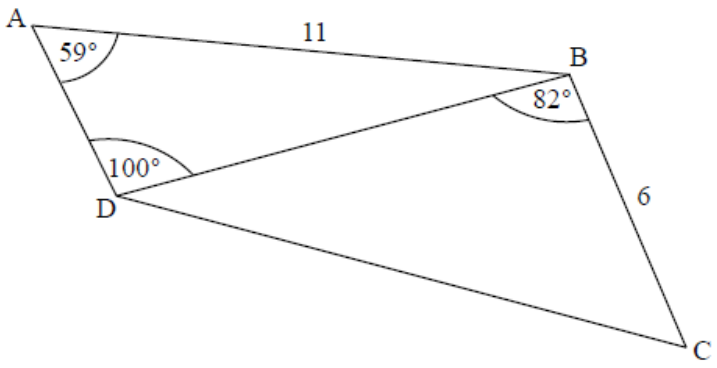
The following diagram shows quadrilateral ABCD.

diagram not to scale



$\mathrm{AB} = 11\,\mathrm{cm}$, $\mathrm{BC} = 6\,\mathrm{cm}$, $\mathrm{B\hat{A}D} = 100°$, and $\mathrm{C\hat{B}D} = 82°$

9a. Find DB. *[3 marks]*

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9b. Find DC. *[3 marks]*

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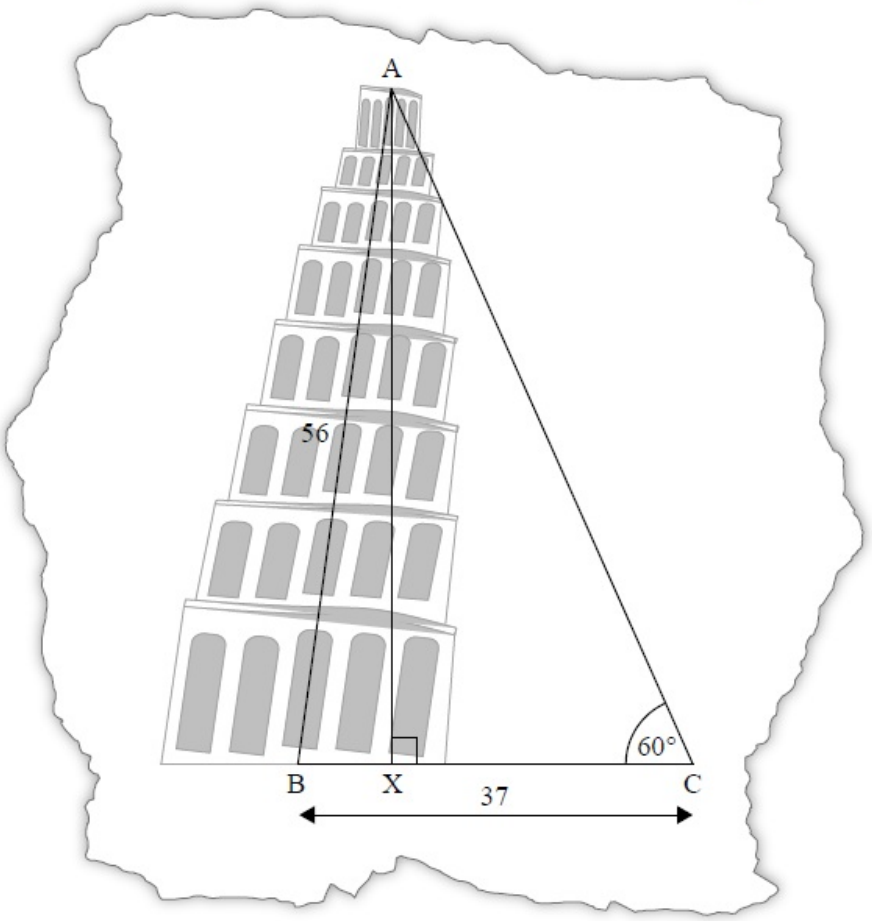
The Tower of Pisa is well known worldwide for how it leans.

Giovanni visits the Tower and wants to investigate how much it is leaning. He draws a diagram showing a non-right triangle, ABC.

On Giovanni's diagram the length of AB is 56 m, the length of BC is 37 m, and angle ACB is 60°. AX is the perpendicular height from A to BC.

diagram not to scale



10a. Use Giovanni's diagram to show that angle ABC, the angle at which the Tower is leaning relative to the horizontal, is 85° to the nearest degree. *[5 marks]*

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10b. Use Giovanni's diagram to calculate the length of AX. *[2 marks]*

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10c. Use Giovanni's diagram to find the length of BX, the horizontal displacement of the Tower. *[2 marks]*

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Giovanni’s tourist guidebook says that the actual horizontal displacement of the Tower, BX, is 3.9 metres.

10d. Find the percentage error on Giovanni’s diagram. *[2 marks]*

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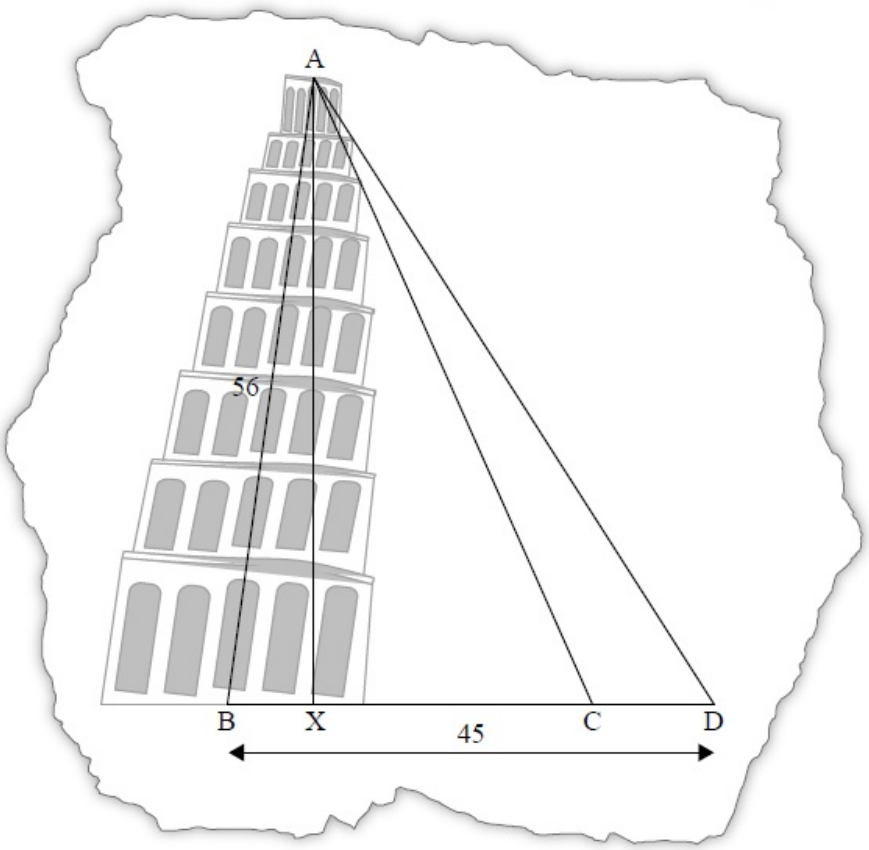
10e. Giovanni adds a point D to his diagram, such that BD = 45 m, and another triangle is formed. *[3 marks]*

diagram not to scale



Find the angle of elevation of A from D.

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