

Exponential equations

We will learn how to solve basic exponential equations.

We will deal with the equations of the form

$$a^{f(x)} = b^{g(x)}$$

where $a, b > 0$ and f, g are real-valued functions. In our examples these will be simple linear or quadratic functions.

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Now we compare the exponents:

$$2x - 1 = 5$$

$$x = 3$$

Example 1

Solve

$$\left(\frac{1}{2}\right)^{x+1} = 4^{x+2}$$

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Compare exponents:

$$\begin{aligned}-x - 1 &= 2x + 4 \\ x &= -\frac{5}{3}\end{aligned}$$

Example 2

Solve:

$$\left(\frac{1}{9}\right)^{x-2} = (\sqrt{3})^{x+6}$$

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Compare exponents:

$$\begin{aligned}-2x + 4 &= \frac{x}{2} + 3 \\ x &= \frac{2}{5}\end{aligned}$$

Example 3

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$$4 \times 8^x = (2\sqrt{2})^{-x}$$

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Solution:

$$\begin{aligned}4 \times 8^x &= (2\sqrt{2})^{-x} \\2^2 \times 2^{3x} &= 2^{-\frac{3}{2}x} \\2^{3x+2} &= 2^{-\frac{3}{2}x} \\3x + 2 &= -\frac{3}{2}x \\x &= -\frac{4}{9}\end{aligned}$$

Example 4

Solve

$$3 \times 81^{x-1} = (\sqrt[3]{3})^{-2x}$$

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$$3 \times 3^{4x-4} = 3^{-\frac{2x}{3}}$$

$$3^{4x-3} = 3^{-\frac{2x}{3}}$$

$$4x - 3 = -\frac{2x}{3}$$

$$x = \frac{9}{14}$$

Example 5

Solve:

$$4 \times \left(\frac{1}{\sqrt{2}} \right)^x = \frac{1}{2} \times 16^{x-1}$$

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Solution:

$$\begin{aligned} 4 \times \left(\frac{1}{\sqrt{2}} \right)^x &= \frac{1}{2} \times 16^{x-1} \\ 2^2 \times 2^{-\frac{x}{2}} &= 2^{-1} \times 2^{4x-4} \\ 2^{2-\frac{x}{2}} &= 2^{4x-5} \\ 2 - \frac{x}{2} &= 4x - 5 \\ x &= \frac{14}{9} \end{aligned}$$

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$$(\sqrt[3]{2})^{3x^2-3} = 4^{x+1}$$

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$$2^{x^2-1} = 2^{2x+2}$$

$$x^2 - 1 = 2x + 2$$

$$x^2 - 2x - 3 = 0$$

$$x^2 - 2x + 1 = 4$$

$$(x - 1)^2 = 4$$

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Solution:

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$$2^{x^2-1} = 2^{2x+2}$$

$$x^2 - 1 = 2x + 2$$

$$x^2 - 2x - 3 = 0$$

$$x^2 - 2x + 1 = 4$$

$$(x - 1)^2 = 4$$

We get $x = 3$ or $x = -1$.

In case of any questions, you can email me at T.J.Lechowski@gmail.com.