1. The acceleration in m s^{-2} of a particle moving in a straight line at time t seconds, $t \geq 0$, is given by the formula $a = -\frac{1}{2}v$. When $t = 0$, the velocity is 40 m s^{-1}.

Find an expression for v in terms of t.

(Total 6 marks)

2. Consider the differential equation $\frac{dy}{dx} = x^2 + y^2$ where $y = 1$ when $x = 0$.

(a) Use Euler's method with step length 0.1 to find an approximate value of y when $x = 0.4$.

(7)

(b) Write down, giving a reason, whether your approximate value for y is greater than or less than the actual value of y.

(1)

(Total 8 marks)

3. Consider the differential equation $\frac{dy}{dx} = \frac{y^2 + x^2}{2x^2}$ for which $y = -1$ when $x = 1$.

(a) Use Euler's method with a step length of 0.25 to find an estimate for the value of y when $x = 2$.

(7)

(b) (i) Solve the differential equation giving your answer in the form $y = f(x)$.

(ii) Find the value of y when $x = 2$.

(13)

(Total 20 marks)

4. A curve that passes through the point (1, 2) is defined by the differential equation

$$\frac{dy}{dx} = 2x(1 + x^2 - y).$$

(a) (i) Use Euler's method to get an approximate value of y when $x = 1.3$, taking steps of 0.1. Show intermediate steps to four decimal places in a table.

(ii) How can a more accurate answer be obtained using Euler's method?

(5)

(b) Solve the differential equation giving your answer in the form $y = f(x)$.

(9)

(Total 14 marks)

5. Consider the differential equation $\frac{dy}{dy} + \frac{xy}{4 - x^2} = 1$, where $|x| < 2$ and $y = 1$ when $x = 0$.

(a) Use Euler's method with $h = 0.25$, to find an approximate value of y when $x = 1$, giving your answer to two decimal places.

(10)

(b) (i) By first finding an integrating factor, solve this differential equation. Give your answer in the form $y = f(x)$.

(ii) Calculate, correct to two decimal places, the value of y when $x = 1$.

(10)

(c) Sketch the graph of $y = f(x)$ for $0 \le x \le 1$. Use your sketch to explain why your approximate value of y is greater than the true value of y.

(4)

(Total 24 marks)

6. Given that $\frac{dy}{dx} + 2y \tan x = \sin x$, and $y = 0$ when $x = \frac{\pi}{3}$, find the maximum value of y.

(Total 11 marks)

7. The variables x and y are related by $\frac{dy}{dx} - y \tan x = \cos x$.

(a) Find the Maclaurin series for y up to and including the term in x^2 given that $y = -\frac{\pi}{2}$ when $x = 0$.

(7)

(b) Solve the differential equation given that $y = 0$ when $x = \pi$. Give the solution in the form $y = f(x)$.

(10)
(Total 17 marks)

8. Solve the following differential equation

$$(x + 1)(x + 2)\frac{dy}{dx} + y = x + 1$$

giving your answer in the form $y = f(x)$.

(Total 11 marks)