Trigonometric equations

This presentation shows how to solve certain types of trigonometric equations, starting from very basic ones and finishing with ones where some trigonometric identities and algebraic manipulations are required.

This presentation shows how to solve certain types of trigonometric equations, starting from very basic ones and finishing with ones where some trigonometric identities and algebraic manipulations are required.

Before you start with this presentation make sure you're very familiar with:

This presentation shows how to solve certain types of trigonometric equations, starting from very basic ones and finishing with ones where some trigonometric identities and algebraic manipulations are required.

Before you start with this presentation make sure you're very familiar with:

- radian measure (in almost all equations we will use radians instead of degrees);

This presentation shows how to solve certain types of trigonometric equations, starting from very basic ones and finishing with ones where some trigonometric identities and algebraic manipulations are required.

Before you start with this presentation make sure you're very familiar with:

- radian measure (in almost all equations we will use radians instead of degrees);
- graphs of trigonometric functions ($\sin x$, $\cos x$, $\tan x$, $\cot x$), including basic properties of these graphs (domain, range, period, etc.)

This presentation shows how to solve certain types of trigonometric equations, starting from very basic ones and finishing with ones where some trigonometric identities and algebraic manipulations are required.

Before you start with this presentation make sure you're very familiar with:

- radian measure (in almost all equations we will use radians instead of degrees);
- graphs of trigonometric functions ($\sin x$, $\cos x$, $\tan x$, $\cot x$), including basic properties of these graphs (domain, range, period, etc.)
- values of trigonometric functions for standard angles ($0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$);

This presentation shows how to solve certain types of trigonometric equations, starting from very basic ones and finishing with ones where some trigonometric identities and algebraic manipulations are required.

Before you start with this presentation make sure you're very familiar with:

- radian measure (in almost all equations we will use radians instead of degrees);
- graphs of trigonometric functions ($\sin x,\ \cos x,\ \tan x,\ \cot x$), including basic properties of these graphs (domain, range, period, etc.)
- values of trigonometric functions for standard angles ($0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$);
- reduction formulae (eg. $\sin(\pi - x) = \sin x$ or $\sin(\frac{\pi}{2} - x) = \cos x$)

This presentation shows how to solve certain types of trigonometric equations, starting from very basic ones and finishing with ones where some trigonometric identities and algebraic manipulations are required.

Before you start with this presentation make sure you're very familiar with:

- radian measure (in almost all equations we will use radians instead of degrees);
- graphs of trigonometric functions ($\sin x$, $\cos x$, $\tan x$, $\cot x$), including basic properties of these graphs (domain, range, period, etc.)
- values of trigonometric functions for standard angles ($0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$);
- reduction formulae (eg. $\sin(\pi - x) = \sin x$ or $\sin(\frac{\pi}{2} - x) = \cos x$)
- trigonometric identities: Pythagorean identity, double angle identities, angle sum and difference identities, sum-to-product identities (the last one is not strictly speaking required by IB, but it will be required in my class as it often helps a lot).

Important note

This presentation is for your use only. Please do not share it publicly. In particular do not post it online anywhere.

Plan

We will cover the following topics:

Plan

We will cover the following topics:

- basic trigonometric equations,

Plan

We will cover the following topics:

- basic trigonometric equations,
- variations of basic trigonometric equations,

Plan

We will cover the following topics:

- basic trigonometric equations,
- variations of basic trigonometric equations,
- factorization of trig equations,

Plan

We will cover the following topics:

- basic trigonometric equations,
- variations of basic trigonometric equations,
- factorization of trig equations,
- using Pythagorean identity,

Plan

We will cover the following topics:

- basic trigonometric equations,
- variations of basic trigonometric equations,
- factorization of trig equations,
- using Pythagorean identity,
- using double angle identities,

Plan

We will cover the following topics:

- basic trigonometric equations,
- variations of basic trigonometric equations,
- factorization of trig equations,
- using Pythagorean identity,
- using double angle identities,
- using angle sum and difference identities,

Plan

We will cover the following topics:

- basic trigonometric equations,
- variations of basic trigonometric equations,
- factorization of trig equations,
- using Pythagorean identity,
- using double angle identities,
- using angle sum and difference identities,
- using sum-to-product identities,

Plan

We will cover the following topics:

- basic trigonometric equations,
- variations of basic trigonometric equations,
- factorization of trig equations,
- using Pythagorean identity,
- using double angle identities,
- using angle sum and difference identities,
- using sum-to-product identities,
- some harder examples,

Plan

We will cover the following topics:

- basic trigonometric equations,
- variations of basic trigonometric equations,
- factorization of trig equations,
- using Pythagorean identity,
- using double angle identities,
- using angle sum and difference identities,
- using sum-to-product identities,
- some harder examples,
- exam questions from Polish matura,

Plan

We will cover the following topics:

- basic trigonometric equations,
- variations of basic trigonometric equations,
- factorization of trig equations,
- using Pythagorean identity,
- using double angle identities,
- using angle sum and difference identities,
- using sum-to-product identities,
- some harder examples,
- exam questions from Polish matura,
- IB exam questions.

Basic trigonometric equations - example 1

We will start with the following equation:

$$\sin x = \frac{\sqrt{3}}{2}$$

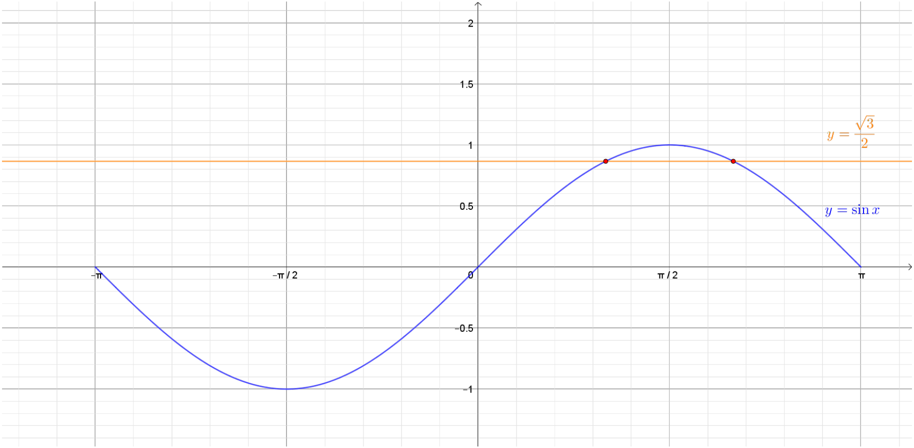
Basic trigonometric equations - example 1

We will start with the following equation:

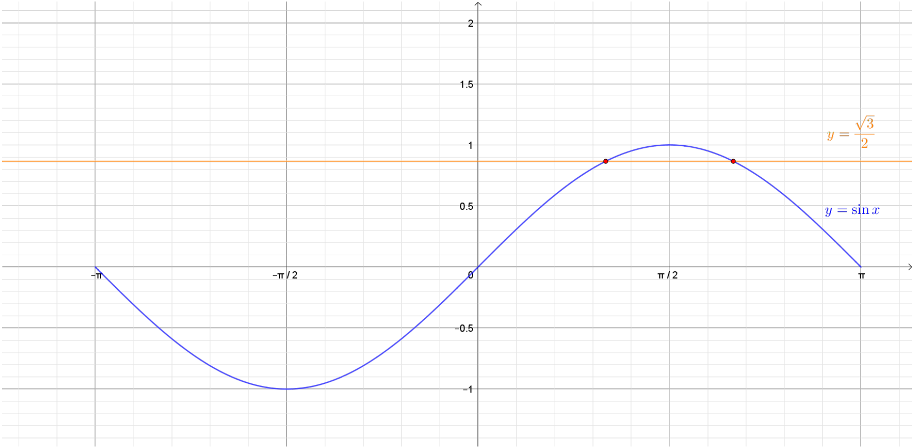
$$\sin x = \frac{\sqrt{3}}{2}$$

We want to draw one period of the sine function (eg. from $-\pi$ to π) and the line $y = \frac{\sqrt{3}}{2}$.

Basic trigonometric equations - example 1

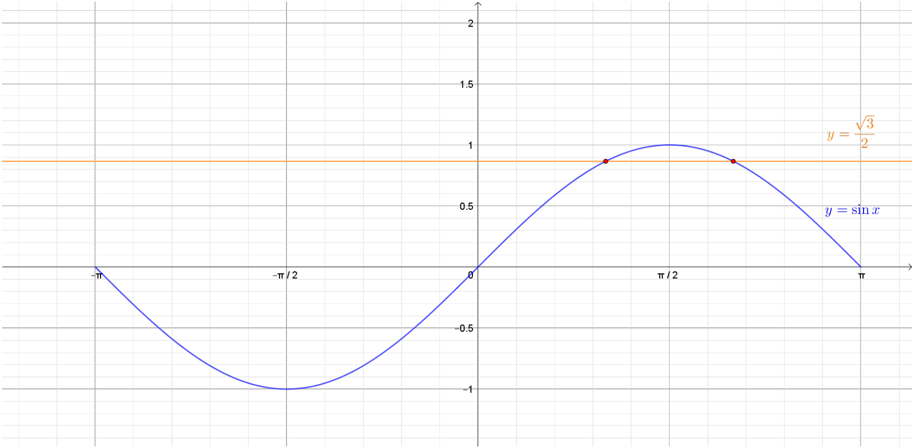


Basic trigonometric equations - example 1



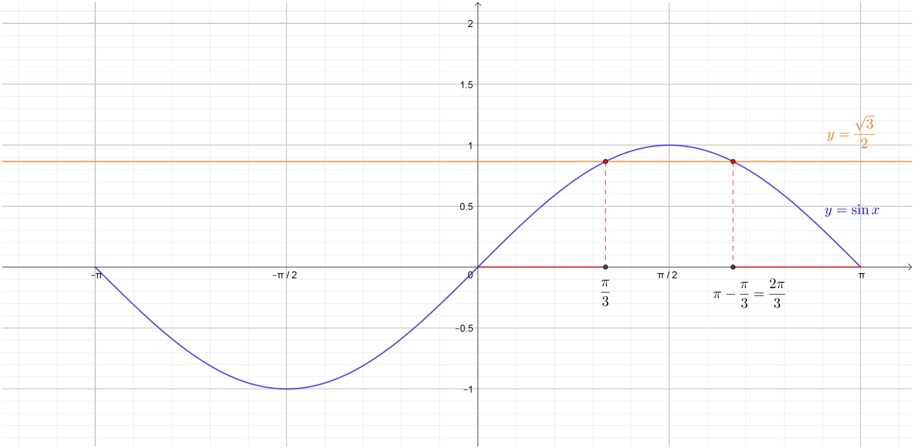
We can see two solutions (red points).

Basic trigonometric equations - example 1



We can see two solutions (red points). We should know one of those (from tables of values of standard angles), we can find the other one using symmetries of the graph.

Basic trigonometric equations - example 1



Our solutions are $x = \frac{\pi}{3}$ and $x = \frac{2\pi}{3}$

Basic trigonometric equations - example 1

So the solutions to

$$\sin x = \frac{\sqrt{3}}{2}$$

are:

Basic trigonometric equations - example 1

So the solutions to

$$\sin x = \frac{\sqrt{3}}{2}$$

are:

$$x = \frac{\pi}{3} + 2k\pi \qquad \text{or} \qquad x = \frac{2\pi}{3} + 2k\pi$$

where $k \in \mathbb{Z}$, so k is an integer.

Basic trigonometric equations - example 1

So the solutions to

$$\sin x = \frac{\sqrt{3}}{2}$$

are:

$$x = \frac{\pi}{3} + 2k\pi \qquad \text{or} \qquad x = \frac{2\pi}{3} + 2k\pi$$

where $k \in \mathbb{Z}$, so k is an integer.

Where does the $2k\pi$ come from?

Basic trigonometric equations - example 1

So the solutions to

$$\sin x = \frac{\sqrt{3}}{2}$$

are:

$$x = \frac{\pi}{3} + 2k\pi \qquad \text{or} \qquad x = \frac{2\pi}{3} + 2k\pi$$

where $k \in \mathbb{Z}$, so k is an integer.

Where does the $2k\pi$ come from? We only drew one period of *sine*, the values repeat themselves every 2π, so adding or subtracting any multiple of 2π to x will not change the value of the function.

Basic trigonometric equations - example 2

Now we want to solve:

$$\cos x = \frac{\sqrt{2}}{2}$$

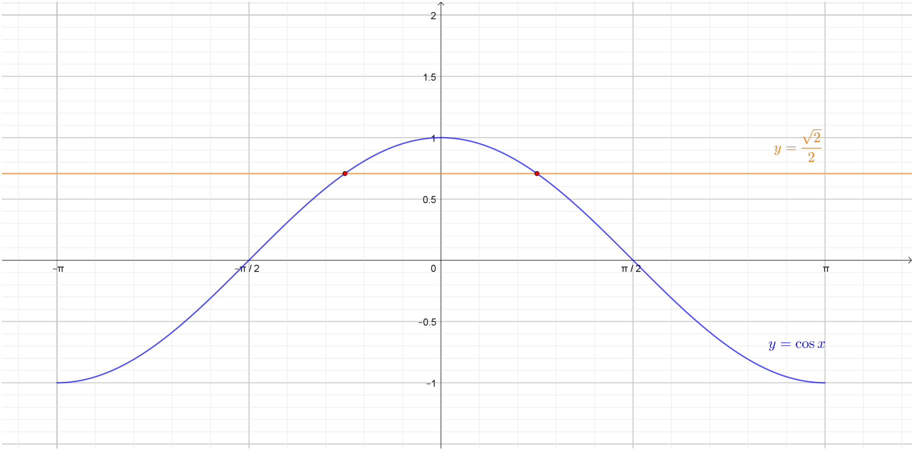
Basic trigonometric equations - example 2

Now we want to solve:

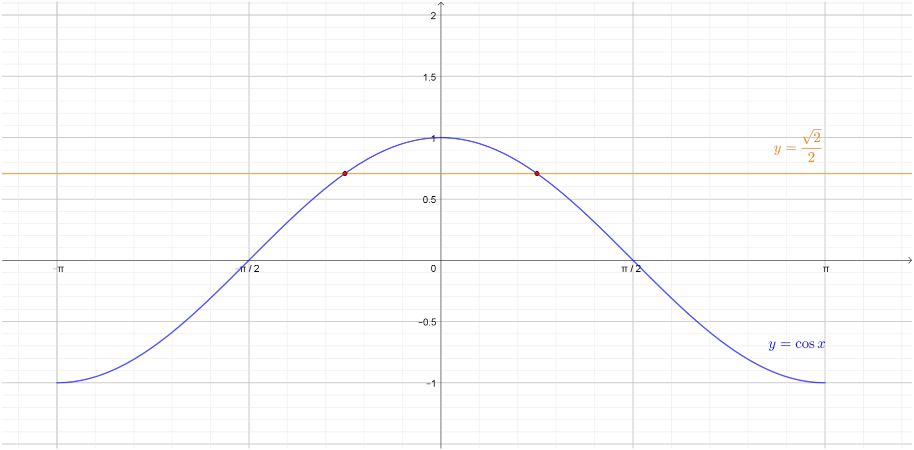
$$\cos x = \frac{\sqrt{2}}{2}$$

We draw one period of the cosine function (again it can be from $-\pi$ to π) and the line $y = \frac{\sqrt{2}}{2}$.

Basic trigonometric equations - example 2

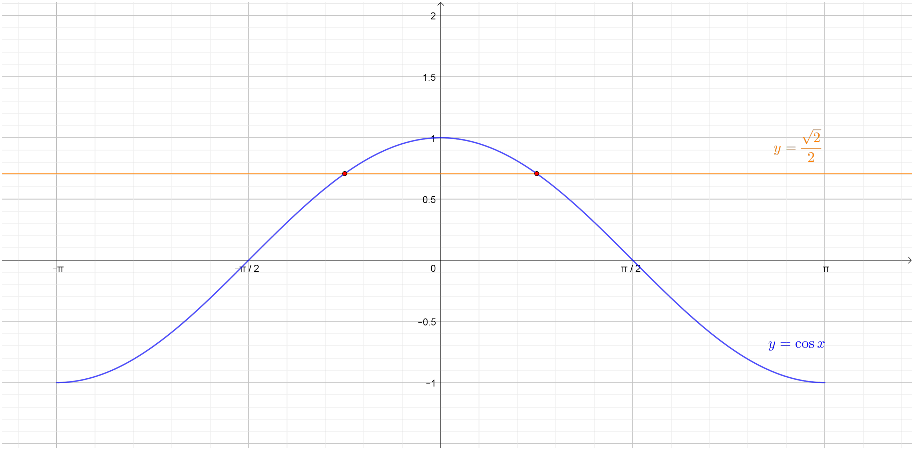


Basic trigonometric equations - example 2



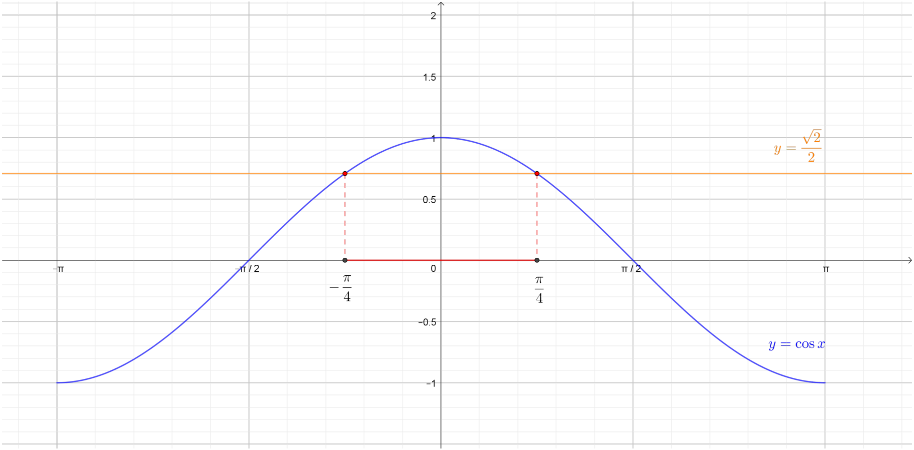
We can see two solutions (red points).

Basic trigonometric equations - example 2



We can see two solutions (red points). We should know one of those solutions and we can find the other one using symmetries of the graph.

Basic trigonometric equations - example 2



One solution is $x = \frac{\pi}{4}$, the other is of course $x = -\frac{\pi}{4}$.

Basic trigonometric equations - example 2

Finally we get that the solutions to the equation

$$\cos x = \frac{\sqrt{2}}{2}$$

are:

Basic trigonometric equations - example 2

Finally we get that the solutions to the equation

$$\cos x = \frac{\sqrt{2}}{2}$$

are:

$$x = \frac{\pi}{4} + 2k\pi \qquad \text{or} \qquad x = -\frac{\pi}{4} + 2k\pi$$

where $k \in \mathbb{Z}$.

Basic trigonometric equations - example 3

Solve:

$$\tan x = \frac{\sqrt{3}}{3}$$

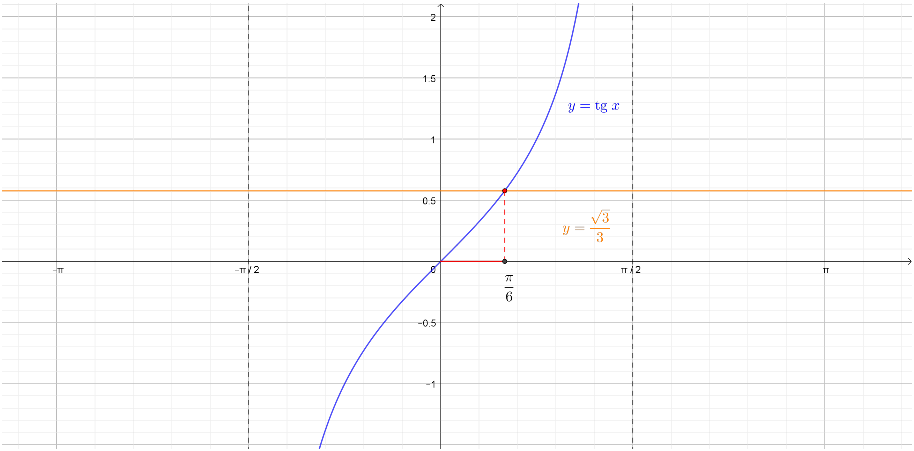
Basic trigonometric equations - example 3

Solve:

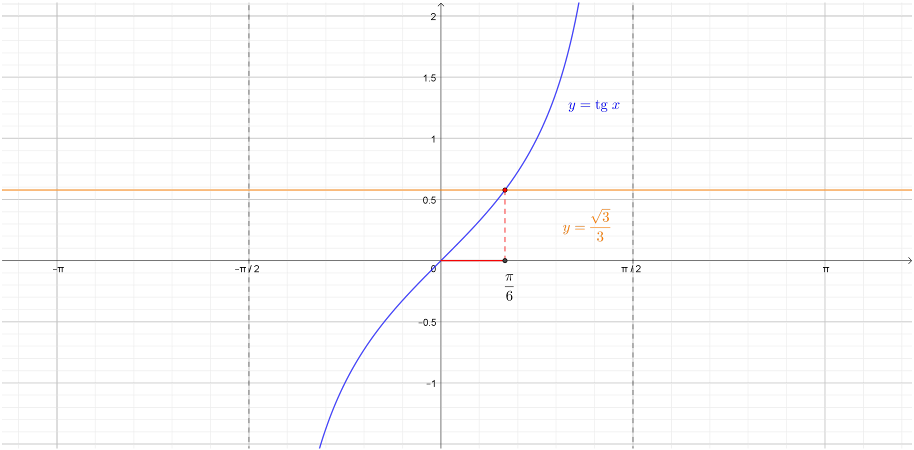
$$\tan x = \frac{\sqrt{3}}{3}$$

We draw one period of tangent function (remember that the period of $\tan x$ is π, it's best to draw it from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$) and the line $y = \frac{\sqrt{3}}{3}$.

Basic trigonometric equations - example 3

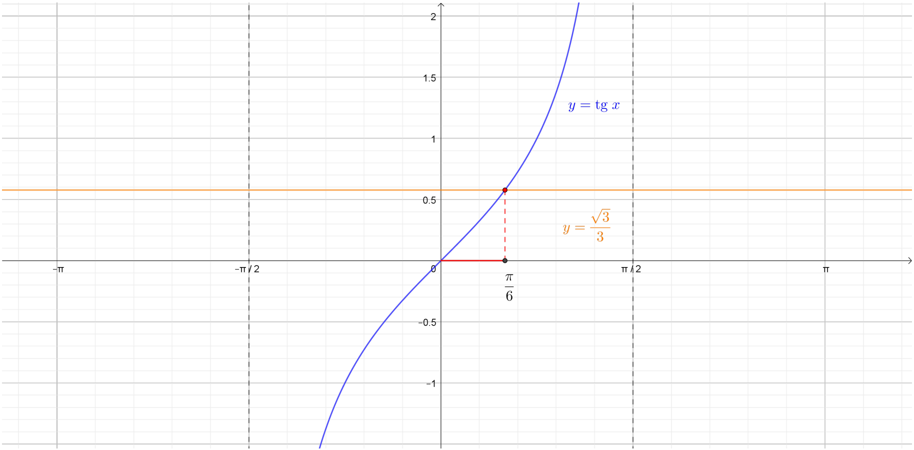


Basic trigonometric equations - example 3



There's one solution (red point).

Basic trigonometric equations - example 3



There's one solution (red point). We know it from the table of standard angles, $x = \frac{\pi}{6}$.

Basic trigonometric equations - example 3

We get that the solutions to the equations

$$\tan x = \frac{\sqrt{3}}{3}$$

are:

Basic trigonometric equations - example 3

We get that the solutions to the equations

$$\tan x = \frac{\sqrt{3}}{3}$$

are:

$$x = \frac{\pi}{6} + k\pi$$

where $k \in \mathbb{Z}$.

Basic trigonometric equations - example 4

Solve:

$$\cot x = 1$$

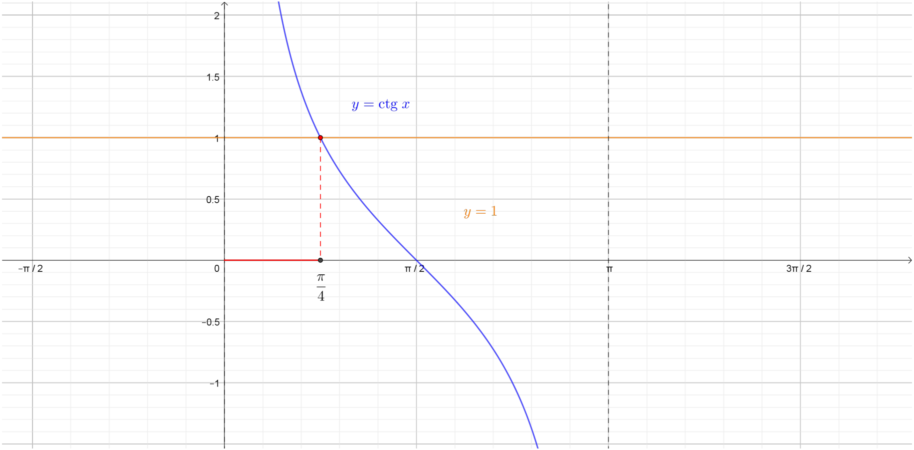
Basic trigonometric equations - example 4

Solve:

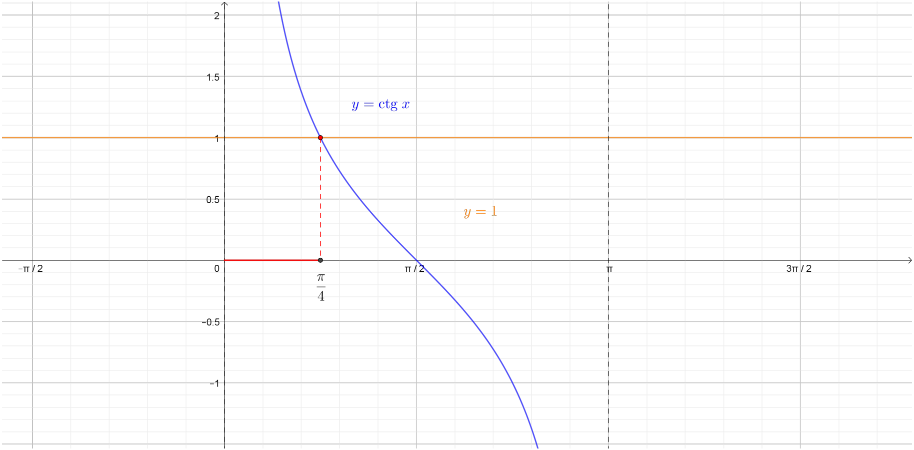
$$\cot x = 1$$

We draw one period of cotangent function (the period is π, we'll draw it between 0 and π) and the line $y = 1$.

Basic trigonometric equations - example 4

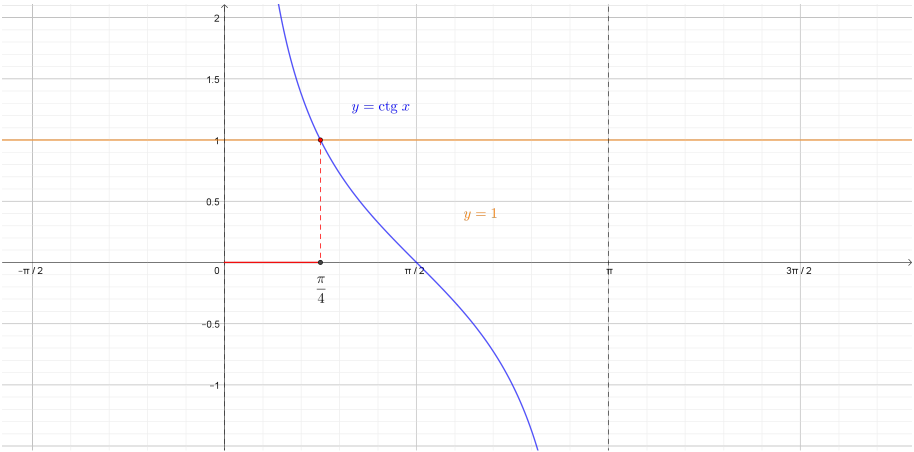


Basic trigonometric equations - example 4



We can see one solution (red point).

Basic trigonometric equations - example 4



We can see one solution (red point). It is $x = \frac{\pi}{4}$.

Basic trigonometric equations - example 4

Therefore the solutions to

$$\cot x = 1$$

are:

Basic trigonometric equations - example 4

Therefore the solutions to

$$\cot x = 1$$

are:

$$x = \frac{\pi}{4} + k\pi$$

where $k \in \mathbb{Z}$.

Basic trigonometric equations - exercises

Solve the following equations:

Basic trigonometric equations - exercises

Solve the following equations:

- Equation:

$$\sin x = \frac{1}{2}$$

Basic trigonometric equations - exercises

Solve the following equations:

- Equation:

$$\sin x = \frac{1}{2}$$

Solution:

Basic trigonometric equations - exercises

Solve the following equations:

- Equation:

$$\sin x = \frac{1}{2}$$

Solution:

$$x = \frac{\pi}{6} + 2k\pi \qquad \text{or} \qquad x = \frac{5\pi}{6} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Basic trigonometric equations - exercises

Solve the following equations:

- Equation:

$$\sin x = \frac{1}{2}$$

Solution:

$$x = \frac{\pi}{6} + 2k\pi \qquad \text{or} \qquad x = \frac{5\pi}{6} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$\cos x = 0$$

Basic trigonometric equations - exercises

Solve the following equations:

- Equation:

$$\sin x = \frac{1}{2}$$

  Solution:

$$x = \frac{\pi}{6} + 2k\pi \qquad \text{or} \qquad x = \frac{5\pi}{6} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$\cos x = 0$$

  Solution:

Basic trigonometric equations - exercises

Solve the following equations:

- Equation:

$$\sin x = \frac{1}{2}$$

Solution:

$$x = \frac{\pi}{6} + 2k\pi \qquad \text{or} \qquad x = \frac{5\pi}{6} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$\cos x = 0$$

Solution:

$$x = \frac{\pi}{2} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Basic trigonometric equations - exercises

- Equation:

$$\tan x = \sqrt{3}$$

Basic trigonometric equations - exercises

- Equation:

$$\tan x = \sqrt{3}$$

  Solution:

Basic trigonometric equations - exercises

- Equation:

$$\tan x = \sqrt{3}$$

Solution:

$$x = \frac{\pi}{3} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Basic trigonometric equations - exercises

- Equation:

$$\tan x = \sqrt{3}$$

  Solution:

$$x = \frac{\pi}{3} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$\cot x = 0$$

Basic trigonometric equations - exercises

- Equation:

$$\tan x = \sqrt{3}$$

  Solution:

$$x = \frac{\pi}{3} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$\cot x = 0$$

  Solution:

Basic trigonometric equations - exercises

- Equation:

$$\tan x = \sqrt{3}$$

Solution:

$$x = \frac{\pi}{3} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$\cot x = 0$$

Solution:

$$x = \frac{\pi}{2} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Basic trigonometric equations - example 5

Solve the equation:

$$\sin x = -1$$

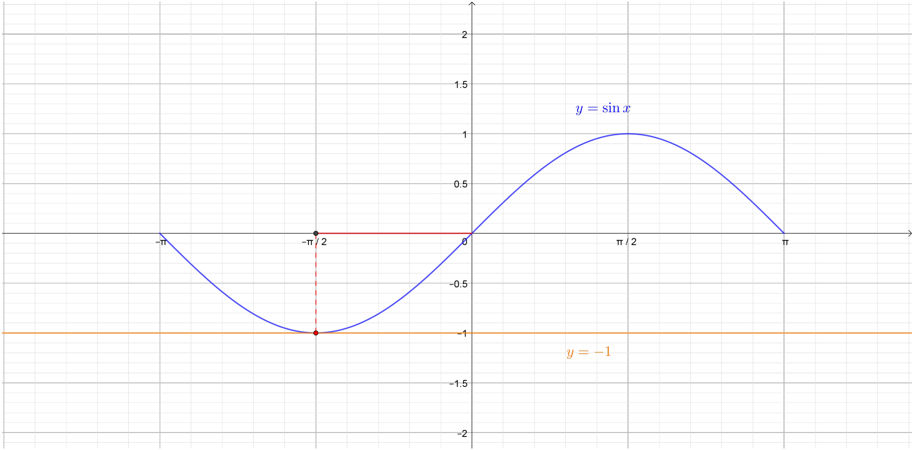
Basic trigonometric equations - example 5

Solve the equation:

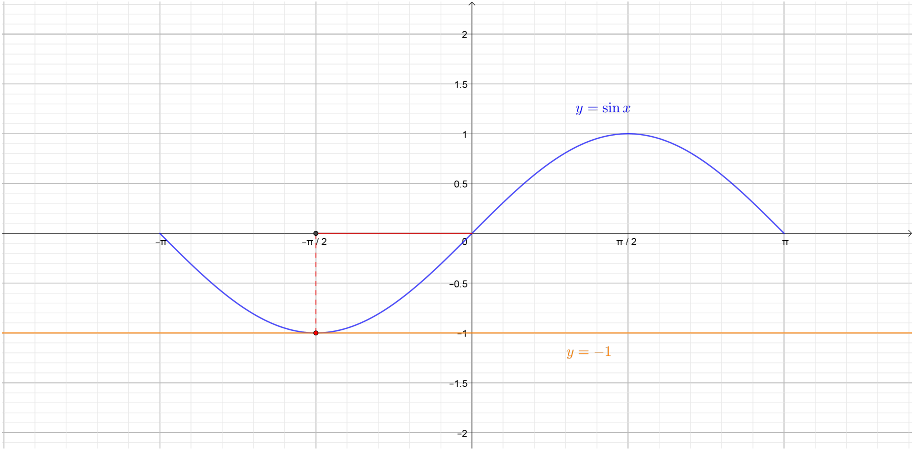
$$\sin x = -1$$

We draw one period of sine function and the line $y = -1$.

Basic trigonometric equations - example 5



Basic trigonometric equations - example 5



We can see one solution, it's of course $x = -\frac{\pi}{2}$.

Basic trigonometric equations - example 5

The solutions to

$$\sin x = -1$$

are:

Basic trigonometric equations - example 5

The solutions to

$$\sin x = -1$$

are:

$$x = -\frac{\pi}{2} + 2k\pi$$

where $k \in \mathbb{Z}$.

Basic trigonometric equations - example 6

Solve:

$$\cos x = -\frac{1}{2}$$

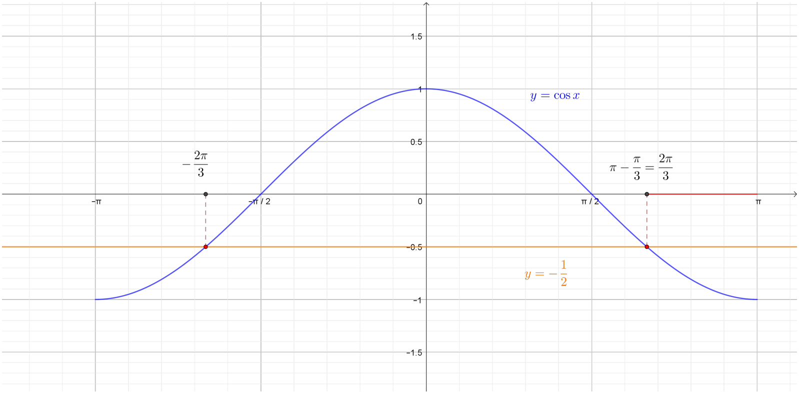
Basic trigonometric equations - example 6

Solve:

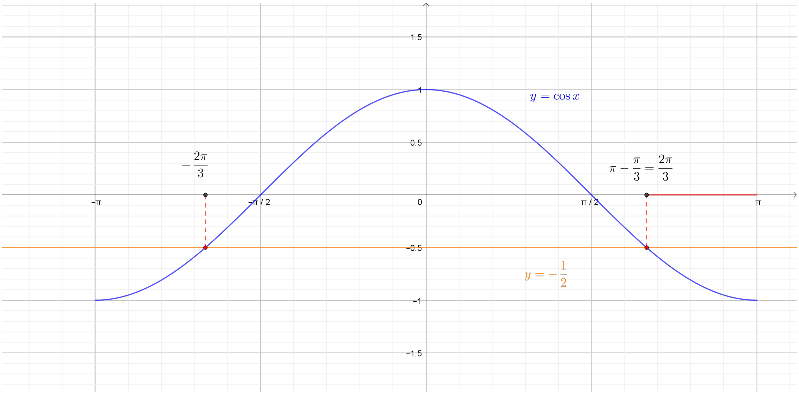
$$\cos x = -\frac{1}{2}$$

We draw one period of cosine function and the line $y = -\frac{1}{2}$.

Basic trigonometric equations - example 6



Basic trigonometric equations - example 6



We can see two solutions. If we were to solve $\cos x = \frac{1}{2}$, then we would know that $x = \frac{\pi}{3}$ is one of the solutions, here we can use the symmetry to get $x = \frac{2\pi}{3}$ as a solution and then we also get $x = -\frac{2\pi}{3}$.

Basic trigonometric equations - example 6

We get that the solutions to

$$\cos x = -\frac{1}{2}$$

are:

Basic trigonometric equations - example 6

We get that the solutions to

$$\cos x = -\frac{1}{2}$$

are:

$$x = -\frac{2\pi}{3} + 2k\pi \qquad \text{or} \qquad x = \frac{2\pi}{3} + 2k\pi$$

where $k \in \mathbb{Z}$.

Basic trigonometric equations - example 7

Solve:

$$\tan x = -1$$

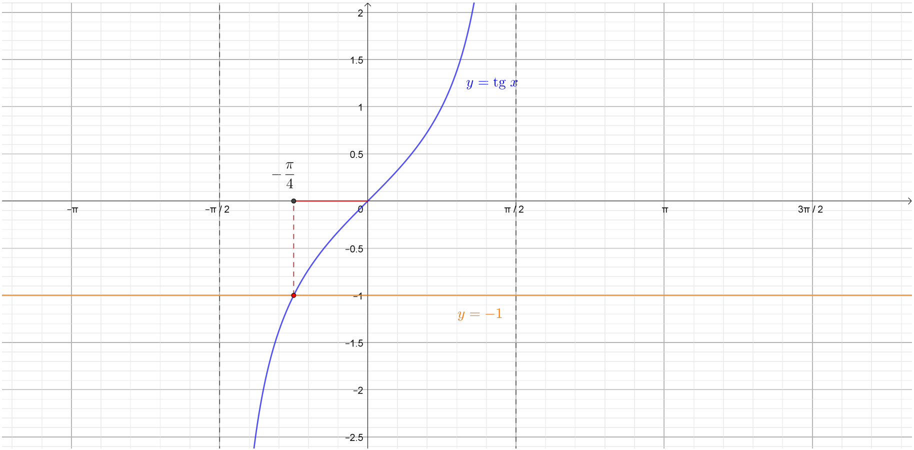
Basic trigonometric equations - example 7

Solve:

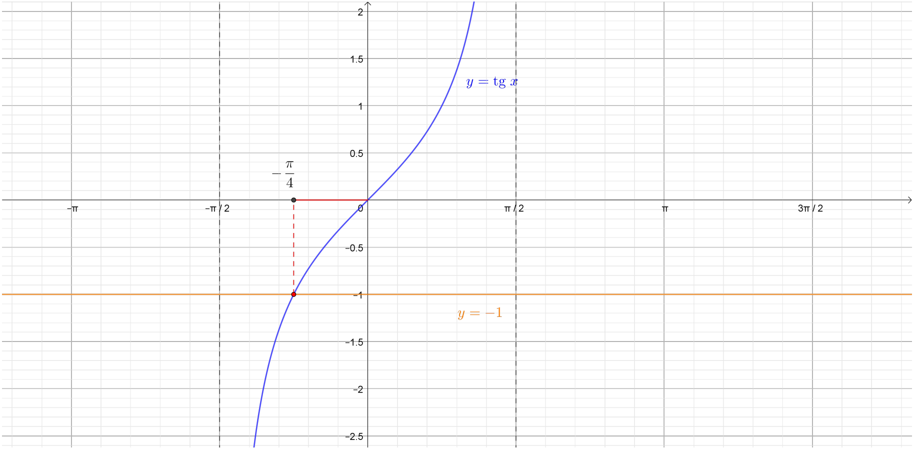
$$\tan x = -1$$

As always we draw one period of tangent function and the line $y = -1$.

Basic trigonometric equations - example 7



Basic trigonometric equations - example 7



There's one solution. If we were to solve $\tan x = 1$, the solution would be $x = \frac{\pi}{4}$, so here we of course have $x = -\frac{\pi}{4}$

Basic trigonometric equations - example 7

So the solutions to

$$\tan x = -1$$

are:

So the solutions to

$$\tan x = -1$$

are:

$$x = -\frac{\pi}{4} + k\pi$$

where $k \in \mathbb{Z}$.

Basic trigonometric equations - example 8

Solve:

$$\cot x = -\sqrt{3}$$

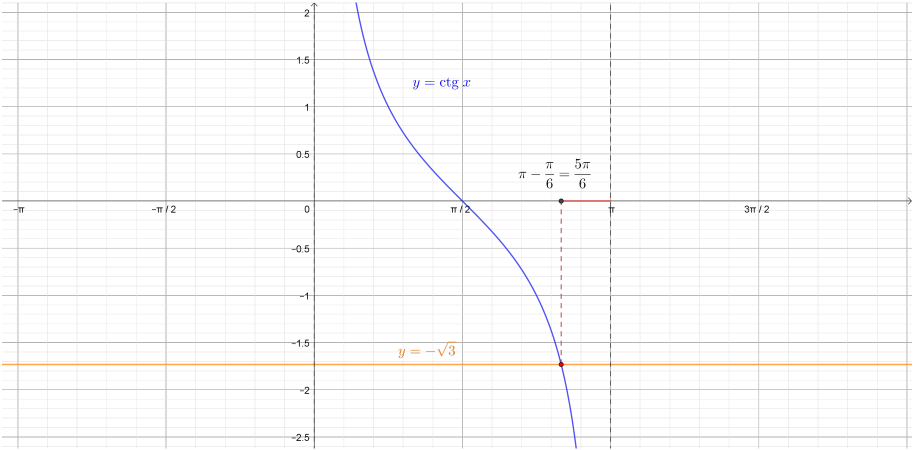
Basic trigonometric equations - example 8

Solve:

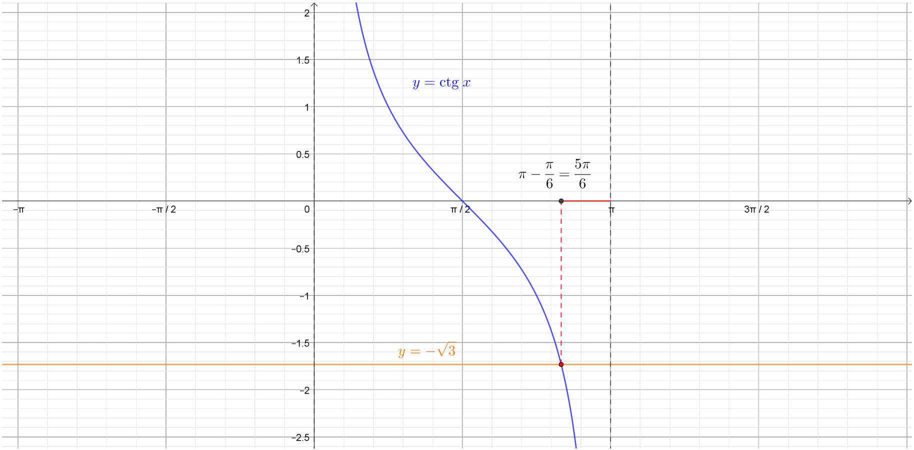
$$\cot x = -\sqrt{3}$$

We draw one period of cotangent function and the line $y = -\sqrt{3}$.

Basic trigonometric equations - example 8



Basic trigonometric equations - example 8



There's one solution. Solving $\cot x = \sqrt{3}$ would give us $x = \frac{\pi}{6}$, so here we have $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Basic trigonometric equations - example 8

So the solutions to

$$\cot x = -\sqrt{3}$$

are:

Basic trigonometric equations - example 8

So the solutions to

$$\cot x = -\sqrt{3}$$

are:

$$x = \frac{5\pi}{6} + k\pi$$

where $k \in \mathbb{Z}$.

Basic trigonometric equations - exercises

Solve the following equations:

Basic trigonometric equations - exercises

Solve the following equations:

- Equation:

$$\sin x = -\frac{\sqrt{3}}{2}$$

Basic trigonometric equations - exercises

Solve the following equations:

- Equation:

$$\sin x = -\frac{\sqrt{3}}{2}$$

  Solutions:

Basic trigonometric equations - exercises

Solve the following equations:

- Equation:

$$\sin x = -\frac{\sqrt{3}}{2}$$

Solutions:

$$x = -\frac{\pi}{3} + 2k\pi \qquad \text{or} \qquad x = -\frac{2\pi}{3} + 2k\pi \qquad \text{wheree } k \in \mathbb{Z}$$

Basic trigonometric equations - exercises

Solve the following equations:

- Equation:

$$\sin x = -\frac{\sqrt{3}}{2}$$

  Solutions:

$$x = -\frac{\pi}{3} + 2k\pi \qquad \text{or} \qquad x = -\frac{2\pi}{3} + 2k\pi \qquad \text{wheree } k \in \mathbb{Z}$$

- Equation:

$$\cos x = -\frac{\sqrt{2}}{2}$$

Basic trigonometric equations - exercises

Solve the following equations:

- Equation:

$$\sin x = -\frac{\sqrt{3}}{2}$$

  Solutions:

$$x = -\frac{\pi}{3} + 2k\pi \qquad \text{or} \qquad x = -\frac{2\pi}{3} + 2k\pi \qquad \text{wheree } k \in \mathbb{Z}$$

- Equation:

$$\cos x = -\frac{\sqrt{2}}{2}$$

  Solutions:

Basic trigonometric equations - exercises

Solve the following equations:

- Equation:

$$\sin x = -\frac{\sqrt{3}}{2}$$

Solutions:

$$x = -\frac{\pi}{3} + 2k\pi \qquad \text{or} \qquad x = -\frac{2\pi}{3} + 2k\pi \qquad \text{wheree } k \in \mathbb{Z}$$

- Equation:

$$\cos x = -\frac{\sqrt{2}}{2}$$

Solutions:

$$x = -\frac{3\pi}{4} + 2k\pi \qquad \text{or} \qquad x = \frac{3\pi}{4} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Basic trigonometric equations - exercises

- Equation:

$$\tan x = -\frac{\sqrt{3}}{3}$$

Basic trigonometric equations - exercises

- Equation:

$$\tan x = -\frac{\sqrt{3}}{3}$$

  Solutions:

Basic trigonometric equations - exercises

- Equation:

$$\tan x = -\frac{\sqrt{3}}{3}$$

Solutions:

$$x = -\frac{\pi}{6} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Basic trigonometric equations - exercises

- Equation:

$$\tan x = -\frac{\sqrt{3}}{3}$$

Solutions:

$$x = -\frac{\pi}{6} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$\cot x = -1$$

Basic trigonometric equations - exercises

- Equation:

$$\tan x = -\frac{\sqrt{3}}{3}$$

Solutions:

$$x = -\frac{\pi}{6} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$\cot x = -1$$

Solutions:

Basic trigonometric equations - exercises

- Equation:

$$\tan x = -\frac{\sqrt{3}}{3}$$

Solutions:

$$x = -\frac{\pi}{6} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$\cot x = -1$$

Solutions:

$$x = \frac{3\pi}{4} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Basic trigonometric equations - example 9

In the examples above we found **all** solutions to a given equation. However in almost all IB trig equation questions you'll be required to find solutions that are in a specific interval.

Basic trigonometric equations - example 9

In the examples above we found **all** solutions to a given equation. However in almost all IB trig equation questions you'll be required to find solutions that are in a specific interval.
Solve

$$\sin x = \frac{\sqrt{2}}{2}$$

for $0 \leq x \leq 3\pi$.

Basic trigonometric equations - example 9

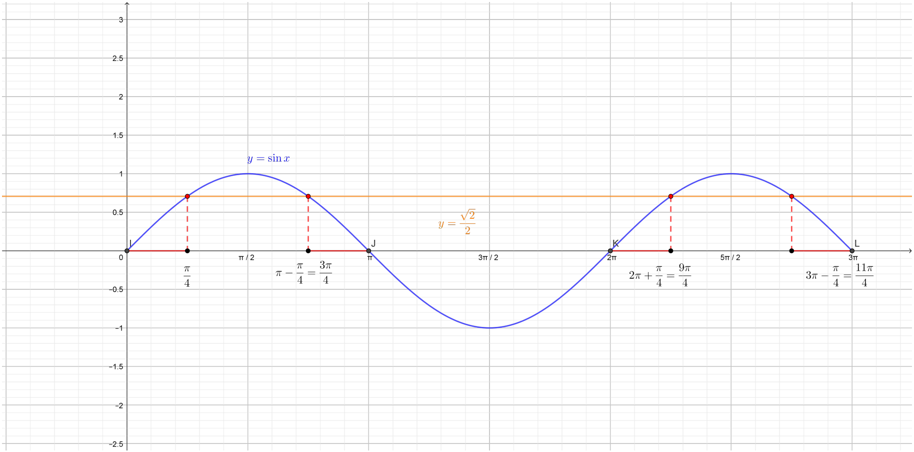
In the examples above we found **all** solutions to a given equation. However in almost all IB trig equation questions you'll be required to find solutions that are in a specific interval.
Solve

$$\sin x = \frac{\sqrt{2}}{2}$$

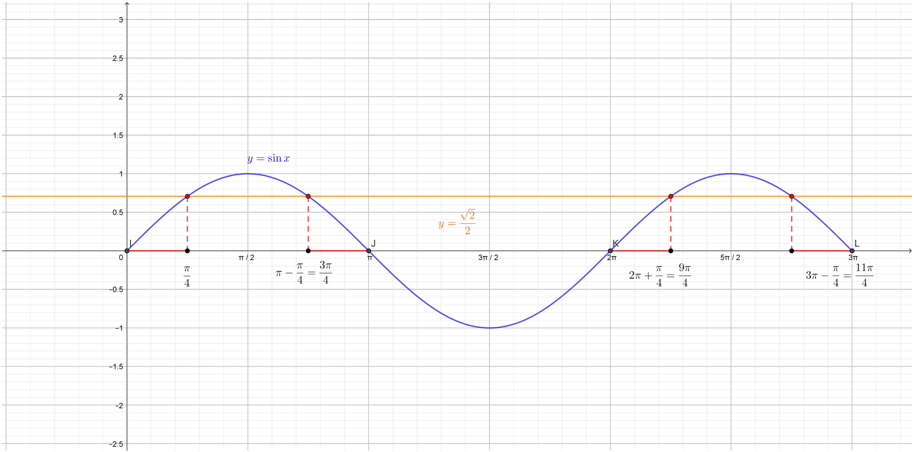
for $0 \leq x \leq 3\pi$.

This is even simpler. We draw $y = \frac{\sqrt{2}}{2}$ and $y = \sin x$, but only for $0 \leq x \leq 3\pi$.

Basic trigonometric equations - example 9

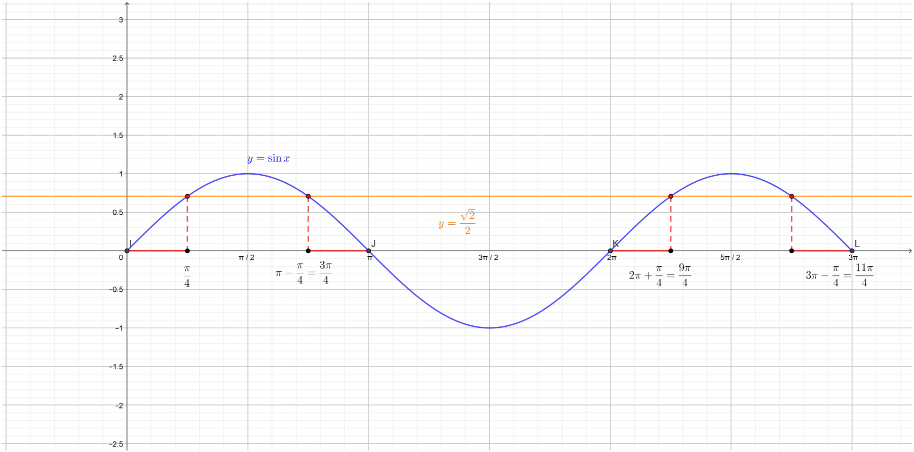


Basic trigonometric equations - example 9



We have four solutions.

Basic trigonometric equations - example 9



We have four solutions. We should know one from the table and find the rest using symmetries and periodicity of the graph.

Basic trigonometric equations - example 9

The solutions to

$$\sin x = \frac{\sqrt{2}}{2}$$

for $0 \leq x \leq 3\pi$ are

Basic trigonometric equations - example 9

The solutions to

$$\sin x = \frac{\sqrt{2}}{2}$$

for $0 \leq x \leq 3\pi$ are $x = \frac{\pi}{4}$ or $x = \frac{3\pi}{4}$ or $x = \frac{9\pi}{4}$ or $x = \frac{11\pi}{4}$.

Basic trigonometric equations - example 10

Solve

$$\cos x = -\frac{\sqrt{3}}{2}$$

for $-2\pi \leq x \leq \pi$.

Basic trigonometric equations - example 10

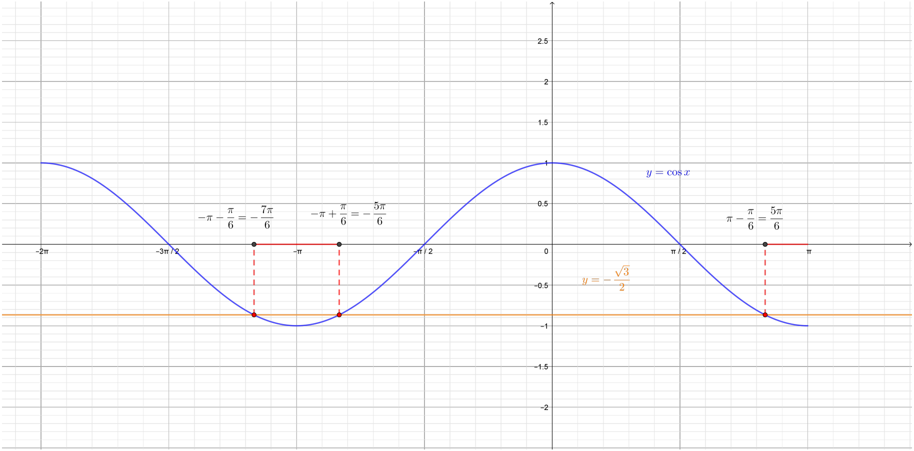
Solve

$$\cos x = -\frac{\sqrt{3}}{2}$$

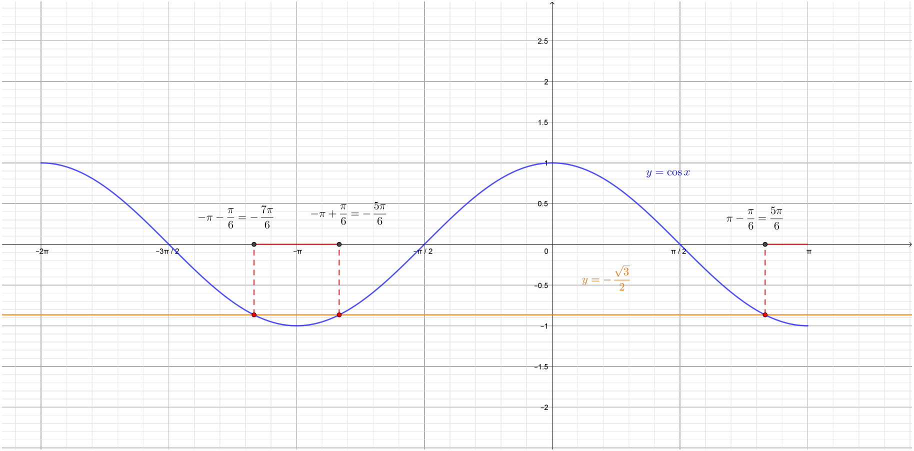
for $-2\pi \leq x \leq \pi$.

We draw $y = -\frac{\sqrt{3}}{2}$ and $y = \cos x$, but only for $-2\pi \leq x \leq \pi$.

Basic trigonometric equations - example 10

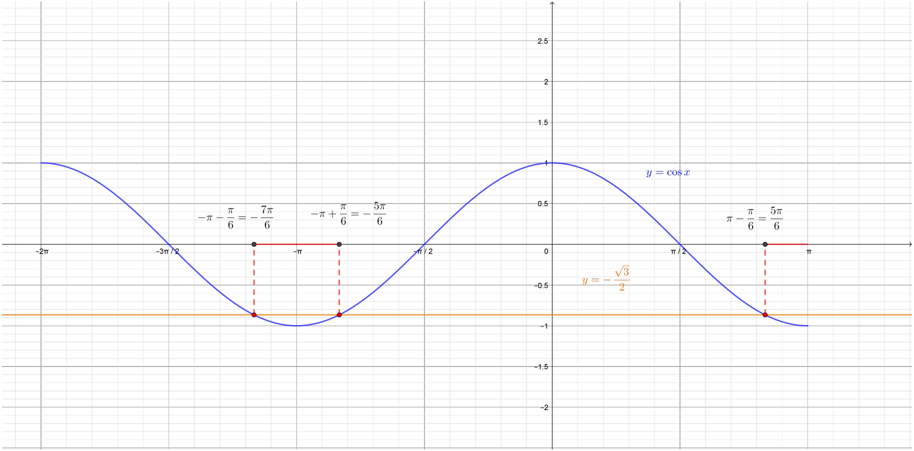


Basic trigonometric equations - example 10



We have 3 solutions.

Basic trigonometric equations - example 10



We have 3 solutions. If we were solving $\cos x = \frac{\sqrt{3}}{2}$, then we would have $x = \frac{\pi}{6}$ as a solution, based on that and symmetries we can find the actual solutions.

Basic trigonometric equations - example 10

The solutions to

$$\cos x = -\frac{\sqrt{3}}{2}$$

for $-2\pi \leq x \leq \pi$ are

Basic trigonometric equations - example 10

The solutions to

$$\cos x = -\frac{\sqrt{3}}{2}$$

for $-2\pi \leq x \leq \pi$ are $x = -\frac{7\pi}{6}$ or $x = -\frac{5\pi}{6}$ or $x = \frac{5\pi}{6}$.

Basic trigonometric equations - exercises

- Solve:

$$\tan x = -1$$

for $-\pi \leq x \leq \pi$.

Basic trigonometric equations - exercises

- Solve:

$$\tan x = -1$$

for $-\pi \leq x \leq \pi$.
Solution:

Basic trigonometric equations - exercises

- Solve:

$$\tan x = -1$$

for $-\pi \leq x \leq \pi$.
Solution:

$$x = -\frac{\pi}{4} \qquad \text{or} \qquad x = \frac{3\pi}{4}$$

Basic trigonometric equations - exercises

- Solve:

$$\tan x = -1$$

for $-\pi \le x \le \pi$.
Solution:

$$x = -\frac{\pi}{4} \qquad \text{or} \qquad x = \frac{3\pi}{4}$$

- Solve:

$$\sin x = 1$$

for $-\pi \le x \le 3\pi$

Basic trigonometric equations - exercises

- Solve:

$$\tan x = -1$$

for $-\pi \leq x \leq \pi$.
Solution:

$$x = -\frac{\pi}{4} \qquad \text{or} \qquad x = \frac{3\pi}{4}$$

- Solve:

$$\sin x = 1$$

for $-\pi \leq x \leq 3\pi$
Solution:

Basic trigonometric equations - exercises

- Solve:

$$\tan x = -1$$

for $-\pi \leq x \leq \pi$.
Solution:

$$x = -\frac{\pi}{4} \qquad \text{or} \qquad x = \frac{3\pi}{4}$$

- Solve:

$$\sin x = 1$$

for $-\pi \leq x \leq 3\pi$
Solution:

$$x = \frac{\pi}{2} \qquad \text{or} \qquad x = \frac{5\pi}{2}$$

The ability to solve simple trigonometric equations is the basis for more complicated equations.

The ability to solve simple trigonometric equations is the basis for more complicated equations. In the end we will almost always arrive at the simple ones.

The ability to solve simple trigonometric equations is the basis for more complicated equations. In the end we will almost always arrive at the simple ones. In the following examples I'll assume that you can solve the basic equations with ease, so make sure you practice those before moving on.

The ability to solve simple trigonometric equations is the basis for more complicated equations. In the end we will almost always arrive at the simple ones. In the following examples I'll assume that you can solve the basic equations with ease, so make sure you practice those before moving on.

On the following slides I'll skip the step with drawing graphs, but you should still do it. It is a very useful habit.

The ability to solve simple trigonometric equations is the basis for more complicated equations. In the end we will almost always arrive at the simple ones. In the following examples I'll assume that you can solve the basic equations with ease, so make sure you practice those before moving on.

On the following slides I'll skip the step with drawing graphs, but you should still do it. It is a very useful habit. What it means is that when you get to a basic trig equation you should solve it as above - quick sketch and read of the solutions.

We move on to equations where some algebraic manipulation is required.

Variations of basic equations - example 1

Solve:

$$2\sin(3x) + 4 = 3$$

Variations of basic equations - example 1

Solve:

$$2\sin(3x)+4=3$$

We rewrite it in the form

$$\sin(3x)=-\frac{1}{2}$$

Variations of basic equations - example 1

Solve:

$$2\sin(3x) + 4 = 3$$

We rewrite it in the form

$$\sin(3x) = -\frac{1}{2}$$

and now we solve as a basic trig equation (but instead of x we have $3x$), so we get:

Variations of basic equations - example 1

Solve:

$$2\sin(3x) + 4 = 3$$

We rewrite it in the form

$$\sin(3x) = -\frac{1}{2}$$

and now we solve as a basic trig equation (but instead of x we have $3x$), so we get:

$$3x = -\frac{\pi}{6} + 2k\pi \qquad \text{or} \qquad 3x = -\frac{5\pi}{6} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Variations of basic equations - example 1

Solve:

$$2\sin(3x) + 4 = 3$$

We rewrite it in the form

$$\sin(3x) = -\frac{1}{2}$$

and now we solve as a basic trig equation (but instead of x we have $3x$), so we get:

$$3x = -\frac{\pi}{6} + 2k\pi \qquad \text{or} \qquad 3x = -\frac{5\pi}{6} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Now we divide by 3, to get x:

$$x = -\frac{\pi}{18} + \frac{2k\pi}{3} \qquad \text{or} \qquad x = -\frac{5\pi}{18} + \frac{2k\pi}{3} \qquad \text{where } k \in \mathbb{Z}$$

and this is our solution.

Note that when solving

$$\sin(3x) = -\frac{1}{2}$$

We don't need to draw $\sin(3x)$ (sine squeezed by a factor of $\frac{1}{3}$). It's better to draw $\sin\alpha$ (so the usual graph of sine), solve for α and then put $3x$ instead of α.

Note that when solving

$$\sin(3x) = -\frac{1}{2}$$

We don't need to draw $\sin(3x)$ (sine squeezed by a factor of $\frac{1}{3}$). It's better to draw $\sin\alpha$ (so the usual graph of sine), solve for α and then put $3x$ instead of α.

We will get back to this in a few slides.

Variations of basic equations - example 2

Solve:

$$\cos(2x - \frac{\pi}{3}) + 1 = 0$$

Variations of basic equations - example 2

Solve:

$$\cos(2x - \frac{\pi}{3}) + 1 = 0$$

We rewrite in the form:

$$\cos(2x - \frac{\pi}{3}) = -1$$

Variations of basic equations - example 2

Solve:

$$\cos(2x - \frac{\pi}{3}) + 1 = 0$$

We rewrite in the form:

$$\cos(2x - \frac{\pi}{3}) = -1$$

and we solve as a basic equation (instead of x we have $2x - \frac{\pi}{3}$), we get:

Variations of basic equations - example 2

Solve:

$$\cos(2x - \frac{\pi}{3}) + 1 = 0$$

We rewrite in the form:

$$\cos(2x - \frac{\pi}{3}) = -1$$

and we solve as a basic equation (instead of x we have $2x - \frac{\pi}{3}$), we get:

$$2x - \frac{\pi}{3} = \pi + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Variations of basic equations - example 2

Solve:

$$\cos(2x - \frac{\pi}{3}) + 1 = 0$$

We rewrite in the form:

$$\cos(2x - \frac{\pi}{3}) = -1$$

and we solve as a basic equation (instead of x we have $2x - \frac{\pi}{3}$), we get:

$$2x - \frac{\pi}{3} = \pi + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

rearrange to find x:

$$x = \frac{2\pi}{3} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

and that's our solution.

Variations of basic equations - example 3

Solve:

$$\tan^2(5x) - 3 = 0$$

Variations of basic equations - example 3

Solve:

$$\tan^2(5x) - 3 = 0$$

We get to:

$$\tan(5x) = -\sqrt{3} \qquad \text{or} \qquad \tan(5x) = \sqrt{3}$$

Variations of basic equations - example 3

Solve:

$$\tan^2(5x) - 3 = 0$$

We get to:

$$\tan(5x) = -\sqrt{3} \qquad \text{or} \qquad \tan(5x) = \sqrt{3}$$

we solve two basic equations (instead of x we have $5x$), we get:

Variations of basic equations - example 3

Solve:

$$\tan^2(5x) - 3 = 0$$

We get to:

$$\tan(5x) = -\sqrt{3} \qquad \text{or} \qquad \tan(5x) = \sqrt{3}$$

we solve two basic equations (instead of x we have $5x$), we get:

$$5x = -\frac{\pi}{3} + k\pi \qquad \text{or} \qquad 5x = \frac{\pi}{3} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Variations of basic equations - example 3

Solve:

$$\tan^2(5x) - 3 = 0$$

We get to:

$$\tan(5x) = -\sqrt{3} \qquad \text{or} \qquad \tan(5x) = \sqrt{3}$$

we solve two basic equations (instead of x we have $5x$), we get:

$$5x = -\frac{\pi}{3} + k\pi \qquad \text{or} \qquad 5x = \frac{\pi}{3} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

rearrange to find x:

$$x = -\frac{\pi}{15} + \frac{k\pi}{5} \qquad \text{or} \qquad x = \frac{\pi}{15} + \frac{k\pi}{5} \qquad \text{where } k \in \mathbb{Z}$$

and that's it.

Variations of basic equations - example 4

Solve:

$$3\cot^2\left(\frac{x}{2}\right) = 1$$

Variations of basic equations - example 4

Solve:

$$3\cot^2\left(\frac{x}{2}\right) = 1$$

Rearrange and get to:

$$\cot\left(\frac{x}{2}\right) = -\frac{\sqrt{3}}{3} \qquad \text{or} \qquad \cot\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{3}$$

Variations of basic equations - example 4

Solve:

$$3\cot^2\left(\frac{x}{2}\right) = 1$$

Rearrange and get to:

$$\cot\left(\frac{x}{2}\right) = -\frac{\sqrt{3}}{3} \qquad \text{or} \qquad \cot\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{3}$$

we now solve two basic equations (instead of x we have $\frac{x}{2}$), we get:

Variations of basic equations - example 4

Solve:

$$3\cot^2\left(\frac{x}{2}\right) = 1$$

Rearrange and get to:

$$\cot\left(\frac{x}{2}\right) = -\frac{\sqrt{3}}{3} \qquad \text{or} \qquad \cot\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{3}$$

we now solve two basic equations (instead of x we have $\frac{x}{2}$), we get:

$$\frac{x}{2} = -\frac{\pi}{3} + k\pi \qquad \text{or} \qquad \frac{x}{2} = \frac{\pi}{3} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Variations of basic equations - example 4

Solve:

$$3\cot^2\left(\frac{x}{2}\right) = 1$$

Rearrange and get to:

$$\cot\left(\frac{x}{2}\right) = -\frac{\sqrt{3}}{3} \qquad \text{or} \qquad \cot\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{3}$$

we now solve two basic equations (instead of x we have $\frac{x}{2}$), we get:

$$\frac{x}{2} = -\frac{\pi}{3} + k\pi \qquad \text{or} \qquad \frac{x}{2} = \frac{\pi}{3} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Multiply by 2 to get x:

$$x = -\frac{2\pi}{3} + 2k\pi \qquad \text{or} \qquad x = \frac{2\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

and that's our solution.

Variations of basic equations - example 5

Solve:

$$|2\cos(3x) - 1| = 1$$

Variations of basic equations - example 5

Solve:

$$|2\cos(3x) - 1| = 1$$

We rearrange and solve to get:

$$\cos(3x) = 0 \qquad \text{or} \qquad \cos(3x) = 1$$

Variations of basic equations - example 5

Solve:

$$|2\cos(3x) - 1| = 1$$

We rearrange and solve to get:

$$\cos(3x) = 0 \qquad \text{or} \qquad \cos(3x) = 1$$

we solve two basic equations (instead of x we have $3x$), we get:

Variations of basic equations - example 5

Solve:

$$|2\cos(3x) - 1| = 1$$

We rearrange and solve to get:

$$\cos(3x) = 0 \qquad \text{or} \qquad \cos(3x) = 1$$

we solve two basic equations (instead of x we have $3x$), we get:

$$3x = \frac{\pi}{2} + k\pi \qquad \text{or} \qquad 3x = 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Variations of basic equations - example 5

Solve:

$$|2\cos(3x) - 1| = 1$$

We rearrange and solve to get:

$$\cos(3x) = 0 \qquad \text{or} \qquad \cos(3x) = 1$$

we solve two basic equations (instead of x we have $3x$), we get:

$$3x = \frac{\pi}{2} + k\pi \qquad \text{or} \qquad 3x = 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

divide by 3 to get x:

$$x = \frac{\pi}{6} + \frac{k\pi}{3} \qquad \text{or} \qquad x = \frac{2k\pi}{3} \qquad \text{where } k \in \mathbb{Z}$$

and we have the solution.

Variations of basic equations - example 6

Solve:

$$|2\sin(7x) + 1| = 2$$

Variations of basic equations - example 6

Solve:

$$|2\sin(7x) + 1| = 2$$

We rearrange to get:

$$\sin(7x) = -\frac{3}{2} \qquad \text{or} \qquad \sin(7x) = \frac{1}{2}$$

Variations of basic equations - example 6

Solve:

$$|2\sin(7x)+1| = 2$$

We rearrange to get:

$$\sin(7x) = -\frac{3}{2} \qquad \text{or} \qquad \sin(7x) = \frac{1}{2}$$

the first equation has no solutions, we solve the second one (instead of x we have $7x$), we get:

Variations of basic equations - example 6

Solve:

$$|2\sin(7x) + 1| = 2$$

We rearrange to get:

$$\sin(7x) = -\frac{3}{2} \qquad \text{or} \qquad \sin(7x) = \frac{1}{2}$$

the first equation has no solutions, we solve the second one (instead of x we have $7x$), we get:

$$7x = \frac{\pi}{6} + 2k\pi \qquad \text{or} \qquad 7x = \frac{5\pi}{6} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Variations of basic equations - example 6

Solve:

$$|2\sin(7x)+1|=2$$

We rearrange to get:

$$\sin(7x)=-\frac{3}{2} \qquad \text{or} \qquad \sin(7x)=\frac{1}{2}$$

the first equation has no solutions, we solve the second one (instead of x we have $7x$), we get:

$$7x=\frac{\pi}{6}+2k\pi \qquad \text{or} \qquad 7x=\frac{5\pi}{6}+2k\pi \qquad \text{where } k\in\mathbb{Z}$$

divide by 7 to get x:

$$x=\frac{\pi}{42}+\frac{2k\pi}{7} \qquad \text{or} \qquad x=\frac{5\pi}{42}+\frac{2k\pi}{7} \qquad \text{where } k\in\mathbb{Z}$$

and that's our solution.

Variations of basic equations - exercises

- Equation:

$$2\sin^2(5x) - 1 = 0$$

Variations of basic equations - exercises

- Equation:

$$2\sin^2(5x) - 1 = 0$$

Solution:

Variations of basic equations - exercises

- Equation:

$$2\sin^2(5x) - 1 = 0$$

Solution:

$$x = \frac{\pi}{20} + \frac{k\pi}{5} \qquad \text{or} \qquad x = \frac{3\pi}{20} + \frac{k\pi}{5} \qquad \text{where } k \in \mathbb{Z}$$

Variations of basic equations - exercises

- Equation:

$$2\sin^2(5x) - 1 = 0$$

Solution:

$$x = \frac{\pi}{20} + \frac{k\pi}{5} \qquad \text{or} \qquad x = \frac{3\pi}{20} + \frac{k\pi}{5} \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$\left|2\cos\left(\frac{x}{3}\right) - 3\right| = 2$$

Variations of basic equations - exercises

- Equation:

$$2\sin^2(5x) - 1 = 0$$

Solution:

$$x = \frac{\pi}{20} + \frac{k\pi}{5} \qquad \text{or} \qquad x = \frac{3\pi}{20} + \frac{k\pi}{5} \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$\left|2\cos\left(\frac{x}{3}\right) - 3\right| = 2$$

Solution:

Variations of basic equations - exercises

- Equation:

$$2\sin^2(5x) - 1 = 0$$

Solution:

$$x = \frac{\pi}{20} + \frac{k\pi}{5} \qquad \text{or} \qquad x = \frac{3\pi}{20} + \frac{k\pi}{5} \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$\left|2\cos\left(\frac{x}{3}\right) - 3\right| = 2$$

Solution:

$$x = -\pi + 6k\pi \qquad \text{or} \qquad x = \pi + 6k\pi \qquad \text{where } k \in \mathbb{Z}$$

Variations of basic equations - exercises

- Equation:

$$3\tan^2(2x - \frac{\pi}{2}) - 1 = 0$$

Variations of basic equations - exercises

- Equation:

$$3\tan^2(2x - \frac{\pi}{2}) - 1 = 0$$

  Solution:

Variations of basic equations - exercises

- Equation:

$$3\tan^2(2x - \frac{\pi}{2}) - 1 = 0$$

Solution:

$$x = \frac{\pi}{6} + \frac{k\pi}{2} \qquad \text{or} \qquad x = \frac{\pi}{3} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

Variations of basic equations - exercises

- Equation:

$$3\tan^2(2x - \frac{\pi}{2}) - 1 = 0$$

  Solution:

$$x = \frac{\pi}{6} + \frac{k\pi}{2} \qquad \text{or} \qquad x = \frac{\pi}{3} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$|2\cot(4x) - 1| = 1$$

Variations of basic equations - exercises

- Equation:

$$3\tan^2(2x - \frac{\pi}{2}) - 1 = 0$$

Solution:

$$x = \frac{\pi}{6} + \frac{k\pi}{2} \qquad \text{or} \qquad x = \frac{\pi}{3} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$|2\cot(4x) - 1| = 1$$

Solution:

Variations of basic equations - exercises

- Equation:

$$3\tan^2(2x - \frac{\pi}{2}) - 1 = 0$$

Solution:

$$x = \frac{\pi}{6} + \frac{k\pi}{2} \qquad \text{or} \qquad x = \frac{\pi}{3} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$|2\cot(4x) - 1| = 1$$

Solution:

$$x = \frac{\pi}{16} + \frac{k\pi}{4} \qquad \text{or} \qquad x = \frac{\pi}{8} + \frac{k\pi}{4} \qquad \text{where } k \in \mathbb{Z}$$

Variations of basic equations - example 7

Again, we will usually have a specified interval for x:

Variations of basic equations - example 7

Again, we will usually have a specified interval for x:
Solve:

$$2\cos 4x - 1 = 0$$

for $0 \leq x \leq \pi$.

Variations of basic equations - example 7

Again, we will usually have a specified interval for x:
Solve:

$$2\cos 4x - 1 = 0$$

for $0 \leq x \leq \pi$.

Here we can use two methods (I recommend the latter).

Variations of basic equations - example 7

Again, we will usually have a specified interval for x:
Solve:

$$2\cos 4x - 1 = 0$$

for $0 \leq x \leq \pi$.

Here we can use two methods (I recommend the latter). The first is to forget about the interval for a moment and solve as above

Variations of basic equations - example 7

Again, we will usually have a specified interval for x:
Solve:

$$2\cos 4x - 1 = 0$$

for $0 \leq x \leq \pi$.

Here we can use two methods (I recommend the latter). The first is to forget about the interval for a moment and solve as above We get:

Variations of basic equations - example 7

Again, we will usually have a specified interval for x:
Solve:

$$2\cos 4x - 1 = 0$$

for $0 \leq x \leq \pi$.

Here we can use two methods (I recommend the latter). The first is to forget about the interval for a moment and solve as above We get:

$$4x = \frac{\pi}{3} + 2k\pi \qquad \text{or} \qquad 4x = -\frac{\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Variations of basic equations - example 7

Again, we will usually have a specified interval for x:
Solve:

$$2\cos 4x - 1 = 0$$

for $0 \leq x \leq \pi$.

Here we can use two methods (I recommend the latter). The first is to forget about the interval for a moment and solve as above We get:

$$4x = \frac{\pi}{3} + 2k\pi \qquad \text{or} \qquad 4x = -\frac{\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

so:

Variations of basic equations - example 7

Again, we will usually have a specified interval for x:
Solve:

$$2\cos 4x - 1 = 0$$

for $0 \leq x \leq \pi$.

Here we can use two methods (I recommend the latter). The first is to forget about the interval for a moment and solve as above We get:

$$4x = \frac{\pi}{3} + 2k\pi \qquad \text{or} \qquad 4x = -\frac{\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

so:

$$x = \frac{\pi}{12} + \frac{k\pi}{2} \qquad \text{or} \qquad x = -\frac{\pi}{12} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

Variations of basic equations - example 7

$$x = \frac{\pi}{12} + \frac{k\pi}{2} \qquad \text{or} \qquad x = -\frac{\pi}{12} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

Variations of basic equations - example 7

$$x = \frac{\pi}{12} + \frac{k\pi}{2} \qquad \text{or} \qquad x = -\frac{\pi}{12} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

Now we need to choose values of k, so that our solutions will satisfy $0 \leq x \leq \pi$.

Variations of basic equations - example 7

$$x = \frac{\pi}{12} + \frac{k\pi}{2} \qquad \text{or} \qquad x = -\frac{\pi}{12} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

Now we need to choose values of k, so that our solutions will satisfy $0 \leq x \leq \pi$. After brief deliberation we get: $x = \frac{\pi}{12}$ or $x = \frac{\pi}{12} + \frac{\pi}{2} = \frac{7\pi}{12}$ or $x = -\frac{\pi}{12} + \frac{\pi}{2} = \frac{5\pi}{12}$ or $x = -\frac{\pi}{12} + \frac{2\pi}{2} = \frac{11\pi}{12}$.

Variations of basic equations - example 7

$$x = \frac{\pi}{12} + \frac{k\pi}{2} \qquad \text{or} \qquad x = -\frac{\pi}{12} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

Now we need to choose values of k, so that our solutions will satisfy $0 \leq x \leq \pi$. After brief deliberation we get: $x = \frac{\pi}{12}$ or $x = \frac{\pi}{12} + \frac{\pi}{2} = \frac{7\pi}{12}$ or $x = -\frac{\pi}{12} + \frac{\pi}{2} = \frac{5\pi}{12}$ or $x = -\frac{\pi}{12} + \frac{2\pi}{2} = \frac{11\pi}{12}$. So we have four solutions.

Variations of basic equations - example 7

$$x = \frac{\pi}{12} + \frac{k\pi}{2} \qquad \text{or} \qquad x = -\frac{\pi}{12} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

Now we need to choose values of k, so that our solutions will satisfy $0 \leq x \leq \pi$. After brief deliberation we get: $x = \frac{\pi}{12}$ or $x = \frac{\pi}{12} + \frac{\pi}{2} = \frac{7\pi}{12}$ or $x = -\frac{\pi}{12} + \frac{\pi}{2} = \frac{5\pi}{12}$ or $x = -\frac{\pi}{12} + \frac{2\pi}{2} = \frac{11\pi}{12}$. So we have four solutions.

Let's go back to the beginning:

$$2\cos 4x - 1 = 0$$

with $0 \leq x \leq \pi$.

Variations of basic equations - example 7

$$x = \frac{\pi}{12} + \frac{k\pi}{2} \qquad \text{or} \qquad x = -\frac{\pi}{12} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

Now we need to choose values of k, so that our solutions will satisfy $0 \leq x \leq \pi$. After brief deliberation we get: $x = \frac{\pi}{12}$ or $x = \frac{\pi}{12} + \frac{\pi}{2} = \frac{7\pi}{12}$ or $x = -\frac{\pi}{12} + \frac{\pi}{2} = \frac{5\pi}{12}$ or $x = -\frac{\pi}{12} + \frac{2\pi}{2} = \frac{11\pi}{12}$. So we have four solutions.

Let's go back to the beginning:

$$2\cos 4x - 1 = 0$$

with $0 \leq x \leq \pi$.
The second method is to set $\alpha = 4x$, now we solve:

$$2\cos \alpha - 1 = 0$$

but $0 \leq \alpha \leq 4\pi$.

Variations of basic equations - example 7

$$x = \frac{\pi}{12} + \frac{k\pi}{2} \qquad \text{or} \qquad x = -\frac{\pi}{12} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

Now we need to choose values of k, so that our solutions will satisfy $0 \leq x \leq \pi$. After brief deliberation we get: $x = \frac{\pi}{12}$ or $x = \frac{\pi}{12} + \frac{\pi}{2} = \frac{7\pi}{12}$ or $x = -\frac{\pi}{12} + \frac{\pi}{2} = \frac{5\pi}{12}$ or $x = -\frac{\pi}{12} + \frac{2\pi}{2} = \frac{11\pi}{12}$. So we have four solutions.

Let's go back to the beginning:

$$2\cos 4x - 1 = 0$$

with $0 \leq x \leq \pi$.
The second method is to set $\alpha = 4x$, now we solve:

$$2\cos\alpha - 1 = 0$$

but $0 \leq \alpha \leq 4\pi$. Remember to change the interval!

Variations of basic equations - example 7

$$2\cos\alpha - 1 = 0$$

with $0 \leq \alpha \leq 4\pi$.

Variations of basic equations - example 7

$$2\cos\alpha - 1 = 0$$

with $0 \leq \alpha \leq 4\pi$.

We draw $\cos\alpha$ for $0 \leq \alpha \leq 4\pi$ and we find the solutions:

$$\alpha = \frac{\pi}{3} \quad \text{or} \quad \alpha = \frac{5\pi}{3} \quad \text{or} \quad \alpha = \frac{7\pi}{3} \quad \text{or} \quad \alpha = \frac{11\pi}{3}$$

Variations of basic equations - example 7

$$2\cos\alpha - 1 = 0$$

with $0 \leq \alpha \leq 4\pi$.

We draw $\cos\alpha$ for $0 \leq \alpha \leq 4\pi$ and we find the solutions:

$$\alpha = \frac{\pi}{3} \quad \text{or} \quad \alpha = \frac{5\pi}{3} \quad \text{or} \quad \alpha = \frac{7\pi}{3} \quad \text{or} \quad \alpha = \frac{11\pi}{3}$$

We go back to x, we have $\alpha = 4x$, so $x = \frac{\alpha}{4}$ and we get:

Variations of basic equations - example 7

$$2\cos\alpha - 1 = 0$$

with $0 \leq \alpha \leq 4\pi$.

We draw $\cos\alpha$ for $0 \leq \alpha \leq 4\pi$ and we find the solutions:

$$\alpha = \frac{\pi}{3} \quad \text{or} \quad \alpha = \frac{5\pi}{3} \quad \text{or} \quad \alpha = \frac{7\pi}{3} \quad \text{or} \quad \alpha = \frac{11\pi}{3}$$

We go back to x, we have $\alpha = 4x$, so $x = \frac{\alpha}{4}$ and we get:

$$x = \frac{\pi}{12} \quad \text{or} \quad x = \frac{5\pi}{12} \quad \text{or} \quad x = \frac{7\pi}{12} \quad \text{or} \quad x = \frac{11\pi}{12}$$

Variations of basic equations - example 8

Solve:

$$\sin^2\left(\frac{x}{2}\right) = \frac{3}{4}$$

for $-2\pi \leq x \leq 6\pi$.

Variations of basic equations - example 8

Solve:

$$\sin^2\left(\frac{x}{2}\right) = \frac{3}{4}$$

for $-2\pi \leq x \leq 6\pi$.

We let $\alpha = \frac{x}{2}$ and get:

Variations of basic equations - example 8

Solve:

$$\sin^2\left(\frac{x}{2}\right) = \frac{3}{4}$$

for $-2\pi \leq x \leq 6\pi$.

We let $\alpha = \frac{x}{2}$ and get:

$$\sin^2 \alpha = \frac{3}{4}$$

with $-\pi \leq \alpha \leq 3\pi$.

Variations of basic equations - example 8

Solve:

$$\sin^2\left(\frac{x}{2}\right) = \frac{3}{4}$$

for $-2\pi \leq x \leq 6\pi$.

We let $\alpha = \frac{x}{2}$ and get:

$$\sin^2 \alpha = \frac{3}{4}$$

with $-\pi \leq \alpha \leq 3\pi$.
This gives:

$$\sin \alpha = \pm\frac{\sqrt{3}}{2}$$

Variations of basic equations - example 8

We're solving

$$\sin\alpha = \pm\frac{\sqrt{3}}{2}$$

for $-\pi \leq \alpha \leq 3\pi$.

Variations of basic equations - example 8

We're solving

$$\sin\alpha = \pm\frac{\sqrt{3}}{2}$$

for $-\pi \leq \alpha \leq 3\pi$. We draw the graph of *sine* and the lines $y = \pm\frac{\sqrt{3}}{2}$ in the given interval and find the solutions.

Variations of basic equations - example 8

We're solving

$$\sin\alpha = \pm\frac{\sqrt{3}}{2}$$

for $-\pi \leq \alpha \leq 3\pi$. We draw the graph of *sine* and the lines $y = \pm\frac{\sqrt{3}}{2}$ in the given interval and find the solutions. There should be 8 of them:

Variations of basic equations - example 8

We're solving

$$\sin\alpha = \pm\frac{\sqrt{3}}{2}$$

for $-\pi \leq \alpha \leq 3\pi$. We draw the graph of *sine* and the lines $y = \pm\frac{\sqrt{3}}{2}$ in the given interval and find the solutions. There should be 8 of them:

$$\alpha \in \left\{-\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}\right\}$$

Variations of basic equations - example 8

We're solving

$$\sin\alpha = \pm\frac{\sqrt{3}}{2}$$

for $-\pi \leq \alpha \leq 3\pi$. We draw the graph of *sine* and the lines $y = \pm\frac{\sqrt{3}}{2}$ in the given interval and find the solutions. There should be 8 of them:

$$\alpha \in \left\{-\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}\right\}$$

We go back to x.

Variations of basic equations - example 8

We're solving

$$\sin\alpha = \pm\frac{\sqrt{3}}{2}$$

for $-\pi \leq \alpha \leq 3\pi$. We draw the graph of *sine* and the lines $y = \pm\frac{\sqrt{3}}{2}$ in the given interval and find the solutions. There should be 8 of them:

$$\alpha \in \left\{-\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}\right\}$$

We go back to x. Since $\alpha = \frac{x}{2}$, so we have $x = 2\alpha$, this gives the following solutions:

Variations of basic equations - example 8

We're solving

$$\sin\alpha = \pm\frac{\sqrt{3}}{2}$$

for $-\pi \leq \alpha \leq 3\pi$. We draw the graph of *sine* and the lines $y = \pm\frac{\sqrt{3}}{2}$ in the given interval and find the solutions. There should be 8 of them:

$$\alpha \in \left\{-\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}\right\}$$

We go back to x. Since $\alpha = \frac{x}{2}$, so we have $x = 2\alpha$, this gives the following solutions:

$$x \in \left\{-\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}\right\}$$

Variations of basic equations - exercises

- Solve:

$$\cos^2(3x) = \frac{1}{2}$$

for $0 \leq x \leq \frac{\pi}{2}$.

Variations of basic equations - exercises

- Solve:

$$\cos^2(3x) = \frac{1}{2}$$

for $0 \leq x \leq \frac{\pi}{2}$. Solution:

Variations of basic equations - exercises

- Solve:

$$\cos^2(3x) = \frac{1}{2}$$

for $0 \leq x \leq \frac{\pi}{2}$. Solution:

$$x = \frac{\pi}{12} \quad \text{or} \quad x = \frac{\pi}{4} \quad \text{or} \quad x = \frac{5\pi}{12}$$

Variations of basic equations - exercises

- Solve:

$$\cos^2(3x) = \frac{1}{2}$$

for $0 \leq x \leq \frac{\pi}{2}$. Solution:

$$x = \frac{\pi}{12} \quad \text{or} \quad x = \frac{\pi}{4} \quad \text{or} \quad x = \frac{5\pi}{12}$$

- Solve

$$\tan^2(2x) = 3$$

with $-\pi \leq x \leq \frac{\pi}{2}$.

Variations of basic equations - exercises

- Solve:

$$\cos^2(3x) = \frac{1}{2}$$

for $0 \leq x \leq \frac{\pi}{2}$. Solution:

$$x = \frac{\pi}{12} \quad \text{or} \quad x = \frac{\pi}{4} \quad \text{or} \quad x = \frac{5\pi}{12}$$

- Solve

$$\tan^2(2x) = 3$$

with $-\pi \leq x \leq \frac{\pi}{2}$. Solution:

Variations of basic equations - exercises

- Solve:

$$\cos^2(3x) = \frac{1}{2}$$

for $0 \leq x \leq \frac{\pi}{2}$. Solution:

$$x = \frac{\pi}{12} \quad \text{or} \quad x = \frac{\pi}{4} \quad \text{or} \quad x = \frac{5\pi}{12}$$

- Solve

$$\tan^2(2x) = 3$$

with $-\pi \leq x \leq \frac{\pi}{2}$. Solution:

$$x \in \left\{-\frac{5\pi}{6}, -\frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{3}\right\}$$

Now we move on to equations which can be easily factored resulting in two or more basic equations.

Factoring - example 1

Solve:

$$2\sin^2 x + \sin x - 1 = 0$$

Factoring - example 1

Solve:

$$2\sin^2 x + \sin x - 1 = 0$$

We have a disguised quadratic (we could substitute $s = \sin x$ and solve), let's try factoring. We rewrite the LHS in a factored form:

Factoring - example 1

Solve:

$$2\sin^2 x + \sin x - 1 = 0$$

We have a disguised quadratic (we could substitute $s = \sin x$ and solve), let's try factoring. We rewrite the LHS in a factored form:

$$(2\sin x - 1)(\sin x + 1) = 0$$

Factoring - example 1

Solve:

$$2\sin^2 x + \sin x - 1 = 0$$

We have a disguised quadratic (we could substitute $s = \sin x$ and solve), let's try factoring. We rewrite the LHS in a factored form:

$$(2\sin x - 1)(\sin x + 1) = 0$$

This gives:

$$\sin(x) = \frac{1}{2} \qquad \text{or} \qquad \sin(x) = -1$$

Factoring - example 1

Solve:

$$2\sin^2 x + \sin x - 1 = 0$$

We have a disguised quadratic (we could substitute $s = \sin x$ and solve), let's try factoring. We rewrite the LHS in a factored form:

$$(2\sin x - 1)(\sin x + 1) = 0$$

This gives:

$$\sin(x) = \frac{1}{2} \qquad \text{or} \qquad \sin(x) = -1$$

We solve these basic equations to get:

Factoring - example 1

Solve:

$$2\sin^2 x + \sin x - 1 = 0$$

We have a disguised quadratic (we could substitute $s = \sin x$ and solve), let's try factoring. We rewrite the LHS in a factored form:

$$(2\sin x - 1)(\sin x + 1) = 0$$

This gives:

$$\sin(x) = \frac{1}{2} \qquad \text{or} \qquad \sin(x) = -1$$

We solve these basic equations to get:

$$x = \frac{\pi}{6} + 2k\pi \quad \text{or} \quad x = \frac{5\pi}{6} + 2k\pi \quad \text{or} \quad x = -\frac{\pi}{2} + 2k\pi \quad \text{where } k \in \mathbb{Z}$$

Factoring - example 2

Solve:

$$2\cos^2 x - 3\cos x - 2 = 0$$

Factoring - example 2

Solve:

$$2\cos^2 x - 3\cos x - 2 = 0$$

We factorize:

Factoring - example 2

Solve:

$$2\cos^2 x - 3\cos x - 2 = 0$$

We factorize:

$$(2\cos x + 1)(\cos x - 2) = 0$$

Factoring - example 2

Solve:

$$2\cos^2 x - 3\cos x - 2 = 0$$

We factorize:

$$(2\cos x + 1)(\cos x - 2) = 0$$

We get:

$$\cos(x) = -\frac{1}{2} \qquad \text{lub} \qquad \cos(x) = 2$$

Factoring - example 2

Solve:

$$2\cos^2 x - 3\cos x - 2 = 0$$

We factorize:

$$(2\cos x + 1)(\cos x - 2) = 0$$

We get:

$$\cos(x) = -\frac{1}{2} \qquad \text{lub} \qquad \cos(x) = 2$$

There's no solutions to the second equation, solving the first one gives:

Factoring - example 2

Solve:

$$2\cos^2 x - 3\cos x - 2 = 0$$

We factorize:

$$(2\cos x + 1)(\cos x - 2) = 0$$

We get:

$$\cos(x) = -\frac{1}{2} \qquad \text{lub} \qquad \cos(x) = 2$$

There's no solutions to the second equation, solving the first one gives:

$$x = -\frac{2\pi}{3} + 2k\pi \qquad \text{or} \qquad x = \frac{2\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Factoring - example 3

Solve:

$$2\sin x\cos x - 2\sin x + \cos x - 1 = 0$$

Factoring - example 3

Solve:

$$2\sin x\cos x - 2\sin x + \cos x - 1 = 0$$

We can first factor out $2\sin x$ from the first two terms, this gives:

Factoring - example 3

Solve:

$$2\sin x\cos x - 2\sin x + \cos x - 1 = 0$$

We can first factor out $2\sin x$ from the first two terms, this gives:

$$2\sin x(\cos x - 1) + \cos x - 1 = 0$$

Factoring - example 3

Solve:

$$2\sin x\cos x - 2\sin x + \cos x - 1 = 0$$

We can first factor out $2\sin x$ from the first two terms, this gives:

$$2\sin x(\cos x - 1) + \cos x - 1 = 0$$

So we have:

Factoring - example 3

Solve:

$$2\sin x\cos x - 2\sin x + \cos x - 1 = 0$$

We can first factor out $2\sin x$ from the first two terms, this gives:

$$2\sin x(\cos x - 1) + \cos x - 1 = 0$$

So we have:

$$(2\sin x + 1)(\cos x - 1) = 0$$

Factoring - example 3

Solve:

$$2\sin x\cos x - 2\sin x + \cos x - 1 = 0$$

We can first factor out $2\sin x$ from the first two terms, this gives:

$$2\sin x(\cos x - 1) + \cos x - 1 = 0$$

So we have:

$$(2\sin x + 1)(\cos x - 1) = 0$$

So:

$$\sin(x) = -\frac{1}{2} \qquad \text{lub} \qquad \cos(x) = 1$$

Factoring - example 3

Solve:

$$2\sin x\cos x - 2\sin x + \cos x - 1 = 0$$

We can first factor out $2\sin x$ from the first two terms, this gives:

$$2\sin x(\cos x - 1) + \cos x - 1 = 0$$

So we have:

$$(2\sin x + 1)(\cos x - 1) = 0$$

So:

$$\sin(x) = -\frac{1}{2} \qquad \text{lub} \qquad \cos(x) = 1$$

Now we solve and get:

Factoring - example 3

Solve:

$$2\sin x\cos x - 2\sin x + \cos x - 1 = 0$$

We can first factor out $2\sin x$ from the first two terms, this gives:

$$2\sin x(\cos x - 1) + \cos x - 1 = 0$$

So we have:

$$(2\sin x + 1)(\cos x - 1) = 0$$

So:

$$\sin(x) = -\frac{1}{2} \qquad \text{lub} \qquad \cos(x) = 1$$

Now we solve and get:

$$x = -\frac{\pi}{6} + 2k\pi \quad \text{or} \quad x = -\frac{5\pi}{6} + 2k\pi \quad \text{or} \quad x = 2k\pi \quad \text{where } k \in \mathbb{Z}$$

Factoring - example 4

Solve:

$$3\tan^4 x - 10\tan^2 x + 3 = 0$$

Factoring - example 4

Solve:

$$3\tan^4 x - 10\tan^2 x + 3 = 0$$

We can set $t = \tan^2 x$, but let's try factoring again:

Factoring - example 4

Solve:

$$3\tan^4 x - 10\tan^2 x + 3 = 0$$

We can set $t = \tan^2 x$, but let's try factoring again:

$$(3\tan^2 x - 1)(\tan^2 x - 3) = 0$$

Factoring - example 4

Solve:

$$3\tan^4 x - 10\tan^2 x + 3 = 0$$

We can set $t = \tan^2 x$, but let's try factoring again:

$$(3\tan^2 x - 1)(\tan^2 x - 3) = 0$$

We can continue factoring (using difference of squares) or we can just write that:

Factoring - example 4

Solve:

$$3\tan^4 x - 10\tan^2 x + 3 = 0$$

We can set $t = \tan^2 x$, but let's try factoring again:

$$(3\tan^2 x - 1)(\tan^2 x - 3) = 0$$

We can continue factoring (using difference of squares) or we can just write that:

$$\tan(x) = \pm\frac{\sqrt{3}}{3} \qquad \text{lub} \qquad \tan(x) = \pm\sqrt{3}$$

Factoring - example 4

Solve:

$$3\tan^4 x - 10\tan^2 x + 3 = 0$$

We can set $t = \tan^2 x$, but let's try factoring again:

$$(3\tan^2 x - 1)(\tan^2 x - 3) = 0$$

We can continue factoring (using difference of squares) or we can just write that:

$$\tan(x) = \pm\frac{\sqrt{3}}{3} \qquad \text{lub} \qquad \tan(x) = \pm\sqrt{3}$$

We solve and get:

Factoring - example 4

Solve:

$$3\tan^4 x - 10\tan^2 x + 3 = 0$$

We can set $t = \tan^2 x$, but let's try factoring again:

$$(3\tan^2 x - 1)(\tan^2 x - 3) = 0$$

We can continue factoring (using difference of squares) or we can just write that:

$$\tan(x) = \pm\frac{\sqrt{3}}{3} \qquad \text{lub} \qquad \tan(x) = \pm\sqrt{3}$$

We solve and get:

$$x = \frac{\pi}{6} + \frac{k\pi}{2} \qquad \text{or} \qquad x = \frac{\pi}{3} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

Factoring - example 4

Solve:

$$3\tan^4 x - 10\tan^2 x + 3 = 0$$

We can set $t = \tan^2 x$, but let's try factoring again:

$$(3\tan^2 x - 1)(\tan^2 x - 3) = 0$$

We can continue factoring (using difference of squares) or we can just write that:

$$\tan(x) = \pm\frac{\sqrt{3}}{3} \qquad \text{lub} \qquad \tan(x) = \pm\sqrt{3}$$

We solve and get:

$$x = \frac{\pi}{6} + \frac{k\pi}{2} \qquad \text{or} \qquad x = \frac{\pi}{3} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

Think about this solution. Make sure you understand where it came from.

Factoring - exercises

- Solve:

$$2\cos^2 x - \cos x - 3 = 0$$

Factoring - exercises

- Solve:

$$2\cos^2 x - \cos x - 3 = 0$$

Solution:

$$x = \pi + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Factoring - exercises

- Solve:

$$2\cos^2 x - \cos x - 3 = 0$$

  Solution:

$$x = \pi + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Solve:

$$\sin x \cos x + \sin x - \cos x - 1 = 0$$

Factoring - exercises

- Solve:

$$2\cos^2 x - \cos x - 3 = 0$$

Solution:

$$x = \pi + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Solve:

$$\sin x \cos x + \sin x - \cos x - 1 = 0$$

Solution:

$$x = \frac{\pi}{2} + 2k\pi \qquad \text{or} \qquad x = \pi + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Factoring - exercises

- Solve:

$$\cot^3 x - \cot^2 x - 3\cot x + 3 = 0$$

Factoring - exercises

- Solve:

$$\cot^3 x - \cot^2 x - 3\cot x + 3 = 0$$

Solution:

$$x = \frac{\pi}{4} + k\pi \quad \text{or} \quad x = \frac{\pi}{6} + k\pi \quad \text{or} \quad x = \frac{5\pi}{6} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Factoring - exercises

- Solve:

$$\cot^3 x - \cot^2 x - 3\cot x + 3 = 0$$

Solution:

$$x = \frac{\pi}{4} + k\pi \quad \text{or} \quad x = \frac{\pi}{6} + k\pi \quad \text{or} \quad x = \frac{5\pi}{6} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Solve:

$$\sin^3 x - 4\sin^2 x - \sin x + 4 = 0$$

Factoring - exercises

- Solve:

$$\cot^3 x - \cot^2 x - 3\cot x + 3 = 0$$

Solution:

$$x = \frac{\pi}{4} + k\pi \quad \text{or} \quad x = \frac{\pi}{6} + k\pi \quad \text{or} \quad x = \frac{5\pi}{6} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Solve:

$$\sin^3 x - 4\sin^2 x - \sin x + 4 = 0$$

Solution:

$$x = \frac{\pi}{2} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

We increase the difficulty slightly.

We increase the difficulty slightly. We add the Pythagorean identity to our arsenal.

Pythagorean identity - introduction

The Pythagorean identity is probably the most famous trigonometric identity. For any angle x we have:

$$\sin^2 x + \cos^2 x = 1$$

Pythagorean identity - introduction

The Pythagorean identity is probably the most famous trigonometric identity. For any angle x we have:

$$\sin^2 x + \cos^2 x = 1$$

We can use it to solve simple problems like:

Pythagorean identity - introduction

The Pythagorean identity is probably the most famous trigonometric identity. For any angle x we have:

$$\sin^2 x + \cos^2 x = 1$$

We can use it to solve simple problems like:

Simple problem

Given an angle α, such that $\cos\alpha = \frac{1}{3}$ and $\frac{3\pi}{2} < \alpha < 2\pi$, calculate $\sin\alpha$ and $\cot\alpha$.

Pythagorean identity - introduction

The Pythagorean identity is probably the most famous trigonometric identity. For any angle x we have:

$$\sin^2 x + \cos^2 x = 1$$

We can use it to solve simple problems like:

Simple problem

Given an angle α, such that $\cos\alpha = \frac{1}{3}$ and $\frac{3\pi}{2} < \alpha < 2\pi$, calculate $\sin\alpha$ and $\cot\alpha$.

We have $\sin x = -\frac{2\sqrt{2}}{3}$ and $\cot x = -\frac{\sqrt{2}}{4}$.

Pythagorean identity - introduction

The Pythagorean identity is probably the most famous trigonometric identity. For any angle x we have:

$$\sin^2 x + \cos^2 x = 1$$

We can use it to solve simple problems like:

Simple problem

Given an angle α, such that $\cos\alpha = \frac{1}{3}$ and $\frac{3\pi}{2} < \alpha < 2\pi$, calculate $\sin\alpha$ and $\cot\alpha$.

We have $\sin x = -\frac{2\sqrt{2}}{3}$ and $\cot x = -\frac{\sqrt{2}}{4}$. Refer to chapter 8E in Core HL if you forgot about these types of problems.

Pythagorean identity - introduction

Remember that the Pythagorean identity works for any angle x, so we have

$$\sin^2 31^\circ + \cos^2 31^\circ = 1$$

Pythagorean identity - introduction

Remember that the Pythagorean identity works for any angle x, so we have

$$\sin^2 31^\circ + \cos^2 31^\circ = 1$$

$$\sin^2 \frac{\pi}{7} + \cos^2 \frac{\pi}{7} = 1$$

Pythagorean identity - introduction

Remember that the Pythagorean identity works for any angle x, so we have

$$\sin^2 31^\circ + \cos^2 31^\circ = 1$$

$$\sin^2 \frac{\pi}{7} + \cos^2 \frac{\pi}{7} = 1$$

but also:

$$\sin^2(3\alpha) + \cos^2(3\alpha) = 1$$

Pythagorean identity - introduction

Remember that the Pythagorean identity works for any angle x, so we have

$$\sin^2 31^\circ + \cos^2 31^\circ = 1$$

$$\sin^2 \frac{\pi}{7} + \cos^2 \frac{\pi}{7} = 1$$

but also:

$$\sin^2(3\alpha) + \cos^2(3\alpha) = 1$$

$$\sin^2\left(\frac{x}{2} - \pi\right) + \cos^2\left(\frac{x}{2} - \pi\right) = 1$$

Pythagorean identity - introduction

Remember that the Pythagorean identity works for any angle x, so we have

$$\sin^2 31^\circ + \cos^2 31^\circ = 1$$

$$\sin^2 \frac{\pi}{7} + \cos^2 \frac{\pi}{7} = 1$$

but also:

$$\sin^2(3\alpha) + \cos^2(3\alpha) = 1$$

$$\sin^2\left(\frac{x}{2} - \pi\right) + \cos^2\left(\frac{x}{2} - \pi\right) = 1$$

Solving trig equations using Pythagorean identity boils down to simplifying the equation so that it can be solved using previous methods.

Pythagorean identity - introduction

Note that there are two very simple consequences of Pythagorean identity, namely:

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Pythagorean identity - introduction

Note that there are two very simple consequences of Pythagorean identity, namely:

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

They of course can be derived by dividing the Pythagorean identity by $\cos^2 x$ and $\sin^2 x$ respectively.

Pythagorean identity - example 1

Solve:

$$5\sin x - 2\cos^2 x = 1$$

Pythagorean identity - example 1

Solve:

$$5\sin x - 2\cos^2 x = 1$$

We use Pythagorean identity to replace $-2\cos^2 x$ with $2\sin^2 x - 2$ and we get:

Pythagorean identity - example 1

Solve:

$$5\sin x - 2\cos^2 x = 1$$

We use Pythagorean identity to replace $-2\cos^2 x$ with $2\sin^2 x - 2$ and we get:

$$2\sin^2 x + 5\sin x - 3 = 0$$

Factorize:

Pythagorean identity - example 1

Solve:

$$5\sin x - 2\cos^2 x = 1$$

We use Pythagorean identity to replace $-2\cos^2 x$ with $2\sin^2 x - 2$ and we get:

$$2\sin^2 x + 5\sin x - 3 = 0$$

Factorize:

$$(2\sin x - 1)(\sin x + 3) = 0$$

We now solve and get:

Pythagorean identity - example 1

Solve:

$$5\sin x - 2\cos^2 x = 1$$

We use Pythagorean identity to replace $-2\cos^2 x$ with $2\sin^2 x - 2$ and we get:

$$2\sin^2 x + 5\sin x - 3 = 0$$

Factorize:

$$(2\sin x - 1)(\sin x + 3) = 0$$

We now solve and get:

$$x = \frac{\pi}{6} + 2k\pi \qquad \text{or} \qquad x = \frac{5\pi}{6} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Pythagorean identity - example 2

Solve the equation:

$$2\sin^2 3x + \cos 3x = 1$$

Pythagorean identity - example 2

Solve the equation:

$$2\sin^2 3x + \cos 3x = 1$$

We use the Pythagorean identity to change $2\sin^2 3x$ into $2 - 2\cos^2 3x$, and we get:

Pythagorean identity - example 2

Solve the equation:

$$2\sin^2 3x + \cos 3x = 1$$

We use the Pythagorean identity to change $2\sin^2 3x$ into $2 - 2\cos^2 3x$, and we get:

$$2\cos^2 3x - \cos 3x - 1 = 0$$

Factorize:

Pythagorean identity - example 2

Solve the equation:

$$2\sin^2 3x + \cos 3x = 1$$

We use the Pythagorean identity to change $2\sin^2 3x$ into $2 - 2\cos^2 3x$, and we get:

$$2\cos^2 3x - \cos 3x - 1 = 0$$

Factorize:

$$(2\cos 3x + 1)(\cos 3x - 1) = 0$$

we now solve and get that:

Pythagorean identity - example 2

Solve the equation:

$$2\sin^2 3x + \cos 3x = 1$$

We use the Pythagorean identity to change $2\sin^2 3x$ into $2 - 2\cos^2 3x$, and we get:

$$2\cos^2 3x - \cos 3x - 1 = 0$$

Factorize:

$$(2\cos 3x + 1)(\cos 3x - 1) = 0$$

we now solve and get that:

$$x = -\frac{2\pi}{9} + \frac{2k\pi}{3} \quad \text{or} \quad x = \frac{2\pi}{9} + \frac{2k\pi}{3} \quad \text{or} \quad x = \frac{2k\pi}{3} \qquad \text{where } k \in \mathbb{Z}$$

Pythagorean identity - exercises

- Equation:

$$2\sin x = 2 + \cos^2 x$$

Pythagorean identity - exercises

- Equation:

$$2\sin x = 2 + \cos^2 x$$

Solution:

Pythagorean identity - exercises

- Equation:

$$2\sin x = 2 + \cos^2 x$$

Solution:

$$x = \frac{\pi}{2} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Pythagorean identity - exercises

- Equation:

$$2\sin x = 2 + \cos^2 x$$

Solution:

$$x = \frac{\pi}{2} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$2\cos^2 2x + 7\sin 2x + 2 = 0$$

Pythagorean identity - exercises

- Equation:

$$2\sin x = 2 + \cos^2 x$$

Solution:

$$x = \frac{\pi}{2} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$2\cos^2 2x + 7\sin 2x + 2 = 0$$

Solution:

Pythagorean identity - exercises

- Equation:

$$2\sin x = 2 + \cos^2 x$$

Solution:

$$x = \frac{\pi}{2} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$2\cos^2 2x + 7\sin 2x + 2 = 0$$

Solution:

$$x = -\frac{\pi}{12} + k\pi \qquad \text{or} \qquad x = -\frac{5\pi}{12} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

We increase the difficulty.

We increase the difficulty. It may happen that we get different angles in the same equation.

We increase the difficulty. It may happen that we get different angles in the same equation. If the equation is something like:

$$(\sin 3x - 1)(2\cos 2x - 1) = 0$$

We increase the difficulty. It may happen that we get different angles in the same equation. If the equation is something like:

$$(\sin 3x - 1)(2\cos 2x - 1) = 0$$

then there's no problem.

We increase the difficulty. It may happen that we get different angles in the same equation. If the equation is something like:

$$(\sin 3x - 1)(2\cos 2x - 1) = 0$$

then there's no problem. We have $3x$ and $2x$, but we easily get two basic equations.

We increase the difficulty. It may happen that we get different angles in the same equation. If the equation is something like:

$$(\sin 3x - 1)(2\cos 2x - 1) = 0$$

then there's no problem. We have $3x$ and $2x$, but we easily get two basic equations. The solutions are:

We increase the difficulty. It may happen that we get different angles in the same equation. If the equation is something like:

$$(\sin 3x - 1)(2\cos 2x - 1) = 0$$

then there's no problem. We have $3x$ and $2x$, but we easily get two basic equations. The solutions are:

$$x = \frac{\pi}{6} + \frac{2k\pi}{3} \quad \text{or} \quad x = -\frac{\pi}{6} + k\pi \quad \text{or} \quad x = \frac{\pi}{6} + k\pi \qquad \text{gdzie } k \in \mathbb{Z}$$

We increase the difficulty. It may happen that we get different angles in the same equation. If the equation is something like:

$$(\sin 3x - 1)(2\cos 2x - 1) = 0$$

then there's no problem. We have $3x$ and $2x$, but we easily get two basic equations. The solutions are:

$$x = \frac{\pi}{6} + \frac{2k\pi}{3} \quad \text{or} \quad x = -\frac{\pi}{6} + k\pi \quad \text{or} \quad x = \frac{\pi}{6} + k\pi \qquad \text{gdzie } k \in \mathbb{Z}$$

It may however happen that it's not so simple and then the goal would be to make sure that we have the same angle everywhere.

Double angle formulae - introduction

Formulae that you **have to** remember:

Double angle formulae - introduction

Formulae that you **have to** remember:

$$\sin 2x = 2\sin x\cos x$$

Double angle formulae - introduction

Formulae that you **have to** remember:

$$\sin 2x = 2\sin x\cos x$$

$$\begin{aligned}\cos 2x = \cos^2 x - \sin^2 x = \\ = 2\cos^2 x - 1 = \\ = 1 - 2\sin^2 x\end{aligned}$$

Double angle formulae - introduction

Formulae that you **have to** remember:

$$\sin 2x = 2\sin x \cos x$$

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x = \\ &= 2\cos^2 x - 1 = \\ &= 1 - 2\sin^2 x\end{aligned}$$

In case of *cosine* we in fact have 3 formulae and we use the one which suits us.

Double angle formulae - introduction

Formulae that you **have to** remember:

$$\sin 2x = 2\sin x\cos x$$

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x = \\ &= 2\cos^2 x - 1 = \\ &= 1 - 2\sin^2 x\end{aligned}$$

In case of *cosine* we in fact have 3 formulae and we use the one which suits us.

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

Double angle formulae - introduction

Remember that these formulae work regardless of the angle, so in particular we have:

$$\sin 10^\circ = 2\sin 5^\circ \cos 5^\circ$$

Double angle formulae - introduction

Remember that these formulae work regardless of the angle, so in particular we have:

$$\sin 10^\circ = 2\sin 5^\circ \cos 5^\circ$$

$$\sin 8\alpha = 2\sin 4\alpha \cos 4\alpha$$

Double angle formulae - introduction

Remember that these formulae work regardless of the angle, so in particular we have:

$$\sin 10^\circ = 2\sin 5^\circ \cos 5^\circ$$

$$\sin 8\alpha = 2\sin 4\alpha \cos 4\alpha$$

$$\cos(10x) = 1 - 2\sin^2(5x)$$

Double angle formulae - introduction

Remember that these formulae work regardless of the angle, so in particular we have:

$$\sin 10^\circ = 2\sin 5^\circ \cos 5^\circ$$

$$\sin 8\alpha = 2\sin 4\alpha \cos 4\alpha$$

$$\cos(10x) = 1 - 2\sin^2(5x)$$

$$\cos(x) = \cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)$$

Double angle formulae - introduction

Remember that these formulae work regardless of the angle, so in particular we have:

$$\sin 10^\circ = 2\sin 5^\circ \cos 5^\circ$$

$$\sin 8\alpha = 2\sin 4\alpha \cos 4\alpha$$

$$\cos(10x) = 1 - 2\sin^2(5x)$$

$$\cos(x) = \cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{2\tan(\frac{\theta}{4})}{1-\tan^2(\frac{\theta}{4})}$$

Double angle formulae - introduction

Remember that these formulae work regardless of the angle, so in particular we have:

$$\sin 10^\circ = 2\sin 5^\circ \cos 5^\circ$$

$$\sin 8\alpha = 2\sin 4\alpha \cos 4\alpha$$

$$\cos(10x) = 1 - 2\sin^2(5x)$$

$$\cos(x) = \cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{2\tan(\frac{\theta}{4})}{1-\tan^2(\frac{\theta}{4})}$$

The angle on the left hand side has to be twice the angle on the right hand side.

Double angle - example 1

Solve:

$$\sin 2x + \sin x = 0$$

Double angle - example 1

Solve:

$$\sin 2x + \sin x = 0$$

We use double angle formula for sine ($\sin 2x = 2\sin x\cos x$) and get:

Double angle - example 1

Solve:

$$\sin 2x + \sin x = 0$$

We use double angle formula for sine ($\sin 2x = 2\sin x \cos x$) and get:

$$2\sin x \cos x + \sin x = 0$$

We factor out $\sin x$ and we get:

Double angle - example 1

Solve:

$$\sin 2x + \sin x = 0$$

We use double angle formula for sine ($\sin 2x = 2\sin x \cos x$) and get:

$$2\sin x \cos x + \sin x = 0$$

We factor out $\sin x$ and we get:

$$\sin x(2\cos x + 1) = 0$$

solve the above to get:

Double angle - example 1

Solve:

$$\sin 2x + \sin x = 0$$

We use double angle formula for sine ($\sin 2x = 2\sin x \cos x$) and get:

$$2\sin x \cos x + \sin x = 0$$

We factor out $\sin x$ and we get:

$$\sin x(2\cos x + 1) = 0$$

solve the above to get:

$$x = k\pi \quad \text{or} \quad x = -\frac{2\pi}{3} + 2k\pi \quad \text{or} \quad x = \frac{2\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Double angle - example 2

Solve

$$\cos(6x) - 3\cos(3x) + 1 = 0$$

Double angle - example 2

Solve

$$\cos(6x) - 3\cos(3x) + 1 = 0$$

We use double angle formula for cosine ($\cos 6x = 2\cos^2(3x) - 1$), we get:

Double angle - example 2

Solve

$$\cos(6x) - 3\cos(3x) + 1 = 0$$

We use double angle formula for cosine ($\cos 6x = 2\cos^2(3x) - 1$), we get:

$$2\cos^2(3x) - 3\cos(3x) = 0$$

Factor out $\cos(3x)$:

Double angle - example 2

Solve

$$\cos(6x) - 3\cos(3x) + 1 = 0$$

We use double angle formula for cosine ($\cos 6x = 2\cos^2(3x) - 1$), we get:

$$2\cos^2(3x) - 3\cos(3x) = 0$$

Factor out $\cos(3x)$:

$$\cos(3x)(2\cos(3x) - 3) = 0$$

solve to get:

Double angle - example 2

Solve

$$\cos(6x) - 3\cos(3x) + 1 = 0$$

We use double angle formula for cosine ($\cos 6x = 2\cos^2(3x) - 1$), we get:

$$2\cos^2(3x) - 3\cos(3x) = 0$$

Factor out $\cos(3x)$:

$$\cos(3x)(2\cos(3x) - 3) = 0$$

solve to get:

$$3x = \frac{\pi}{2} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Double angle - example 2

Solve

$$\cos(6x) - 3\cos(3x) + 1 = 0$$

We use double angle formula for cosine ($\cos 6x = 2\cos^2(3x) - 1$), we get:

$$2\cos^2(3x) - 3\cos(3x) = 0$$

Factor out $\cos(3x)$:

$$\cos(3x)(2\cos(3x) - 3) = 0$$

solve to get:

$$3x = \frac{\pi}{2} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

so:

$$x = \frac{\pi}{6} + \frac{k\pi}{3} \qquad \text{where } k \in \mathbb{Z}$$

Double angle - example 3

Solve:

$$\cos 4x + 4\sin 2x + 5 = 0$$

Double angle - example 3

Solve:

$$\cos 4x + 4\sin 2x + 5 = 0$$

We use double angle formula ($\cos 4x = 1 - 2\sin^2 2x$), we get:

Double angle - example 3

Solve:

$$\cos 4x + 4\sin 2x + 5 = 0$$

We use double angle formula ($\cos 4x = 1 - 2\sin^2 2x$), we get:

$$-2\sin^2 2x + 4\sin 2x + 6 = 0$$

Divide by -2 and factorize:

Double angle - example 3

Solve:

$$\cos 4x + 4\sin 2x + 5 = 0$$

We use double angle formula ($\cos 4x = 1 - 2\sin^2 2x$), we get:

$$-2\sin^2 2x + 4\sin 2x + 6 = 0$$

Divide by -2 and factorize:

$$(\sin 2x + 1)(\sin 2x - 3) = 0$$

solve and get (the second equation has no solutions):

Double angle - example 3

Solve:

$$\cos 4x + 4\sin 2x + 5 = 0$$

We use double angle formula ($\cos 4x = 1 - 2\sin^2 2x$), we get:

$$-2\sin^2 2x + 4\sin 2x + 6 = 0$$

Divide by -2 and factorize:

$$(\sin 2x + 1)(\sin 2x - 3) = 0$$

solve and get (the second equation has no solutions):

$$2x = \frac{3\pi}{2} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Double angle - example 3

Solve:

$$\cos 4x + 4\sin 2x + 5 = 0$$

We use double angle formula ($\cos 4x = 1 - 2\sin^2 2x$), we get:

$$-2\sin^2 2x + 4\sin 2x + 6 = 0$$

Divide by -2 and factorize:

$$(\sin 2x + 1)(\sin 2x - 3) = 0$$

solve and get (the second equation has no solutions):

$$2x = \frac{3\pi}{2} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

so:

$$x = \frac{3\pi}{4} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Double angle - exercises

- Equation:

$$\sin x - 2\cos\frac{x}{2} = 0$$

Double angle - exercises

- Equation:

$$\sin x - 2\cos\frac{x}{2} = 0$$

Solution:

Double angle - exercises

- Equation:

$$\sin x - 2\cos\frac{x}{2} = 0$$

Solution:

$$x = \pi + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Double angle - exercises

- Equation:

$$\sin x - 2\cos\frac{x}{2} = 0$$

Solution:

$$x = \pi + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$\cos 4x + 2\sin 2x\cos 2x + 1 = 0$$

Double angle - exercises

- Equation:

$$\sin x - 2\cos\frac{x}{2} = 0$$

Solution:

$$x = \pi + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$\cos 4x + 2\sin 2x\cos 2x + 1 = 0$$

Solution:

Double angle - exercises

- Equation:

$$\sin x - 2\cos\frac{x}{2} = 0$$

Solution:

$$x = \pi + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$\cos 4x + 2\sin 2x\cos 2x + 1 = 0$$

Solution:

$$x = \frac{\pi}{4} + \frac{k\pi}{2} \qquad \text{or} \qquad x = -\frac{\pi}{8} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

Formula for *sine/cosine* of sum/difference of angles - introduction

We have the following formulae for sine and cosine of sum and difference of angles:

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Formula for *sine/cosine* of sum/difference of angles - introduction

We have the following formulae for sine and cosine of sum and difference of angles:

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

They can be used to calculate for example $\sin(\frac{7\pi}{12})$ or $\cos 15^\circ$:

Formula for *sine/cosine* of sum/difference of angles - introduction

We have the following formulae for sine and cosine of sum and difference of angles:

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

They can be used to calculate for example $\sin(\frac{7\pi}{12})$ or $\cos 15^\circ$:

$$\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) =$$
$$= \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3} = \frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{2}+\sqrt{6}}{4}$$

Formula for *sine/cosine* of sum/difference of angles - introduction

$$\cos 15^\circ = \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ =$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

Formula for *sine/cosine* of sum/difference of angles - introduction

$$\cos 15^\circ = \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ =$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

We get the same result. This, of course, is no accident, we have $\frac{7\pi}{12} = 105^\circ$, so
$\sin 105^\circ = \sin(180 - 75^\circ) = \sin 75^\circ = \cos(90^\circ - 75^\circ) = \cos 15^\circ$.

Formula for *sine/cosine* of sum/difference of angles - introduction

$$\cos 15^\circ = \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ =$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

We get the same result. This, of course, is no accident, we have $\frac{7\pi}{12} = 105^\circ$, so
$\sin 105^\circ = \sin(180 - 75^\circ) = \sin 75^\circ = \cos(90^\circ - 75^\circ) = \cos 15^\circ$.

When solving equations we will in most cases use the formulae in the opposite direction.

Formula for *sine/cosine* of sum/difference of angles - example 1

Solve

$$\sin x + \sqrt{3}\cos x = \sqrt{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 1

Solve

$$\sin x + \sqrt{3}\cos x = \sqrt{2}$$

We have a sum so it's appropriate to change it to cosine of a difference or sine of a sum. We will do the later. We want to change 1 into cosine and $\sqrt{3}$ into a sine. By drawing an appropriate triangle we can see that the hypotenuse is 2 (so we need to divide both sides by 2) and the required angle is $\alpha = \frac{\pi}{3}$:

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 1

Solve

$$\sin x + \sqrt{3}\cos x = \sqrt{2}$$

We have a sum so it's appropriate to change it to cosine of a difference or sine of a sum. We will do the later. We want to change 1 into cosine and $\sqrt{3}$ into a sine. By drawing an appropriate triangle we can see that the hypotenuse is 2 (so we need to divide both sides by 2) and the required angle is $\alpha = \frac{\pi}{3}$:

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

So we get:

Formula for *sine/cosine* of sum/difference of angles - example 1

Solve

$$\sin x + \sqrt{3}\cos x = \sqrt{2}$$

We have a sum so it's appropriate to change it to cosine of a difference or sine of a sum. We will do the later. We want to change 1 into cosine and $\sqrt{3}$ into a sine. By drawing an appropriate triangle we can see that the hypotenuse is 2 (so we need to divide both sides by 2) and the required angle is $\alpha = \frac{\pi}{3}$:

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

So we get:

$$\cos\frac{\pi}{3}\sin x + \sin\frac{\pi}{3}\cos x = \frac{\sqrt{2}}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 1

Now we can apply the formula for the sine of the sum of angles to get:

$$\sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

Note that we could have tried to use the formula for cosine of a difference of the angles. In which case we would need to change 1 into sine and $\sqrt{3}$ into cosine. The hypotenuse is still 2, but the angle is $\alpha = \frac{\pi}{6}$, so we would get:

Formula for *sine/cosine* of sum/difference of angles - example 1

Now we can apply the formula for the sine of the sum of angles to get:

$$\sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

Note that we could have tried to use the formula for cosine of a difference of the angles. In which case we would need to change 1 into sine and $\sqrt{3}$ into cosine. The hypotenuse is still 2, but the angle is $\alpha = \dfrac{\pi}{6}$, so we would get:

$$\sin\frac{\pi}{6}\sin x + \cos\frac{\pi}{6}\cos x = \frac{\sqrt{2}}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 1

Now we can apply the formula for the sine of the sum of angles to get:

$$\sin\left(x+\frac{\pi}{3}\right)=\frac{\sqrt{2}}{2}$$

Note that we could have tried to use the formula for cosine of a difference of the angles. In which case we would need to change 1 into sine and $\sqrt{3}$ into cosine. The hypotenuse is still 2, but the angle is $\alpha=\frac{\pi}{6}$, so we would get:

$$\sin\frac{\pi}{6}\sin x+\cos\frac{\pi}{6}\cos x=\frac{\sqrt{2}}{2}$$

Applying the formula for the cosine of a difference we get:

$$\cos\left(x-\frac{\pi}{6}\right)=\frac{\sqrt{2}}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 1

Going back, we have:

$$\sin\left(x+\frac{\pi}{3}\right)=\frac{\sqrt{2}}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 1

Going back, we have:

$$\sin\left(x+\frac{\pi}{3}\right)=\frac{\sqrt{2}}{2}$$

This is simple now, we have:

$$x+\frac{\pi}{3}=\frac{\pi}{4}+2k\pi \qquad \text{lub} \qquad x+\frac{\pi}{3}=\frac{3\pi}{4}+2k\pi \qquad \text{gdzie } k\in\mathbb{Z}$$

Formula for *sine/cosine* of sum/difference of angles - example 1

Going back, we have:

$$\sin\left(x+\frac{\pi}{3}\right)=\frac{\sqrt{2}}{2}$$

This is simple now, we have:

$$x+\frac{\pi}{3}=\frac{\pi}{4}+2k\pi \qquad \text{lub} \qquad x+\frac{\pi}{3}=\frac{3\pi}{4}+2k\pi \qquad \text{gdzie } k\in\mathbb{Z}$$

so:

$$x=-\frac{\pi}{12}+2k\pi \qquad \text{lub} \qquad x=\frac{5\pi}{12}+2k\pi \qquad \text{gdzie } k\in\mathbb{Z}$$

Formula for *sine/cosine* of sum/difference of angles - example 1

If we used the formula for cosine of a difference we would end up with:

$$\cos\left(x - \frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 1

If we used the formula for cosine of a difference we would end up with:

$$\cos\left(x-\frac{\pi}{6}\right)=\frac{\sqrt{2}}{2}$$

This gives:

$$x-\frac{\pi}{6}=-\frac{\pi}{4}+2k\pi \qquad \text{lub} \qquad x-\frac{\pi}{6}=\frac{\pi}{4}+2k\pi \qquad \text{gdzie } k\in\mathbb{Z}$$

Formula for *sine/cosine* of sum/difference of angles - example 1

If we used the formula for cosine of a difference we would end up with:

$$\cos\left(x - \frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$$

This gives:

$$x - \frac{\pi}{6} = -\frac{\pi}{4} + 2k\pi \qquad \text{lub} \qquad x - \frac{\pi}{6} = \frac{\pi}{4} + 2k\pi \qquad \text{gdzie } k \in \mathbb{Z}$$

and the final answer is of course the same.

Formula for *sine/cosine* of sum/difference of angles - example 2

Solve:

$$\sin x - \cos x = \sqrt{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 2

Solve:

$$\sin x - \cos x = \sqrt{2}$$

We can use sine of a difference here. We draw an appropriate triangle, the hypotenuse is $\sqrt{2}$ and the angle is $\alpha = \frac{\pi}{4}$. So we divide both side by $\sqrt{2}$.

$$\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x = 1$$

so:

Formula for *sine/cosine* of sum/difference of angles - example 2

Solve:

$$\sin x - \cos x = \sqrt{2}$$

We can use sine of a difference here. We draw an appropriate triangle, the hypotenuse is $\sqrt{2}$ and the angle is $\alpha = \frac{\pi}{4}$. So we divide both side by $\sqrt{2}$.

$$\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x = 1$$

so:

$$\cos\frac{\pi}{4}\sin x - \sin\frac{\pi}{4}\cos x = 1$$

Formula for *sine/cosine* of sum/difference of angles - example 2

Solve:

$$\sin x - \cos x = \sqrt{2}$$

We can use sine of a difference here. We draw an appropriate triangle, the hypotenuse is $\sqrt{2}$ and the angle is $\alpha = \frac{\pi}{4}$. So we divide both side by $\sqrt{2}$.

$$\frac{1}{\sqrt{2}} \sin x - \frac{1}{\sqrt{2}} \cos x = 1$$

so:

$$\cos \frac{\pi}{4} \sin x - \sin \frac{\pi}{4} \cos x = 1$$

Now apply the formula for sine of a difference:

$$\sin\left(x - \frac{\pi}{4}\right) = 1$$

Formula for *sine/cosine* of sum/difference of angles - example 2

$$\sin\left(x - \frac{\pi}{4}\right) = 1$$

Formula for *sine/cosine* of sum/difference of angles - example 2

$$\sin\left(x - \frac{\pi}{4}\right) = 1$$

this gives:

$$x - \frac{\pi}{4} = \frac{\pi}{2} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Formula for *sine/cosine* of sum/difference of angles - example 2

$$\sin\left(x - \frac{\pi}{4}\right) = 1$$

this gives:

$$x - \frac{\pi}{4} = \frac{\pi}{2} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

So finally we get:

$$x = \frac{3\pi}{4} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Formula for *sine/cosine* of sum/difference of angles - example 3

Solve:

$$\sqrt{3}\sin x + \cos x = 1$$

Formula for *sine/cosine* of sum/difference of angles - example 3

Solve:

$$\sqrt{3}\sin x + \cos x = 1$$

We can apply the formula for sine of a sum. We draw a triangle with adjacent side $\sqrt{3}$ and opposite side 1. The hypotenuse is 2 and the angle is $\alpha = \frac{\pi}{6}$. So we divide both sides by 2:

$$\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \frac{1}{2}$$

so:

Formula for *sine/cosine* of sum/difference of angles - example 3

Solve:

$$\sqrt{3}\sin x + \cos x = 1$$

We can apply the formula for sine of a sum. We draw a triangle with adjacent side $\sqrt{3}$ and opposite side 1. The hypotenuse is 2 and the angle is $\alpha = \frac{\pi}{6}$. So we divide both sides by 2:

$$\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \frac{1}{2}$$

so:

$$\cos\frac{\pi}{6}\sin x + \sin\frac{\pi}{6}\cos x = \frac{1}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 3

Solve:

$$\sqrt{3}\sin x + \cos x = 1$$

We can apply the formula for sine of a sum. We draw a triangle with adjacent side $\sqrt{3}$ and opposite side 1. The hypotenuse is 2 and the angle is $\alpha = \frac{\pi}{6}$. So we divide both sides by 2:

$$\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \frac{1}{2}$$

so:

$$\cos\frac{\pi}{6}\sin x + \sin\frac{\pi}{6}\cos x = \frac{1}{2}$$

Applying the formula we get:

$$\sin\left(x + \frac{\pi}{6}\right) = \frac{1}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 3

$$\sin\left(x + \frac{\pi}{6}\right) = \frac{1}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 3

$$\sin\left(x+\frac{\pi}{6}\right)=\frac{1}{2}$$

this gives:

$$x+\frac{\pi}{6}=\frac{\pi}{6}+2k\pi \qquad \text{lub} \qquad x+\frac{\pi}{6}=\frac{5\pi}{6}+2k\pi \qquad \text{gdzie } k\in\mathbb{Z}$$

Formula for *sine/cosine* of sum/difference of angles - example 3

$$\sin\left(x+\frac{\pi}{6}\right)=\frac{1}{2}$$

this gives:

$$x+\frac{\pi}{6}=\frac{\pi}{6}+2k\pi \qquad \text{lub} \qquad x+\frac{\pi}{6}=\frac{5\pi}{6}+2k\pi \qquad \text{gdzie } k\in\mathbb{Z}$$

so finally we get:

$$x=2k\pi \qquad \text{lub} \qquad x=\frac{2\pi}{3}+2k\pi \qquad \text{gdzie } k\in\mathbb{Z}$$

Formula for *sine/cosine* of sum/difference of angles - example 4

Solve:

$$\sin 3x + \cos 3x = -\frac{\sqrt{6}}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 4

Solve:

$$\sin 3x + \cos 3x = -\frac{\sqrt{6}}{2}$$

The fact that instead of x we have $3x$ makes no difference.

Formula for *sine/cosine* of sum/difference of angles - example 4

Solve:

$$\sin 3x + \cos 3x = -\frac{\sqrt{6}}{2}$$

The fact that instead of x we have $3x$ makes no difference. We draw a triangle with both adjacent and opposite sides being 1. The hypotenuse is $\sqrt{2}$ and the angle is $\alpha = \frac{\pi}{6}$. We divide both side by $\sqrt{2}$.

$$\frac{1}{\sqrt{2}}\sin 3x + \frac{1}{\sqrt{2}}\cos 3x = -\frac{\sqrt{12}}{4}$$

so:

Formula for *sine/cosine* of sum/difference of angles - example 4

Solve:

$$\sin 3x + \cos 3x = -\frac{\sqrt{6}}{2}$$

The fact that instead of x we have $3x$ makes no difference. We draw a triangle with both adjacent and opposite sides being 1. The hypotenuse is $\sqrt{2}$ and the angle is $\alpha = \frac{\pi}{6}$. We divide both side by $\sqrt{2}$.

$$\frac{1}{\sqrt{2}}\sin 3x + \frac{1}{\sqrt{2}}\cos 3x = -\frac{\sqrt{12}}{4}$$

so:

$$\cos\frac{\pi}{4}\sin 3x + \sin\frac{\pi}{4}\cos 3x = -\frac{\sqrt{3}}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 4

Solve:

$$\sin 3x + \cos 3x = -\frac{\sqrt{6}}{2}$$

The fact that instead of x we have $3x$ makes no difference. We draw a triangle with both adjacent and opposite sides being 1. The hypotenuse is $\sqrt{2}$ and the angle is $\alpha = \frac{\pi}{6}$. We divide both side by $\sqrt{2}$.

$$\frac{1}{\sqrt{2}} \sin 3x + \frac{1}{\sqrt{2}} \cos 3x = -\frac{\sqrt{12}}{4}$$

so:

$$\cos \frac{\pi}{4} \sin 3x + \sin \frac{\pi}{4} \cos 3x = -\frac{\sqrt{3}}{2}$$

Using formula for sine of a sum we get:

$$\sin\left(3x + \frac{\pi}{4}\right) = -\frac{\sqrt{3}}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 4

$$\sin\left(3x + \frac{\pi}{4}\right) = -\frac{\sqrt{3}}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 4

$$\sin\left(3x+\frac{\pi}{4}\right)=-\frac{\sqrt{3}}{2}$$

we get

$$3x+\frac{\pi}{4}=-\frac{\pi}{3}+2k\pi \qquad \text{or} \qquad 3x+\frac{\pi}{4}=-\frac{2\pi}{3}+2k\pi \qquad \text{where } k\in\mathbb{Z}$$

Formula for *sine/cosine* of sum/difference of angles - example 4

$$\sin\left(3x+\frac{\pi}{4}\right)=-\frac{\sqrt{3}}{2}$$

we get

$$3x+\frac{\pi}{4}=-\frac{\pi}{3}+2k\pi \qquad \text{or} \qquad 3x+\frac{\pi}{4}=-\frac{2\pi}{3}+2k\pi \qquad \text{where } k\in\mathbb{Z}$$

So in the end we get:

$$x=-\frac{7\pi}{36}+\frac{2k\pi}{3} \qquad \text{or} \qquad x=-\frac{11\pi}{36}+\frac{2k\pi}{3} \qquad \text{where } k\in\mathbb{Z}$$

Formula for *sine/cosine* of sum/difference of angles - exercise

- Solve:

$$\sin 2x - \sqrt{3}\cos 2x = 1$$

Formula for *sine/cosine* of sum/difference of angles - exercise

- Solve:

$$\sin 2x - \sqrt{3}\cos 2x = 1$$

Solution:

Formula for *sine/cosine* of sum/difference of angles - exercise

- Solve:

$$\sin 2x - \sqrt{3}\cos 2x = 1$$

Solution:

$$x = \frac{\pi}{4} + k\pi \qquad \text{or} \qquad x = \frac{7\pi}{12} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Now we move to the final set of examples, where we apply formulae for sums and differences of sines and cosines.

Formulae for sum and difference of *sine/cosine* - introduction

These are not required by IB, but nevertheless useful. They're **not** included in the formula booklet so you should learn them by heart (in fact it's best to learn the ones in the formula booklet by heart as well).

Formulae for sum and difference of *sine/cosine* - introduction

These are not required by IB, but nevertheless useful. They're **not** included in the formula booklet so you should learn them by heart (in fact it's best to learn the ones in the formula booklet by heart as well).

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

Formulae for sum and difference of *sine/cosine* - example 1

Solve:

$$\sin x + \sin 2x = 0$$

Formulae for sum and difference of *sine/cosine* - example 1

Solve:

$$\sin x + \sin 2x = 0$$

We of course use the formula for the sum of the sine:

$$2\sin\frac{3x}{2}\cos\frac{x}{2} = 0$$

so we get:

Formulae for sum and difference of *sine/cosine* - example 1

Solve:

$$\sin x + \sin 2x = 0$$

We of course use the formula for the sum of the sine:

$$2\sin\frac{3x}{2}\cos\frac{x}{2} = 0$$

so we get:

$$\frac{3x}{2} = k\pi \qquad \text{or} \qquad \frac{x}{2} = \frac{\pi}{2} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Formulae for sum and difference of *sine/cosine* - example 1

Solve:

$$\sin x + \sin 2x = 0$$

We of course use the formula for the sum of the sine:

$$2\sin\frac{3x}{2}\cos\frac{x}{2} = 0$$

so we get:

$$\frac{3x}{2} = k\pi \qquad \text{or} \qquad \frac{x}{2} = \frac{\pi}{2} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

and finally:

$$x = \frac{2k\pi}{3} \qquad \text{or} \qquad x = \pi + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Formulae for sum and difference of *sine/cosine* - example 1

Solve:

$$\sin x + \sin 2x = 0$$

We of course use the formula for the sum of the sine:

$$2\sin\frac{3x}{2}\cos\frac{x}{2} = 0$$

so we get:

$$\frac{3x}{2} = k\pi \qquad \text{or} \qquad \frac{x}{2} = \frac{\pi}{2} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

and finally:

$$x = \frac{2k\pi}{3} \qquad \text{or} \qquad x = \pi + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

We've solve the above equation earlier using $\sin 2x = 2\sin x\cos x$.

Formulae for sum and difference of *sine/cosine* - example 1

Solve:

$$\sin x + \sin 2x = 0$$

We of course use the formula for the sum of the sine:

$$2\sin\frac{3x}{2}\cos\frac{x}{2} = 0$$

so we get:

$$\frac{3x}{2} = k\pi \qquad \text{or} \qquad \frac{x}{2} = \frac{\pi}{2} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

and finally:

$$x = \frac{2k\pi}{3} \qquad \text{or} \qquad x = \pi + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

We've solve the above equation earlier using $\sin 2x = 2\sin x\cos x$.
Compare the answers.

Formulae for sum and difference of *sine/cosine* - example 1

Solve:

$$\sin x + \sin 2x = 0$$

We of course use the formula for the sum of the sine:

$$2\sin\frac{3x}{2}\cos\frac{x}{2} = 0$$

so we get:

$$\frac{3x}{2} = k\pi \qquad \text{or} \qquad \frac{x}{2} = \frac{\pi}{2} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

and finally:

$$x = \frac{2k\pi}{3} \qquad \text{or} \qquad x = \pi + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

We've solve the above equation earlier using $\sin 2x = 2\sin x\cos x$. Compare the answers. At first glance you may think that we got different solutions, but if you study it carefully you will see that they are indeed the same.

Formulae for sum and difference of *sine/cosine* - example 2

Solve:

$$\cos x + \cos 2x + \cos 3x = 0$$

Formulae for sum and difference of *sine/cosine* - example 2

Solve:

$$\cos x + \cos 2x + \cos 3x = 0$$

We use the formula for the sum of cosine to $\cos x + \cos 3x$.

Formulae for sum and difference of *sine/cosine* - example 2

Solve:

$$\cos x + \cos 2x + \cos 3x = 0$$

We use the formula for the sum of cosine to $\cos x + \cos 3x$. Why?

Formulae for sum and difference of *sine/cosine* - example 2

Solve:

$$\cos x + \cos 2x + \cos 3x = 0$$

We use the formula for the sum of cosine to $\cos x + \cos 3x$. Why? Because we get $2\cos 2x \cos(-x)$ i $\cos 2x$ and we will be able to factorize the expression:

$$2\cos 2x \cos(-x) + \cos 2x = 0$$

factor out $\cos 2x$ (and change $\cos(-x)$ to $\cos x$):

Formulae for sum and difference of *sine/cosine* - example 2

Solve:

$$\cos x + \cos 2x + \cos 3x = 0$$

We use the formula for the sum of cosine to $\cos x + \cos 3x$. Why? Because we get $2\cos 2x \cos(-x)$ i $\cos 2x$ and we will be able to factorize the expression:

$$2\cos 2x \cos(-x) + \cos 2x = 0$$

factor out $\cos 2x$ (and change $\cos(-x)$ to $\cos x$):

$$\cos 2x(2\cos x + 1) = 0$$

Formulae for sum and difference of *sine/cosine* - example 2

Solve:

$$\cos x + \cos 2x + \cos 3x = 0$$

We use the formula for the sum of cosine to $\cos x + \cos 3x$. Why? Because we get $2\cos 2x \cos(-x)$ i $\cos 2x$ and we will be able to factorize the expression:

$$2\cos 2x \cos(-x) + \cos 2x = 0$$

factor out $\cos 2x$ (and change $\cos(-x)$ to $\cos x$):

$$\cos 2x(2\cos x + 1) = 0$$

This gives:

Formulae for sum and difference of *sine/cosine* - example 2

Solve:

$$\cos x + \cos 2x + \cos 3x = 0$$

We use the formula for the sum of cosine to $\cos x + \cos 3x$. Why? Because we get $2\cos 2x \cos(-x)$ i $\cos 2x$ and we will be able to factorize the expression:

$$2\cos 2x \cos(-x) + \cos 2x = 0$$

factor out $\cos 2x$ (and change $\cos(-x)$ to $\cos x$):

$$\cos 2x(2\cos x + 1) = 0$$

This gives:

$$2x = \frac{\pi}{2} + k\pi \quad \text{or} \quad x = -\frac{2\pi}{3} + 2k\pi \quad \text{or} \quad x = \frac{2\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Formulae for sum and difference of *sine/cosine* - example 2

Solve:

$$\cos x + \cos 2x + \cos 3x = 0$$

We use the formula for the sum of cosine to $\cos x + \cos 3x$. Why? Because we get $2\cos 2x \cos(-x)$ i $\cos 2x$ and we will be able to factorize the expression:

$$2\cos 2x \cos(-x) + \cos 2x = 0$$

factor out $\cos 2x$ (and change $\cos(-x)$ to $\cos x$):

$$\cos 2x(2\cos x + 1) = 0$$

This gives:

$$2x = \frac{\pi}{2} + k\pi \quad \text{or} \quad x = -\frac{2\pi}{3} + 2k\pi \quad \text{or} \quad x = \frac{2\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

In the end:

Formulae for sum and difference of *sine/cosine* - example 2

Solve:

$$\cos x + \cos 2x + \cos 3x = 0$$

We use the formula for the sum of cosine to $\cos x + \cos 3x$. Why? Because we get $2\cos 2x \cos(-x)$ i $\cos 2x$ and we will be able to factorize the expression:

$$2\cos 2x \cos(-x) + \cos 2x = 0$$

factor out $\cos 2x$ (and change $\cos(-x)$ to $\cos x$):

$$\cos 2x(2\cos x + 1) = 0$$

This gives:

$$2x = \frac{\pi}{2} + k\pi \quad \text{or} \quad x = -\frac{2\pi}{3} + 2k\pi \quad \text{or} \quad x = \frac{2\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

In the end:

$$x = \frac{\pi}{4} + \frac{k\pi}{2} \quad \text{or} \quad x = -\frac{2\pi}{3} + 2k\pi \quad \text{or} \quad x = \frac{2\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Formulae for sum and difference of *sine/cosine* - example 3

Solve:

$$\sin 2x - \cos 3x = 0$$

Formulae for sum and difference of *sine/cosine* - example 3

Solve:

$$\sin 2x - \cos 3x = 0$$

This may seem problematic at first as there is no obvious formula that applies here, but we can simply change cosine into sine using the formula that changes a function into co-function:

Formulae for sum and difference of *sine/cosine* - example 3

Solve:

$$\sin 2x - \cos 3x = 0$$

This may seem problematic at first as there is no obvious formula that applies here, but we can simply change cosine into sine using the formula that changes a function into co-function:

$$\sin 2x - \sin(\frac{\pi}{2} - 3x) = 0$$

Formulae for sum and difference of *sine/cosine* - example 3

Solve:

$$\sin 2x - \cos 3x = 0$$

This may seem problematic at first as there is no obvious formula that applies here, but we can simply change cosine into sine using the formula that changes a function into co-function:

$$\sin 2x - \sin(\frac{\pi}{2} - 3x) = 0$$

Now apply the formula for difference of sines:

Formulae for sum and difference of *sine/cosine* - example 3

Solve:

$$\sin 2x - \cos 3x = 0$$

This may seem problematic at first as there is no obvious formula that applies here, but we can simply change cosine into sine using the formula that changes a function into co-function:

$$\sin 2x - \sin(\frac{\pi}{2} - 3x) = 0$$

Now apply the formula for difference of sines:

$$2\sin\left(\frac{2x - (\frac{\pi}{2} - 3x)}{2}\right)\cos\left(\frac{2x + (\frac{\pi}{2} - 3x)}{2}\right) = 0$$

Formulae for sum and difference of *sine/cosine* - example 3

$$2\sin\left(\frac{2x-(\frac{\pi}{2}-3x)}{2}\right)\cos\left(\frac{2x+(\frac{\pi}{2}-3x)}{2}\right)=0$$

Formulae for sum and difference of *sine/cosine* - example 3

$$2\sin\left(\frac{2x-(\frac{\pi}{2}-3x)}{2}\right)\cos\left(\frac{2x+(\frac{\pi}{2}-3x)}{2}\right)=0$$

Simplify to get:

Formulae for sum and difference of *sine/cosine* - example 3

$$2\sin\left(\frac{2x-(\frac{\pi}{2}-3x)}{2}\right)\cos\left(\frac{2x+(\frac{\pi}{2}-3x)}{2}\right)=0$$

Simplify to get:

$$2\sin\left(\frac{5x}{2}-\frac{\pi}{4}\right)\cos\left(-\frac{x}{2}+\frac{\pi}{4}\right)=0$$

Formulae for sum and difference of *sine/cosine* - example 3

$$2\sin\left(\frac{2x-(\frac{\pi}{2}-3x)}{2}\right)\cos\left(\frac{2x+(\frac{\pi}{2}-3x)}{2}\right)=0$$

Simplify to get:

$$2\sin\left(\frac{5x}{2}-\frac{\pi}{4}\right)\cos\left(-\frac{x}{2}+\frac{\pi}{4}\right)=0$$

This gives:

Formulae for sum and difference of *sine/cosine* - example 3

$$2\sin\left(\frac{2x-(\frac{\pi}{2}-3x)}{2}\right)\cos\left(\frac{2x+(\frac{\pi}{2}-3x)}{2}\right)=0$$

Simplify to get:

$$2\sin\left(\frac{5x}{2}-\frac{\pi}{4}\right)\cos\left(-\frac{x}{2}+\frac{\pi}{4}\right)=0$$

This gives:

$$\frac{5x}{2}-\frac{\pi}{4}=k\pi \qquad \text{or} \qquad -\frac{x}{2}+\frac{\pi}{4}=\frac{\pi}{2}+k\pi \qquad \text{where } k\in\mathbb{Z}$$

Formulae for sum and difference of *sine/cosine* - example 3

$$2\sin\left(\frac{2x-(\frac{\pi}{2}-3x)}{2}\right)\cos\left(\frac{2x+(\frac{\pi}{2}-3x)}{2}\right)=0$$

Simplify to get:

$$2\sin\left(\frac{5x}{2}-\frac{\pi}{4}\right)\cos\left(-\frac{x}{2}+\frac{\pi}{4}\right)=0$$

This gives:

$$\frac{5x}{2}-\frac{\pi}{4}=k\pi \qquad \text{or} \qquad -\frac{x}{2}+\frac{\pi}{4}=\frac{\pi}{2}+k\pi \qquad \text{where } k\in\mathbb{Z}$$

And finally:

$$x=\frac{\pi}{10}+\frac{2k\pi}{5} \qquad \text{or} \qquad x=-\frac{\pi}{2}+2k\pi \qquad \text{where } k\in\mathbb{Z}$$

Formulae for sum and difference of *sine/cosine* - exercises

- Equation:

$$\sin x = \sin 3x$$

- Equation:

$$\sin x = \sin 3x$$

Solution:

$$x = k\pi \qquad \text{or} \qquad x = \frac{\pi}{4} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

Formulae for sum and difference of *sine/cosine* - exercises

- Equation:

$$\sin x = \sin 3x$$

Solution:

$$x = k\pi \qquad \text{or} \qquad x = \frac{\pi}{4} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$\sin x + \sin 3x = \sin 2x$$

Formulae for sum and difference of *sine/cosine* - exercises

- Equation:

$$\sin x = \sin 3x$$

Solution:

$$x = k\pi \qquad \text{or} \qquad x = \frac{\pi}{4} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$\sin x + \sin 3x = \sin 2x$$

Solution:

$$x = \frac{k\pi}{2} \quad \text{or} \quad x = -\frac{\pi}{3} + 2k\pi \quad \text{or} \quad x = \frac{\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Advanced examples

On the next slides we will look at some advanced examples, where we need to make some important observations.

Advanced examples

On the next slides we will look at some advanced examples, where we need to make some important observations.

Make sure you think about these example before looking at the solutions. There may be multiple ways to solve those.

Advanced problems - example 1

Solve:

$$\sin^4 x + \cos^4 x = \cos 2x$$

Advanced problems - example 1

Solve:

$$\sin^4 x + \cos^4 x = \cos 2x$$

The first observation is that the left hand side can be written as $(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$,

Advanced problems - example 1

Solve:

$$\sin^4 x + \cos^4 x = \cos 2x$$

The first observation is that the left hand side can be written as $(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$, and the bracket is just 1 (using Pythagorean identity).

Advanced problems - example 1

Solve:

$$\sin^4 x + \cos^4 x = \cos 2x$$

The first observation is that the left hand side can be written as $(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$, and the bracket is just 1 (using Pythagorean identity). So we get:

$$1 - 2\sin^2 x \cos^2 x = \cos 2x$$

Advanced problems - example 1

Solve:

$$\sin^4 x + \cos^4 x = \cos 2x$$

The first observation is that the left hand side can be written as $(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$, and the bracket is just 1 (using Pythagorean identity). So we get:

$$1 - 2\sin^2 x \cos^2 x = \cos 2x$$

Now we have two options. We can try to make all angles equal $2x$ or make them all equal x.

Advanced problems - example 1

Solve:

$$\sin^4 x + \cos^4 x = \cos 2x$$

The first observation is that the left hand side can be written as $(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$, and the bracket is just 1 (using Pythagorean identity). So we get:

$$1 - 2\sin^2 x \cos^2 x = \cos 2x$$

Now we have two options. We can try to make all angles equal $2x$ or make them all equal x. Let's analyse both options.

Advanced problems - example 1

$$1 - 2\sin^2 x \cos^2 x = \cos 2x$$

Advanced problems - example 1

$$1 - 2\sin^2 x \cos^2 x = \cos 2x$$

We try to make all angles equal to $2x$. Recall that $\sin 2x = 2\sin x \cos x$, so $\sin^2 2x = 4\sin^2 x \cos^2 x$.

Advanced problems - example 1

$$1 - 2\sin^2 x \cos^2 x = \cos 2x$$

We try to make all angles equal to $2x$. Recall that $\sin 2x = 2\sin x \cos x$, so $\sin^2 2x = 4\sin^2 x \cos^2 x$. It makes sense to multiply our equation by 2 to get:

Advanced problems - example 1

$$1 - 2\sin^2 x \cos^2 x = \cos 2x$$

We try to make all angles equal to $2x$. Recall that $\sin 2x = 2\sin x \cos x$, so $\sin^2 2x = 4\sin^2 x \cos^2 x$. It makes sense to multiply our equation by 2 to get:

$$2 - 4\sin^2 x \cos^2 x = 2\cos 2x$$

Advanced problems - example 1

$$1 - 2\sin^2 x \cos^2 x = \cos 2x$$

We try to make all angles equal to $2x$. Recall that $\sin 2x = 2\sin x \cos x$, so $\sin^2 2x = 4\sin^2 x \cos^2 x$. It makes sense to multiply our equation by 2 to get:

$$2 - 4\sin^2 x \cos^2 x = 2\cos 2x$$

so we get:

$$2 - \sin^2 2x = 2\cos 2x$$

Advanced problems - example 1

$$1 - 2\sin^2 x \cos^2 x = \cos 2x$$

We try to make all angles equal to $2x$. Recall that $\sin 2x = 2\sin x \cos x$, so $\sin^2 2x = 4\sin^2 x \cos^2 x$. It makes sense to multiply our equation by 2 to get:

$$2 - 4\sin^2 x \cos^2 x = 2\cos 2x$$

so we get:

$$2 - \sin^2 2x = 2\cos 2x$$

Now we can use Pythagorean identity to change $-\sin^2 2x$ into $\cos^2 2x - 1$,

Advanced problems - example 1

$$1 - 2\sin^2 x \cos^2 x = \cos 2x$$

We try to make all angles equal to $2x$. Recall that $\sin 2x = 2\sin x \cos x$, so $\sin^2 2x = 4\sin^2 x \cos^2 x$. It makes sense to multiply our equation by 2 to get:

$$2 - 4\sin^2 x \cos^2 x = 2\cos 2x$$

so we get:

$$2 - \sin^2 2x = 2\cos 2x$$

Now we can use Pythagorean identity to change $-\sin^2 2x$ into $\cos^2 2x - 1$,and we get (moving all terms to the left hand side):

$$\cos^2 2x - 2\cos 2x + 1 = 0$$

Advanced problems - example 1

$$1 - 2\sin^2 x \cos^2 x = \cos 2x$$

We try to make all angles equal to $2x$. Recall that $\sin 2x = 2\sin x \cos x$, so $\sin^2 2x = 4\sin^2 x \cos^2 x$. It makes sense to multiply our equation by 2 to get:

$$2 - 4\sin^2 x \cos^2 x = 2\cos 2x$$

so we get:

$$2 - \sin^2 2x = 2\cos 2x$$

Now we can use Pythagorean identity to change $-\sin^2 2x$ into $\cos^2 2x - 1$,and we get (moving all terms to the left hand side):

$$\cos^2 2x - 2\cos 2x + 1 = 0$$

This is equivalent to:

$$(\cos 2x - 1)^2 = 0$$

Advanced problems - example 1

$$(\cos 2x - 1)^2 = 0$$

Advanced problems - example 1

$$(\cos 2x - 1)^2 = 0$$

so $\cos 2x = 1$, which gives:

$$2x = 2k\pi \qquad \text{gdzie } k \in \mathbb{Z}$$

Advanced problems - example 1

$$(\cos 2x - 1)^2 = 0$$

so $\cos 2x = 1$, which gives:

$$2x = 2k\pi \qquad \text{gdzie } k \in \mathbb{Z}$$

hence:

$$x = k\pi \qquad \text{gdzie } k \in \mathbb{Z}$$

Advanced problems - example 1

Let's go back to:

$$1 - 2\sin^2 x \cos^2 x = \cos 2x$$

Advanced problems - example 1

Let's go back to:

$$1 - 2\sin^2 x \cos^2 x = \cos 2x$$

Let's try to change the angles to x. We have three formulae for $\cos 2x$, we will use $\cos 2x = 1 - 2\sin^2 x$, because this will allow us to cancel 1 on both sides

Advanced problems - example 1

Let's go back to:

$$1 - 2\sin^2 x \cos^2 x = \cos 2x$$

Let's try to change the angles to x. We have three formulae for $\cos 2x$, we will use $\cos 2x = 1 - 2\sin^2 x$, because this will allow us to cancel 1 on both sides After moving all terms to one side we get:

$$2\sin^2 x - 2\sin^2 x \cos^2 x = 0$$

Advanced problems - example 1

Let's go back to:

$$1 - 2\sin^2 x \cos^2 x = \cos 2x$$

Let's try to change the angles to x. We have three formulae for $\cos 2x$, we will use $\cos 2x = 1 - 2\sin^2 x$, because this will allow us to cancel 1 on both sides After moving all terms to one side we get:

$$2\sin^2 x - 2\sin^2 x \cos^2 x = 0$$

We factor out $2\sin^2 x$:

$$2\sin^2 x(1 - \cos^2 x) = 0$$

Advanced problems - example 1

Let's go back to:

$$1 - 2\sin^2 x \cos^2 x = \cos 2x$$

Let's try to change the angles to x. We have three formulae for $\cos 2x$, we will use $\cos 2x = 1 - 2\sin^2 x$, because this will allow us to cancel 1 on both sides After moving all terms to one side we get:

$$2\sin^2 x - 2\sin^2 x \cos^2 x = 0$$

We factor out $2\sin^2 x$:

$$2\sin^2 x(1 - \cos^2 x) = 0$$

so $\sin x = 0$ or $\cos x = \pm 1$,

Advanced problems - example 1

Let's go back to:

$$1 - 2\sin^2 x \cos^2 x = \cos 2x$$

Let's try to change the angles to x. We have three formulae for $\cos 2x$, we will use $\cos 2x = 1 - 2\sin^2 x$, because this will allow us to cancel 1 on both sides After moving all terms to one side we get:

$$2\sin^2 x - 2\sin^2 x \cos^2 x = 0$$

We factor out $2\sin^2 x$:

$$2\sin^2 x(1 - \cos^2 x) = 0$$

so $\sin x = 0$ or $\cos x = \pm 1$, both of these give:

$$x = k\pi \qquad \text{gdzie } k \in \mathbb{Z}$$

Advanced problems - example 2

Solve:

$$\sin 3x + \cos 2x = 1 + 2\sin x \cos 2x$$

Advanced problems - example 2

Solve:

$$\sin 3x + \cos 2x = 1 + 2\sin x \cos 2x$$

We have 3 different angles: $x, 2x$ i $3x$. Let's get rid of $3x$ first.

Advanced problems - example 2

Solve:

$$\sin 3x + \cos 2x = 1 + 2\sin x \cos 2x$$

We have 3 different angles: $x, 2x$ i $3x$. Let's get rid of $3x$ first. We can write $\sin 3x$ as $\sin(x + 2x)$ and we get:

Advanced problems - example 2

Solve:

$$\sin 3x + \cos 2x = 1 + 2\sin x \cos 2x$$

We have 3 different angles: $x, 2x$ i $3x$. Let's get rid of $3x$ first. We can write $\sin 3x$ as $\sin(x + 2x)$ and we get:

$$\sin x \cos 2x + \cos x \sin 2x + \cos 2x = 1 + 2\sin x \cos 2x$$

Advanced problems - example 2

Solve:

$$\sin 3x + \cos 2x = 1 + 2\sin x \cos 2x$$

We have 3 different angles: $x, 2x$ i $3x$. Let's get rid of $3x$ first. We can write $\sin 3x$ as $\sin(x+2x)$ and we get:

$$\sin x \cos 2x + \cos x \sin 2x + \cos 2x = 1 + 2\sin x \cos 2x$$

Moving all terms to one side:

$$\cos x \sin 2x - \sin x \cos 2x + \cos 2x - 1 = 0$$

Advanced problems - example 2

Solve:

$$\sin 3x + \cos 2x = 1 + 2\sin x \cos 2x$$

We have 3 different angles: $x, 2x$ i $3x$. Let's get rid of $3x$ first. We can write $\sin 3x$ as $\sin(x + 2x)$ and we get:

$$\sin x \cos 2x + \cos x \sin 2x + \cos 2x = 1 + 2\sin x \cos 2x$$

Moving all terms to one side:

$$\cos x \sin 2x - \sin x \cos 2x + \cos 2x - 1 = 0$$

The first two term give us a formula for $\sin(2x - x)$, so $\sin x$.

Advanced problems - example 2

Solve:

$$\sin 3x + \cos 2x = 1 + 2\sin x \cos 2x$$

We have 3 different angles: $x, 2x$ i $3x$. Let's get rid of $3x$ first. We can write $\sin 3x$ as $\sin(x + 2x)$ and we get:

$$\sin x \cos 2x + \cos x \sin 2x + \cos 2x = 1 + 2\sin x \cos 2x$$

Moving all terms to one side:

$$\cos x \sin 2x - \sin x \cos 2x + \cos 2x - 1 = 0$$

The first two term give us a formula for $\sin(2x - x)$, so $\sin x$. We change $\cos 2x$ into $1 - 2\sin^2 x$.

Advanced problems - example 2

Solve:

$$\sin 3x + \cos 2x = 1 + 2\sin x \cos 2x$$

We have 3 different angles: $x, 2x$ i $3x$. Let's get rid of $3x$ first. We can write $\sin 3x$ as $\sin(x + 2x)$ and we get:

$$\sin x \cos 2x + \cos x \sin 2x + \cos 2x = 1 + 2\sin x \cos 2x$$

Moving all terms to one side:

$$\cos x \sin 2x - \sin x \cos 2x + \cos 2x - 1 = 0$$

The first two term give us a formula for $\sin(2x - x)$, so $\sin x$. We change $\cos 2x$ into $1 - 2\sin^2 x$. We get:

$$\sin x - 2\sin^2 x = 0$$

Advanced problems - example 2

$$\sin x - 2\sin^2 x = 0$$

Advanced problems - example 2

$$\sin x - 2\sin^2 x = 0$$

We factor out $\sin x$:

$$\sin x(1 - 2\sin x) = 0$$

Advanced problems - example 2

$$\sin x - 2\sin^2 x = 0$$

We factor out $\sin x$:

$$\sin x(1 - 2\sin x) = 0$$

which gives us the following solutions:

$$x = k\pi \quad \text{or} \quad x = \frac{\pi}{6} + 2k\pi \quad \text{or} \quad x = \frac{5\pi}{6} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Advanced problems - example 3

Solve:

$$\sin^3 x + \cos^3 x = 1$$

Advanced problems - example 3

Solve:

$$\sin^3 x + \cos^3 x = 1$$

We will change 1 into $\sin^2 x + \cos^2 x$ and move all terms to one side

Advanced problems - example 3

Solve:

$$\sin^3 x + \cos^3 x = 1$$

We will change 1 into $\sin^2 x + \cos^2 x$ and move all terms to one side

$$\sin^3 x - \sin^2 x + \cos^3 x - \cos^2 x = 0$$

Advanced problems - example 3

Solve:

$$\sin^3 x + \cos^3 x = 1$$

We will change 1 into $\sin^2 x + \cos^2 x$ and move all terms to one side

$$\sin^3 x - \sin^2 x + \cos^3 x - \cos^2 x = 0$$

We factor out $\sin^2 x$ and $\cos^2 x$:

$$\sin^2 x(\sin x - 1) + \cos^2 x(\cos x - 1) = 0$$

Advanced problems - example 3

Solve:

$$\sin^3 x + \cos^3 x = 1$$

We will change 1 into $\sin^2 x + \cos^2 x$ and move all terms to one side

$$\sin^3 x - \sin^2 x + \cos^3 x - \cos^2 x = 0$$

We factor out $\sin^2 x$ and $\cos^2 x$:

$$\sin^2 x(\sin x - 1) + \cos^2 x(\cos x - 1) = 0$$

Now an important observation.

Advanced problems - example 3

Solve:

$$\sin^3 x + \cos^3 x = 1$$

We will change 1 into $\sin^2 x + \cos^2 x$ and move all terms to one side

$$\sin^3 x - \sin^2 x + \cos^3 x - \cos^2 x = 0$$

We factor out $\sin^2 x$ and $\cos^2 x$:

$$\sin^2 x(\sin x - 1) + \cos^2 x(\cos x - 1) = 0$$

Now an important observation. $\sin^2 x \geq 0$,

Advanced problems - example 3

Solve:

$$\sin^3 x + \cos^3 x = 1$$

We will change 1 into $\sin^2 x + \cos^2 x$ and move all terms to one side

$$\sin^3 x - \sin^2 x + \cos^3 x - \cos^2 x = 0$$

We factor out $\sin^2 x$ and $\cos^2 x$:

$$\sin^2 x(\sin x - 1) + \cos^2 x(\cos x - 1) = 0$$

Now an important observation. $\sin^2 x \geq 0$, but $\sin x - 1 \leq 0$, because *sine* cannot be greater than 1.

Advanced problems - example 3

Solve:

$$\sin^3 x + \cos^3 x = 1$$

We will change 1 into $\sin^2 x + \cos^2 x$ and move all terms to one side

$$\sin^3 x - \sin^2 x + \cos^3 x - \cos^2 x = 0$$

We factor out $\sin^2 x$ and $\cos^2 x$:

$$\sin^2 x(\sin x - 1) + \cos^2 x(\cos x - 1) = 0$$

Now an important observation. $\sin^2 x \geq 0$, but $\sin x - 1 \leq 0$, because *sine* cannot be greater than 1. Similarly $\cos^2 x \geq 0$ and $\cos x - 1 \leq 0$.

Advanced problems - example 3

Solve:

$$\sin^3 x + \cos^3 x = 1$$

We will change 1 into $\sin^2 x + \cos^2 x$ and move all terms to one side

$$\sin^3 x - \sin^2 x + \cos^3 x - \cos^2 x = 0$$

We factor out $\sin^2 x$ and $\cos^2 x$:

$$\sin^2 x(\sin x - 1) + \cos^2 x(\cos x - 1) = 0$$

Now an important observation. $\sin^2 x \geq 0$, but $\sin x - 1 \leq 0$, because *sine* cannot be greater than 1. Similarly $\cos^2 x \geq 0$ and $\cos x - 1 \leq 0$.
So $\sin^2 x(\sin x - 1) \leq 0$ oraz $\cos^2 x(\cos x - 1) \leq 0$

Advanced problems - example 3

$$\sin^2 x(\sin x - 1) + \cos^2 x(\cos x - 1) = 0$$

Both terms are non-positive, but their sum is 0, so they both must be 0.

$$\sin^2 x(\sin x - 1) = 0 \qquad \text{and} \qquad \cos^2 x(\cos x - 1) = 0$$

Advanced problems - example 3

$$\sin^2 x(\sin x - 1) + \cos^2 x(\cos x - 1) = 0$$

Both terms are non-positive, but their sum is 0, so they both must be 0.

$$\sin^2 x(\sin x - 1) = 0 \qquad \text{and} \qquad \cos^2 x(\cos x - 1) = 0$$

Solving this gives::

$$x = 2k\pi \qquad \text{or} \qquad x = \frac{\pi}{2} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Advanced problems - example 4

Solve:

$$\cos x - \cos 3x = \sin x - \sin 3x$$

Advanced problems - example 4

Solve:

$$\cos x - \cos 3x = \sin x - \sin 3x$$

Seems obvious that we want to apply the formula for sum of sines and cosines:

Advanced problems - example 4

Solve:

$$\cos x - \cos 3x = \sin x - \sin 3x$$

Seems obvious that we want to apply the formula for sum of sines and cosines:

$$-2\sin 2x \sin(-x) = 2\sin(-x)\cos 2x$$

Advanced problems - example 4

Solve:

$$\cos x - \cos 3x = \sin x - \sin 3x$$

Seems obvious that we want to apply the formula for sum of sines and cosines:

$$-2\sin 2x \sin(-x) = 2\sin(-x)\cos 2x$$

Sine is an odd function, so $\sin(-x) = -\sin x$, using this and moving all terms to one side:

Advanced problems - example 4

Solve:

$$\cos x - \cos 3x = \sin x - \sin 3x$$

Seems obvious that we want to apply the formula for sum of sines and cosines:

$$-2\sin 2x \sin(-x) = 2\sin(-x)\cos 2x$$

Sine is an odd function, so $\sin(-x) = -\sin x$, using this and moving all terms to one side:

$$2\sin 2x \sin x + 2\sin x \cos 2x = 0$$

Advanced problems - example 4

Solve:

$$\cos x - \cos 3x = \sin x - \sin 3x$$

Seems obvious that we want to apply the formula for sum of sines and cosines:

$$-2\sin 2x \sin(-x) = 2\sin(-x)\cos 2x$$

Sine is an odd function, so $\sin(-x) = -\sin x$, using this and moving all terms to one side:

$$2\sin 2x \sin x + 2\sin x \cos 2x = 0$$

Factoring out $2\sin x$:

Advanced problems - example 4

Solve:

$$\cos x - \cos 3x = \sin x - \sin 3x$$

Seems obvious that we want to apply the formula for sum of sines and cosines:

$$-2\sin 2x \sin(-x) = 2\sin(-x)\cos 2x$$

Sine is an odd function, so $\sin(-x) = -\sin x$, using this and moving all terms to one side:

$$2\sin 2x \sin x + 2\sin x \cos 2x = 0$$

Factoring out $2\sin x$:

$$2\sin x(\sin 2x + \cos 2x) = 0$$

Advanced problems - example 4

$$2\sin x(\sin 2x + \cos 2x) = 0$$

Advanced problems - example 4

$$2\sin x(\sin 2x + \cos 2x) = 0$$

So $\sin x = 0$ or $\sin 2x = -\cos 2x$.

Advanced problems - example 4

$$2\sin x(\sin 2x + \cos 2x) = 0$$

So $\sin x = 0$ or $\sin 2x = -\cos 2x$. The second equation can be turned into $\tan 2x = -1$ (by dividing both sides by $\cos x$).

Advanced problems - example 4

$$2\sin x(\sin 2x + \cos 2x) = 0$$

So $\sin x = 0$ or $\sin 2x = -\cos 2x$. The second equation can be turned into $\tan 2x = -1$ (by dividing both sides by $\cos x$). We solve and get:

$$x = k\pi \qquad \text{or} \qquad 2x = -\frac{\pi}{4} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Advanced problems - example 4

$$2\sin x(\sin 2x + \cos 2x) = 0$$

So $\sin x = 0$ or $\sin 2x = -\cos 2x$. The second equation can be turned into $\tan 2x = -1$ (by dividing both sides by $\cos x$). We solve and get:

$$x = k\pi \qquad \text{or} \qquad 2x = -\frac{\pi}{4} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

So finally we have:

$$x = k\pi \qquad \text{or} \qquad x = -\frac{\pi}{8} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

Advanced problems - example 5

Solve:

$$\sin^2 x + \sin^2 2x = \sin^2 3x$$

Advanced problems - example 5

Solve:

$$\sin^2 x + \sin^2 2x = \sin^2 3x$$

Move all terms to one side:

Advanced problems - example 5

Solve:

$$\sin^2 x + \sin^2 2x = \sin^2 3x$$

Move all terms to one side:

$$\sin^2 x - \sin^2 3x + \sin^2 2x = 0$$

Advanced problems - example 5

Solve:

$$\sin^2 x + \sin^2 2x = \sin^2 3x$$

Move all terms to one side:

$$\sin^2 x - \sin^2 3x + \sin^2 2x = 0$$

We can use difference of squares (with the hope that we can get $\sin 2x$ to factor out):

$$(\sin x - \sin 3x)(\sin x + \sin 3x) + \sin^2 2x = 0$$

Advanced problems - example 5

Solve:

$$\sin^2 x + \sin^2 2x = \sin^2 3x$$

Move all terms to one side:

$$\sin^2 x - \sin^2 3x + \sin^2 2x = 0$$

We can use difference of squares (with the hope that we can get $\sin 2x$ to factor out):

$$(\sin x - \sin 3x)(\sin x + \sin 3x) + \sin^2 2x = 0$$

We use the formula for sum and difference of sines:

Advanced problems - example 5

Solve:

$$\sin^2 x + \sin^2 2x = \sin^2 3x$$

Move all terms to one side:

$$\sin^2 x - \sin^2 3x + \sin^2 2x = 0$$

We can use difference of squares (with the hope that we can get $\sin 2x$ to factor out):

$$(\sin x - \sin 3x)(\sin x + \sin 3x) + \sin^2 2x = 0$$

We use the formula for sum and difference of sines:

$$2\sin(-x)\cos 2x \cdot 2\sin 2x \cos x + \sin^2 2x = 0$$

Advanced problems - example 5

$$2\sin(-x)\cos 2x \cdot 2\sin 2x\cos x + \sin^2 2x = 0$$

Advanced problems - example 5

$$2\sin(-x)\cos 2x \cdot 2\sin 2x\cos x + \sin^2 2x = 0$$

We have that $\sin(-x) = -\sin x$ and we get:

$$-4\sin x\cos x\cos 2x\sin 2x + \sin^2 2x = 0$$

Now we have the expression $\sin x\cos x$ which should remind us of the formula $\sin 2x = 2\sin x\cos x$, we use it to get:

Advanced problems - example 5

$$2\sin(-x)\cos 2x \cdot 2\sin 2x\cos x + \sin^2 2x = 0$$

We have that $\sin(-x) = -\sin x$ and we get:

$$-4\sin x\cos x\cos 2x\sin 2x + \sin^2 2x = 0$$

Now we have the expression $\sin x\cos x$ which should remind us of the formula $\sin 2x = 2\sin x\cos x$, we use it to get:

$$-2\cos 2x\sin^2 2x + \sin^2 2x = 0$$

Advanced problems - example 5

$$2\sin(-x)\cos 2x \cdot 2\sin 2x \cos x + \sin^2 2x = 0$$

We have that $\sin(-x) = -\sin x$ and we get:

$$-4\sin x \cos x \cos 2x \sin 2x + \sin^2 2x = 0$$

Now we have the expression $\sin x \cos x$ which should remind us of the formula $\sin 2x = 2\sin x \cos x$, we use it to get:

$$-2\cos 2x \sin^2 2x + \sin^2 2x = 0$$

Now it's a breeze, we factor out $\sin^2 2x$:

$$\sin^2 2x(1 - 2\cos 2x) = 0$$

Advanced problems - example 5

$$\sin^2 2x(1 - 2\cos 2x) = 0$$

Advanced problems - example 5

$$\sin^2 2x(1 - 2\cos 2x) = 0$$

We get:

$$2x = k\pi \quad \text{or} \quad 2x = -\frac{\pi}{3} + 2k\pi \quad \text{or} \quad 2x = \frac{\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Advanced problems - example 5

$$\sin^2 2x(1 - 2\cos 2x) = 0$$

We get:

$$2x = k\pi \quad \text{or} \quad 2x = -\frac{\pi}{3} + 2k\pi \quad \text{or} \quad 2x = \frac{\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

So the final answer is:

$$x = \frac{k\pi}{2} \quad \text{or} \quad x = -\frac{\pi}{6} + k\pi \quad \text{or} \quad x = \frac{\pi}{6} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Advanced problems - example 6

Solve:

$$\cot 8x \cot 10x = -1$$

Advanced problems - example 6

Solve:

$$\cot 8x \cot 10x = -1$$

We will start with the domain (usually the domain of the equation is specified in IB questions, but it's useful to do it anyway)

Advanced problems - example 6

Solve:

$$\cot 8x \cot 10x = -1$$

We will start with the domain (usually the domain of the equation is specified in IB questions, but it's useful to do it anyway)

$$8x \neq k\pi \qquad \text{and} \qquad 10x \neq k\pi$$

so

Advanced problems - example 6

Solve:

$$\cot 8x \cot 10x = -1$$

We will start with the domain (usually the domain of the equation is specified in IB questions, but it's useful to do it anyway)

$$8x \neq k\pi \qquad \text{and} \qquad 10x \neq k\pi$$

so

$$x \neq \frac{k\pi}{8} \qquad \text{and} \qquad x \neq \frac{k\pi}{10}$$

Advanced problems - example 6

Solve:

$$\cot 8x \cot 10x = -1$$

We will start with the domain (usually the domain of the equation is specified in IB questions, but it's useful to do it anyway)

$$8x \neq k\pi \qquad \text{and} \qquad 10x \neq k\pi$$

so

$$x \neq \frac{k\pi}{8} \qquad \text{and} \qquad x \neq \frac{k\pi}{10}$$

Now we will use the fact that $\cot x = \frac{\cos x}{\sin x}$:

Advanced problems - example 6

Solve:

$$\cot 8x \cot 10x = -1$$

We will start with the domain (usually the domain of the equation is specified in IB questions, but it's useful to do it anyway)

$$8x \neq k\pi \qquad \text{and} \qquad 10x \neq k\pi$$

so

$$x \neq \frac{k\pi}{8} \qquad \text{and} \qquad x \neq \frac{k\pi}{10}$$

Now we will use the fact that $\cot x = \frac{\cos x}{\sin x}$:

$$\frac{\cos 8x \cos 10x}{\sin 8x \sin 10x} = -1$$

Advanced problems - example 6

$$\frac{\cos 8x \cos 10x}{\sin 8x \sin 10x} = -1$$

Advanced problems - example 6

$$\frac{\cos 8x \cos 10x}{\sin 8x \sin 10x} = -1$$

Multiply by the denominator (which we know is non-zero) and move to one side to get:

Advanced problems - example 6

$$\frac{\cos 8x \cos 10x}{\sin 8x \sin 10x} = -1$$

Multiply by the denominator (which we know is non-zero) and move to one side to get:

$$\cos 8x \cos 10x + \sin 8x \sin 10x = 0$$

Advanced problems - example 6

$$\frac{\cos 8x \cos 10x}{\sin 8x \sin 10x} = -1$$

Multiply by the denominator (which we know is non-zero) and move to one side to get:

$$\cos 8x \cos 10x + \sin 8x \sin 10x = 0$$

This looks like a formula for a cosine of a difference.

Advanced problems - example 6

$$\frac{\cos 8x \cos 10x}{\sin 8x \sin 10x} = -1$$

Multiply by the denominator (which we know is non-zero) and move to one side to get:

$$\cos 8x \cos 10x + \sin 8x \sin 10x = 0$$

This looks like a formula for a cosine of a difference. We get:

$$\cos 2x = 0$$

Advanced problems - example 6

$$\frac{\cos 8x \cos 10x}{\sin 8x \sin 10x} = -1$$

Multiply by the denominator (which we know is non-zero) and move to one side to get:

$$\cos 8x \cos 10x + \sin 8x \sin 10x = 0$$

This looks like a formula for a cosine of a difference. We get:

$$\cos 2x = 0$$

So:

$$x = \frac{\pi}{4} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

Advanced problems - example 6

$$\frac{\cos 8x \cos 10x}{\sin 8x \sin 10x} = -1$$

Multiply by the denominator (which we know is non-zero) and move to one side to get:

$$\cos 8x \cos 10x + \sin 8x \sin 10x = 0$$

This looks like a formula for a cosine of a difference. We get:

$$\cos 2x = 0$$

So:

$$x = \frac{\pi}{4} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

But beware, all of these solutions are outside of our domain, so in the end our equation has no solutions.

Advanced problems - example 6

$$\frac{\cos 8x \cos 10x}{\sin 8x \sin 10x} = -1$$

Multiply by the denominator (which we know is non-zero) and move to one side to get:

$$\cos 8x \cos 10x + \sin 8x \sin 10x = 0$$

This looks like a formula for a cosine of a difference. We get:

$$\cos 2x = 0$$

So:

$$x = \frac{\pi}{4} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

But beware, all of these solutions are outside of our domain, so in the end our equation has no solutions.

$$x \in \emptyset$$

The next slides include problems that appeared on a Polish Matura (advanced level).

Polish Matura - problem 1

May 2015,

Polish Matura - problem 1

May 2015, simple multiple choice question to begin with:

Zadanie 4. (0–1)

Równanie $2\sin x + 3\cos x = 6$ w przedziale $(0, 2\pi)$

A. nie ma rozwiązań rzeczywistych.

B. ma dokładnie jedno rozwiązanie rzeczywiste.

C. ma dokładnie dwa rozwiązania rzeczywiste.

D. ma więcej niż dwa rozwiązania rzeczywiste.

Polish Matura - problem 1

May 2015, simple multiple choice question to begin with:

Zadanie 4. (0–1)

Równanie $2\sin x + 3\cos x = 6$ w przedziale $(0, 2\pi)$

A. nie ma rozwiązań rzeczywistych.

B. ma dokładnie jedno rozwiązanie rzeczywiste.

C. ma dokładnie dwa rozwiązania rzeczywiste.

D. ma więcej niż dwa rozwiązania rzeczywiste.

This is a very important question, because it shows that it's always important to think about the equation before solving it.

Polish Matura - problem 1

May 2015, simple multiple choice question to begin with:

Zadanie 4. (0–1)

Równanie $2\sin x + 3\cos x = 6$ w przedziale $(0, 2\pi)$

A. nie ma rozwiązań rzeczywistych.

B. ma dokładnie jedno rozwiązanie rzeczywiste.

C. ma dokładnie dwa rozwiązania rzeczywiste.

D. ma więcej niż dwa rozwiązania rzeczywiste.

This is a very important question, because it shows that it's always important to think about the equation before solving it. $\sin x$ is less than or equal to 1, similarly $\cos x$, so the left hand side is certainly not greater than 5 (note that the maximum value of the left hand side is of course

Polish Matura - problem 1

May 2015, simple multiple choice question to begin with:

Zadanie 4. (0–1)

Równanie $2\sin x + 3\cos x = 6$ w przedziale $(0, 2\pi)$

A. nie ma rozwiązań rzeczywistych.

B. ma dokładnie jedno rozwiązanie rzeczywiste.

C. ma dokładnie dwa rozwiązania rzeczywiste.

D. ma więcej niż dwa rozwiązania rzeczywiste.

This is a very important question, because it shows that it's always important to think about the equation before solving it. $\sin x$ is less than or equal to 1, similarly $\cos x$, so the left hand side is certainly not greater than 5 (note that the maximum value of the left hand side is of course (find it)

Polish Matura - problem 1

May 2015, simple multiple choice question to begin with:

Zadanie 4. (0–1)

Równanie $2\sin x + 3\cos x = 6$ w przedziale $(0, 2\pi)$

A. nie ma rozwiązań rzeczywistych.

B. ma dokładnie jedno rozwiązanie rzeczywiste.

C. ma dokładnie dwa rozwiązania rzeczywiste.

D. ma więcej niż dwa rozwiązania rzeczywiste.

This is a very important question, because it shows that it's always important to think about the equation before solving it. $\sin x$ is less than or equal to 1, similarly $\cos x$, so the left hand side is certainly not greater than 5 (note that the maximum value of the left hand side is of course (find it) $\sqrt{13}$),

Polish Matura - problem 1

May 2015, simple multiple choice question to begin with:

Zadanie 4. (0–1)

Równanie $2\sin x + 3\cos x = 6$ w przedziale $(0, 2\pi)$

A. nie ma rozwiązań rzeczywistych.

B. ma dokładnie jedno rozwiązanie rzeczywiste.

C. ma dokładnie dwa rozwiązania rzeczywiste.

D. ma więcej niż dwa rozwiązania rzeczywiste.

This is a very important question, because it shows that it's always important to think about the equation before solving it. $\sin x$ is less than or equal to 1, similarly $\cos x$, so the left hand side is certainly not greater than 5 (note that the maximum value of the left hand side is of course (find it) $\sqrt{13}$), so there'll be no solutions - answer A.

Polish Matura - problem 2

May 2017,

Polish Matura - problem 2

May 2017,

Zadanie 10. (0–4)

Rozwiąż równanie $\cos 2x + 3\cos x = -2$ w przedziale $\langle 0, 2\pi \rangle$.

Polish Matura - problem 2

May 2017,

Zadanie 10. (0–4)

Rozwiąż równanie $\cos 2x + 3\cos x = -2$ w przedziale $\langle 0, 2\pi \rangle$.

We change $\cos 2x$ into $2\cos^2 x - 1$.

Polish Matura - problem 2

May 2017,

Zadanie 10. (0–4)

Rozwiąż równanie $\cos 2x + 3\cos x = -2$ w przedziale $\langle 0, 2\pi \rangle$.

We change $\cos 2x$ into $2\cos^2 x - 1$. Move all terms to one side to get:

$$2\cos^2 x + 3\cos x + 1 = 0$$

Polish Matura - problem 2

May 2017,

Zadanie 10. (0–4)

Rozwiąż równanie $\cos 2x + 3\cos x = -2$ w przedziale $\langle 0, 2\pi \rangle$.

We change $\cos 2x$ into $2\cos^2 x - 1$. Move all terms to one side to get:

$$2\cos^2 x + 3\cos x + 1 = 0$$

Factorize:

Polish Matura - problem 2

May 2017,

Zadanie 10. (0–4)
Rozwiąż równanie $\cos 2x + 3\cos x = -2$ w przedziale $\langle 0, 2\pi \rangle$.

We change $\cos 2x$ into $2\cos^2 x - 1$. Move all terms to one side to get:

$$2\cos^2 x + 3\cos x + 1 = 0$$

Factorize:

$$(2\cos x + 1)(\cos x + 1) = 0$$

Polish Matura - problem 2

May 2017,

Zadanie 10. (0–4)

Rozwiąż równanie $\cos 2x + 3\cos x = -2$ w przedziale $\langle 0, 2\pi \rangle$.

We change $\cos 2x$ into $2\cos^2 x - 1$. Move all terms to one side to get:

$$2\cos^2 x + 3\cos x + 1 = 0$$

Factorize:

$$(2\cos x + 1)(\cos x + 1) = 0$$

Now we sketch the graph of cosine for $0 \leq x \leq 2\pi$.

Polish Matura - problem 2

May 2017,

Zadanie 10. (0–4)

Rozwiąż równanie $\cos 2x + 3\cos x = -2$ w przedziale $\langle 0, 2\pi \rangle$.

We change $\cos 2x$ into $2\cos^2 x - 1$. Move all terms to one side to get:

$$2\cos^2 x + 3\cos x + 1 = 0$$

Factorize:

$$(2\cos x + 1)(\cos x + 1) = 0$$

Now we sketch the graph of cosine for $0 \leq x \leq 2\pi$. We get:

$$x = \frac{2\pi}{3} \qquad \text{or} \qquad x = \pi \qquad \text{or} \qquad x = \frac{4\pi}{3}$$

Polish Matura - problem 3

May 2018,

Polish Matura - problem 3

May 2018,

Zadanie 11. (0–4)

Rozwiąż równanie $\sin 6x + \cos 3x = 2\sin 3x + 1$ w przedziale $\langle 0, \pi \rangle$.

Polish Matura - problem 3

May 2018,

Zadanie 11. (0–4)

Rozwiąż równanie $\sin 6x + \cos 3x = 2\sin 3x + 1$ w przedziale $\langle 0, \pi \rangle$.

We change $\sin 6x$ into $2\sin 3x \cos 3x$:

$$2\sin 3x \cos 3x + \cos 3x = 2\sin 3x + 1$$

Polish Matura - problem 3

May 2018,

Zadanie 11. (0–4)

Rozwiąż równanie $\sin 6x + \cos 3x = 2\sin 3x + 1$ w przedziale $\langle 0, \pi \rangle$.

We change $\sin 6x$ into $2\sin 3x \cos 3x$:

$$2\sin 3x \cos 3x + \cos 3x = 2\sin 3x + 1$$

Factor out $\cos 3x$ and move all terms to one side:

Polish Matura - problem 3

May 2018,

Zadanie 11. (0–4)

Rozwiąż równanie $\sin 6x + \cos 3x = 2\sin 3x + 1$ w przedziale $\langle 0, \pi \rangle$.

We change $\sin 6x$ into $2\sin 3x \cos 3x$:

$$2\sin 3x \cos 3x + \cos 3x = 2\sin 3x + 1$$

Factor out $\cos 3x$ and move all terms to one side:

$$\cos 3x(2\sin 3x + 1) - (2\sin 3x + 1) = 0$$

Polish Matura - problem 3

May 2018,

Zadanie 11. (0–4)

Rozwiąż równanie $\sin 6x + \cos 3x = 2\sin 3x + 1$ w przedziale $\langle 0, \pi \rangle$.

We change $\sin 6x$ into $2\sin 3x \cos 3x$:

$$2\sin 3x \cos 3x + \cos 3x = 2\sin 3x + 1$$

Factor out $\cos 3x$ and move all terms to one side:

$$\cos 3x(2\sin 3x + 1) - (2\sin 3x + 1) = 0$$

Now we can factor out $(2\sin 3x + 1)$:

Polish Matura - problem 3

May 2018,

Zadanie 11. (0–4)

Rozwiąż równanie $\sin 6x + \cos 3x = 2\sin 3x + 1$ w przedziale $\langle 0, \pi \rangle$.

We change $\sin 6x$ into $2\sin 3x \cos 3x$:

$$2\sin 3x \cos 3x + \cos 3x = 2\sin 3x + 1$$

Factor out $\cos 3x$ and move all terms to one side:

$$\cos 3x(2\sin 3x + 1) - (2\sin 3x + 1) = 0$$

Now we can factor out $(2\sin 3x + 1)$:

$$(2\sin 3x + 1)(\cos 3x - 1) = 0$$

Polish Matura - problem 3

$$(2\sin 3x + 1)(\cos 3x - 1) = 0$$

Polish Matura - problem 3

$$(2\sin 3x + 1)(\cos 3x - 1) = 0$$

Now it's fairly simple, beware though that we have $3x$ and the domain is $0 \leq x \leq \pi$.

Polish Matura - problem 3

$$(2\sin 3x + 1)(\cos 3x - 1) = 0$$

Now it's fairly simple, beware though that we have $3x$ and the domain is $0 \leq x \leq \pi$. We subsitute $\alpha = 3x$ and then we have $0 \leq \alpha \leq 3\pi$.

Polish Matura - problem 3

$$(2\sin 3x + 1)(\cos 3x - 1) = 0$$

Now it's fairly simple, beware though that we have $3x$ and the domain is $0 \leq x \leq \pi$. We subsitute $\alpha = 3x$ and then we have $0 \leq \alpha \leq 3\pi$. We now get the following solutions:

Polish Matura - problem 3

$$(2\sin 3x + 1)(\cos 3x - 1) = 0$$

Now it's fairly simple, beware though that we have $3x$ and the domain is $0 \leq x \leq \pi$. We subsitute $\alpha = 3x$ and then we have $0 \leq \alpha \leq 3\pi$. We now get the following solutions:

$$\alpha = \frac{7\pi}{6} \quad \text{or} \quad \alpha = \frac{11\pi}{6} \quad \text{or} \quad \alpha = 0 \quad \text{or} \quad \alpha = 2\pi$$

Polish Matura - problem 3

$$(2\sin 3x + 1)(\cos 3x - 1) = 0$$

Now it's fairly simple, beware though that we have $3x$ and the domain is $0 \leq x \leq \pi$. We subsitute $\alpha = 3x$ and then we have $0 \leq \alpha \leq 3\pi$. We now get the following solutions:

$$\alpha = \frac{7\pi}{6} \quad \text{or} \quad \alpha = \frac{11\pi}{6} \quad \text{or} \quad \alpha = 0 \quad \text{or} \quad \alpha = 2\pi$$

Since $\alpha = 3x$, then $x = \frac{\alpha}{3}$, so:

Polish Matura - problem 3

$$(2\sin 3x + 1)(\cos 3x - 1) = 0$$

Now it's fairly simple, beware though that we have $3x$ and the domain is $0 \leq x \leq \pi$. We subsitute $\alpha = 3x$ and then we have $0 \leq \alpha \leq 3\pi$. We now get the following solutions:

$$\alpha = \frac{7\pi}{6} \quad \text{or} \quad \alpha = \frac{11\pi}{6} \quad \text{or} \quad \alpha = 0 \quad \text{or} \quad \alpha = 2\pi$$

Since $\alpha = 3x$, then $x = \frac{\alpha}{3}$, so:

$$x = \frac{7\pi}{18} \quad \text{or} \quad x = \frac{11\pi}{18} \quad \text{or} \quad x = 0 \quad \text{or} \quad x = \frac{2\pi}{3}$$

Polish Matura - problem 4

May 2019,

Polish Matura - problem 4

May 2019,

Zadanie 2. (0–1)

Liczba $\cos^2 105° - \sin^2 105°$ jest równa

A. $-\frac{\sqrt{3}}{2}$ **B.** $-\frac{1}{2}$ **C.** $\frac{1}{2}$ **D.** $\frac{\sqrt{3}}{2}$

Polish Matura - problem 4

May 2019,

Zadanie 2. (0–1)

Liczba $\cos^2 105° - \sin^2 105°$ jest równa

A. $-\frac{\sqrt{3}}{2}$ **B.** $-\frac{1}{2}$ **C.** $\frac{1}{2}$ **D.** $\frac{\sqrt{3}}{2}$

If we remember the formulae, then we should immediately notice $\cos 2x = \cos^2 x - \sin^2 x$, so we get:

Polish Matura - problem 4

May 2019,

Zadanie 2. (0–1)

Liczba $\cos^2 105° - \sin^2 105°$ jest równa

A. $-\frac{\sqrt{3}}{2}$ **B.** $-\frac{1}{2}$ **C.** $\frac{1}{2}$ **D.** $\frac{\sqrt{3}}{2}$

If we remember the formulae, then we should immediately notice $\cos 2x = \cos^2 x - \sin^2 x$, so we get::

$$\cos^2 105° - \sin^2 105° = \cos 210° = \cos(180° + 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$$

Polish Matura - problem 4

May 2019,

Zadanie 2. (0–1)

Liczba $\cos^2 105° - \sin^2 105°$ jest równa

A. $-\frac{\sqrt{3}}{2}$ **B.** $-\frac{1}{2}$ **C.** $\frac{1}{2}$ **D.** $\frac{\sqrt{3}}{2}$

If we remember the formulae, then we should immediately notice $\cos 2x = \cos^2 x - \sin^2 x$, so we get::

$$\cos^2 105^\circ - \sin^2 105^\circ = \cos 210^\circ = \cos(180^\circ + 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

Answer A.

Polish Matura - problem 5

May 2019,

Polish Matura - problem 5

May 2019,

Zadanie 14. (0–4)

Rozwiąż równanie $(\cos x)\left[\sin\left(x-\frac{\pi}{3}\right)+\sin\left(x+\frac{\pi}{3}\right)\right]=\frac{1}{2}\sin x$.

Polish Matura - problem 5

May 2019,

Zadanie 14. (0–4)

Rozwiąż równanie $(\cos x)\left[\sin\left(x-\frac{\pi}{3}\right)+\sin\left(x+\frac{\pi}{3}\right)\right]=\frac{1}{2}\sin x$.

Looks complicated, but the first step is obvious - we add the sines using appropriate formula. We get:

Polish Matura - problem 5

May 2019,

Zadanie 14. (0–4)

Rozwiąż równanie $(\cos x)\left[\sin\left(x-\frac{\pi}{3}\right)+\sin\left(x+\frac{\pi}{3}\right)\right]=\frac{1}{2}\sin x$.

Looks complicated, but the first step is obvious - we add the sines using appropriate formula. We get:

$$\cos x \cdot 2\sin x \cos\frac{\pi}{3} = \frac{1}{2}\sin x$$

Polish Matura - problem 5

May 2019,

Zadanie 14. (0–4)

Rozwiąż równanie $(\cos x)\left[\sin\left(x-\frac{\pi}{3}\right)+\sin\left(x+\frac{\pi}{3}\right)\right]=\frac{1}{2}\sin x$.

Looks complicated, but the first step is obvious - we add the sines using appropriate formula. We get:

$$\cos x \cdot 2\sin x \cos\frac{\pi}{3} = \frac{1}{2}\sin x$$

Now it becomes very simple.

Polish Matura - problem 5

May 2019,

Zadanie 14. (0–4)

Rozwiąż równanie $(\cos x)\left[\sin\left(x-\frac{\pi}{3}\right)+\sin\left(x+\frac{\pi}{3}\right)\right]=\frac{1}{2}\sin x$.

Looks complicated, but the first step is obvious - we add the sines using appropriate formula. We get:

$$\cos x \cdot 2\sin x \cos\frac{\pi}{3} = \frac{1}{2}\sin x$$

Now it becomes very simple. Of course we have $\cos\frac{\pi}{3} = \frac{1}{2}$.

Polish Matura - problem 5

May 2019,

Zadanie 14. (0–4)

Rozwiąż równanie $(\cos x)\left[\sin\left(x-\frac{\pi}{3}\right)+\sin\left(x+\frac{\pi}{3}\right)\right]=\frac{1}{2}\sin x$.

Looks complicated, but the first step is obvious - we add the sines using appropriate formula. We get:

$$\cos x \cdot 2\sin x \cos\frac{\pi}{3} = \frac{1}{2}\sin x$$

Now it becomes very simple. Of course we have $\cos\frac{\pi}{3} = \frac{1}{2}$. We move all terms to one side and we get:

Polish Matura - problem 5

May 2019,

Zadanie 14. (0–4)

Rozwiąż równanie $(\cos x)\left[\sin\left(x-\frac{\pi}{3}\right)+\sin\left(x+\frac{\pi}{3}\right)\right]=\frac{1}{2}\sin x$.

Looks complicated, but the first step is obvious - we add the sines using appropriate formula. We get:

$$\cos x \cdot 2\sin x \cos\frac{\pi}{3} = \frac{1}{2}\sin x$$

Now it becomes very simple. Of course we have $\cos\frac{\pi}{3} = \frac{1}{2}$. We move all terms to one side and we get:

$$\sin x\cos x - \frac{1}{2}\sin x = 0$$

Polish Matura - problem 5

$$\sin x \cos x - \frac{1}{2} \sin x = 0$$

Polish Matura - problem 5

$$\sin x \cos x - \frac{1}{2}\sin x = 0$$

We factor out $\sin x$:

Polish Matura - problem 5

$$\sin x \cos x - \frac{1}{2} \sin x = 0$$

We factor out $\sin x$:

$$\sin x(\cos x - \frac{1}{2}) = 0$$

Polish Matura - problem 5

$$\sin x \cos x - \frac{1}{2}\sin x = 0$$

We factor out $\sin x$:

$$\sin x(\cos x - \frac{1}{2}) = 0$$

This gives:

Polish Matura - problem 5

$$\sin x \cos x - \frac{1}{2}\sin x = 0$$

We factor out $\sin x$:

$$\sin x(\cos x - \frac{1}{2}) = 0$$

This gives:

$$x = k\pi \quad \text{or} \quad x = -\frac{\pi}{3} + 2k\pi \quad \text{or} \quad x = \frac{\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Now we move on to IB exam questions.

IB exam - problem 1

[Maximum mark: 6]

Find all solutions to the equation $\tan x + \tan 2x = 0$ where $0^\circ \leq x < 360^\circ$.

IB exam - problem 1

[Maximum mark: 6]

Find all solutions to the equation $\tan x + \tan 2x = 0$ where $0^\circ \leq x < 360^\circ$.

Note that the interval is in degrees - this is unusual.

IB exam - problem 1

[Maximum mark: 6]

Find all solutions to the equation $\tan x + \tan 2x = 0$ where $0^\circ \leq x < 360^\circ$.

Note that the interval is in degrees - this is unusual. It's quite obvious that we will use double angle formula for tangent:

IB exam - problem 1

[Maximum mark: 6]

Find all solutions to the equation $\tan x + \tan 2x = 0$ where $0^\circ \leq x < 360^\circ$.

Note that the interval is in degrees - this is unusual. It's quite obvious that we will use double angle formula for tangent:

$$\tan x + \frac{2\tan x}{1-\tan^2 x} = 0$$

IB exam - problem 1

[Maximum mark: 6]

Find all solutions to the equation $\tan x + \tan 2x = 0$ where $0^\circ \le x < 360^\circ$.

Note that the interval is in degrees - this is unusual. It's quite obvious that we will use double angle formula for tangent:

$$\tan x + \frac{2\tan x}{1-\tan^2 x} = 0$$

Now it makes sense to multiply both sides by $1-\tan^2 x$ to get:

IB exam - problem 1

[Maximum mark: 6]

Find all solutions to the equation $\tan x + \tan 2x = 0$ where $0^\circ \leq x < 360^\circ$.

Note that the interval is in degrees - this is unusual. It's quite obvious that we will use double angle formula for tangent:

$$\tan x + \frac{2\tan x}{1-\tan^2 x} = 0$$

Now it makes sense to multiply both sides by $1-\tan^2 x$ to get:

$$\tan x - \tan^3 x + 2\tan x = 0$$

IB exam - problem 1

Now this is very easy, factoring $\tan x$ we get:

IB exam - problem 1

Now this is very easy, factoring $\tan x$ we get:

$$\tan x(3 - \tan^2 x) = 0$$

IB exam - problem 1

Now this is very easy, factoring $\tan x$ we get:

$$\tan x(3 - \tan^2 x) = 0$$

So $\tan x = 0$ or $\tan x = \pm\sqrt{3}$.

IB exam - problem 1

Now this is very easy, factoring $\tan x$ we get:

$$\tan x(3 - \tan^2 x) = 0$$

So $\tan x = 0$ or $\tan x = \pm\sqrt{3}$. We should draw the graph of $\tan x$ for $0° \leq x < 360°$, so that we don't miss any solutions.

IB exam - problem 1

Now this is very easy, factoring $\tan x$ we get:

$$\tan x(3 - \tan^2 x) = 0$$

So $\tan x = 0$ or $\tan x = \pm\sqrt{3}$. We should draw the graph of $\tan x$ for $0^\circ \leq x < 360^\circ$, so that we don't miss any solutions. In the end we get:

$$x \in \{0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ, 300^\circ\}$$

IB exam - problem 2

[Maximum mark: 7]

Solve the equation $\sin 2x - \cos 2x = 1 + \sin x - \cos x$ for $x \in [-\pi, \pi]$.

IB exam - problem 2

[Maximum mark: 7]

Solve the equation $\sin 2x - \cos 2x = 1 + \sin x - \cos x$ for $x \in [-\pi, \pi]$.

We will start by using double angle formulae for sine and cosine. For cosine it makes sense to use $\cos 2x = 2\cos^2 x - 1$, because this will allow us to cancel 1 on both sides.

IB exam - problem 2

[Maximum mark: 7]

Solve the equation $\sin 2x - \cos 2x = 1 + \sin x - \cos x$ for $x \in [-\pi, \pi]$.

We will start by using double angle formulae for sine and cosine. For cosine it makes sense to use $\cos 2x = 2\cos^2 x - 1$, because this will allow us to cancel 1 on both sides. We get:

$$2\sin x \cos x - 2\cos^2 x + 1 = 1 + \sin x - \cos x$$

IB exam - problem 2

[Maximum mark: 7]

Solve the equation $\sin 2x - \cos 2x = 1 + \sin x - \cos x$ for $x \in [-\pi, \pi]$.

We will start by using double angle formulae for sine and cosine. For cosine it makes sense to use $\cos 2x = 2\cos^2 x - 1$, because this will allow us to cancel 1 on both sides. We get:

$$2\sin x \cos x - 2\cos^2 x + 1 = 1 + \sin x - \cos x$$

Moving all terms to one side we get:

IB exam - problem 2

[Maximum mark: 7]

Solve the equation $\sin 2x - \cos 2x = 1 + \sin x - \cos x$ for $x \in [-\pi, \pi]$.

We will start by using double angle formulae for sine and cosine. For cosine it makes sense to use $\cos 2x = 2\cos^2 x - 1$, because this will allow us to cancel 1 on both sides. We get:

$$2\sin x \cos x - 2\cos^2 x + 1 = 1 + \sin x - \cos x$$

Moving all terms to one side we get:

$$2\sin x \cos x - 2\cos^2 x - \sin x + \cos x = 0$$

IB exam - problem 2

This looks doable now. Factor $2\cos x$ from the first two terms and -1 from the next two:

IB exam - problem 2

This looks doable now. Factor $2\cos x$ from the first two terms and -1 from the next two:

$$2\cos x(\sin x - \cos x) - (\sin x - \cos x) = 0$$

IB exam - problem 2

This looks doable now. Factor $2\cos x$ from the first two terms and -1 from the next two:

$$2\cos x(\sin x - \cos x) - (\sin x - \cos x) = 0$$

This gives:

$$(\sin x - \cos x)(2\cos x - 1) = 0$$

So we have $\sin x = \cos x$, which gives $\tan x = 1$ or from the second bracket $\cos x = \frac{1}{2}$.

IB exam - problem 2

This looks doable now. Factor $2\cos x$ from the first two terms and -1 from the next two:

$$2\cos x(\sin x - \cos x) - (\sin x - \cos x) = 0$$

This gives:

$$(\sin x - \cos x)(2\cos x - 1) = 0$$

So we have $\sin x = \cos x$, which gives $\tan x = 1$ or from the second bracket $\cos x = \frac{1}{2}$. These are easy to solve. We draw $\tan x$ and $\cos x$ in the interval $-\pi \leq x \leq \pi$ and get that:

IB exam - problem 2

This looks doable now. Factor $2\cos x$ from the first two terms and -1 from the next two:

$$2\cos x(\sin x - \cos x) - (\sin x - \cos x) = 0$$

This gives:

$$(\sin x - \cos x)(2\cos x - 1) = 0$$

So we have $\sin x = \cos x$, which gives $\tan x = 1$ or from the second bracket $\cos x = \frac{1}{2}$. These are easy to solve. We draw $\tan x$ and $\cos x$ in the interval $-\pi \leq x \leq \pi$ and get that:

$$x \in \left\{-\frac{3\pi}{4}, -\frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{3}\right\}$$

IB exam - problem 3

[Maximum mark: 8]

Consider the equation $\frac{\sqrt{3}-1}{\sin x}+\frac{\sqrt{3}+1}{\cos x}=4\sqrt{2},\ 0<x<\frac{\pi}{2}$. Given that $\sin\left(\frac{\pi}{12}\right)=\frac{\sqrt{6}-\sqrt{2}}{4}$ and $\cos\left(\frac{\pi}{12}\right)=\frac{\sqrt{6}+\sqrt{2}}{4}$

(a) verify that $x=\frac{\pi}{12}$ is a solution to the equation; [3]

(b) hence find the other solution to the equation for $0<x<\frac{\pi}{2}$. [5]

IB exam - problem 3

[Maximum mark: 8]

Consider the equation $\dfrac{\sqrt{3}-1}{\sin x}+\dfrac{\sqrt{3}+1}{\cos x}=4\sqrt{2},\ 0<x<\dfrac{\pi}{2}$. Given that $\sin\left(\dfrac{\pi}{12}\right)=\dfrac{\sqrt{6}-\sqrt{2}}{4}$ and $\cos\left(\dfrac{\pi}{12}\right)=\dfrac{\sqrt{6}+\sqrt{2}}{4}$

(a) verify that $x=\dfrac{\pi}{12}$ is a solution to the equation; [3]

(b) hence find the other solution to the equation for $0<x<\dfrac{\pi}{2}$. [5]

The first part is easy, since we're given all the information.

IB exam - problem 3 (a)

We will start with the left hand side:

$$\begin{aligned} LHS &= \frac{\sqrt{3}-1}{\sin\frac{\pi}{12}} + \frac{\sqrt{3}+1}{\cos\frac{\pi}{12}} = \\ &= \frac{4(\sqrt{3}-1)}{\sqrt{6}-\sqrt{2}} + \frac{4(\sqrt{3}+1)}{\sqrt{6}+\sqrt{2}} = \\ &= \frac{4(\sqrt{3}-1)}{\sqrt{2}(\sqrt{3}-1)} + \frac{4(\sqrt{3}+1)}{\sqrt{2}(\sqrt{3}+1)} = \\ &= \frac{4}{\sqrt{2}} + \frac{4}{\sqrt{2}} = \\ &= \frac{8}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2} = RHS \end{aligned}$$

so $x = \dfrac{\pi}{12}$ is a solution.

IB exam - problem 3 (b)

We can multiply both sides by $\sin x \cos x$ to get rid of denominators. We get

IB exam - problem 3 (b)

We can multiply both sides by $\sin x \cos x$ to get rid of denominators. We get

$$(\sqrt{3}-1)\cos x + (\sqrt{3}+1)\sin x = 4\sqrt{2}\sin x \cos x$$

IB exam - problem 3 (b)

We can multiply both sides by $\sin x \cos x$ to get rid of denominators. We get

$$(\sqrt{3}-1)\cos x + (\sqrt{3}+1)\sin x = 4\sqrt{2}\sin x \cos x$$

Now we can try to use the formula for sine of sums on the left hand side. The opposite side is $\sqrt{3}-1$, the adjacent side is $\sqrt{3}+1$. The hypotenuse is $2\sqrt{2}$.

IB exam - problem 3 (b)

We can multiply both sides by $\sin x \cos x$ to get rid of denominators. We get

$$(\sqrt{3}-1)\cos x + (\sqrt{3}+1)\sin x = 4\sqrt{2}\sin x \cos x$$

Now we can try to use the formula for sine of sums on the left hand side. The opposite side is $\sqrt{3}-1$, the adjacent side is $\sqrt{3}+1$. The hypotenuse is $2\sqrt{2}$. The angle then becomes $\frac{\pi}{12}$.

IB exam - problem 3 (b)

We can multiply both sides by $\sin x \cos x$ to get rid of denominators. We get

$$(\sqrt{3}-1)\cos x + (\sqrt{3}+1)\sin x = 4\sqrt{2}\sin x \cos x$$

Now we can try to use the formula for sine of sums on the left hand side. The opposite side is $\sqrt{3}-1$, the adjacent side is $\sqrt{3}+1$. The hypotenuse is $2\sqrt{2}$. The angle then becomes $\frac{\pi}{12}$. We divide both sides by $2\sqrt{2}$:

IB exam - problem 3 (b)

We can multiply both sides by $\sin x \cos x$ to get rid of denominators. We get

$$(\sqrt{3}-1)\cos x + (\sqrt{3}+1)\sin x = 4\sqrt{2}\sin x \cos x$$

Now we can try to use the formula for sine of sums on the left hand side. The opposite side is $\sqrt{3}-1$, the adjacent side is $\sqrt{3}+1$. The hypotenuse is $2\sqrt{2}$. The angle then becomes $\frac{\pi}{12}$. We divide both sides by $2\sqrt{2}$:

$$\frac{\sqrt{3}-1}{2\sqrt{2}}\cos x + \frac{\sqrt{3}+1}{2\sqrt{2}}\sin x = 2\sin x \cos x$$

IB exam - problem 3 (b)

We can multiply both sides by $\sin x \cos x$ to get rid of denominators. We get

$$(\sqrt{3}-1)\cos x + (\sqrt{3}+1)\sin x = 4\sqrt{2}\sin x \cos x$$

Now we can try to use the formula for sine of sums on the left hand side. The opposite side is $\sqrt{3}-1$, the adjacent side is $\sqrt{3}+1$. The hypotenuse is $2\sqrt{2}$. The angle then becomes $\frac{\pi}{12}$. We divide both sides by $2\sqrt{2}$:

$$\frac{\sqrt{3}-1}{2\sqrt{2}}\cos x + \frac{\sqrt{3}+1}{2\sqrt{2}}\sin x = 2\sin x \cos x$$

This is happens to be perfect for the right hand side as well.

IB exam - problem 3 (b)

We get:

IB exam - problem 3 (b)

We get:

$$\sin\frac{\pi}{12}\cos x + \cos\frac{\pi}{12}\sin x = \sin 2x$$

IB exam - problem 3 (b)

We get:

$$\sin\frac{\pi}{12}\cos x+\cos\frac{\pi}{12}\sin x=\sin 2x$$

Apply the formula for the sine of sum:

IB exam - problem 3 (b)

We get:

$$\sin\frac{\pi}{12}\cos x + \cos\frac{\pi}{12}\sin x = \sin 2x$$

Apply the formula for the sine of sum:

$$\sin\left(\frac{\pi}{12} + x\right) = \sin 2x$$

IB exam - problem 3 (b)

We get:

$$\sin\frac{\pi}{12}\cos x + \cos\frac{\pi}{12}\sin x = \sin 2x$$

Apply the formula for the sine of sum:

$$\sin\left(\frac{\pi}{12} + x\right) = \sin 2x$$

Now there are two ways to proceed.

IB exam - problem 3 (b)

We get:

$$\sin\frac{\pi}{12}\cos x + \cos\frac{\pi}{12}\sin x = \sin 2x$$

Apply the formula for the sine of sum:

$$\sin\left(\frac{\pi}{12} + x\right) = \sin 2x$$

Now there are two ways to proceed. We can move all terms to one side and use the formula for difference of sines:

$$\sin\left(\frac{\pi}{12} + x\right) - \sin 2x = 0$$

IB exam - problem 3 (b)

We get:

$$\sin\frac{\pi}{12}\cos x+\cos\frac{\pi}{12}\sin x=\sin 2x$$

Apply the formula for the sine of sum:

$$\sin\left(\frac{\pi}{12}+x\right)=\sin 2x$$

Now there are two ways to proceed. We can move all terms to one side and use the formula for difference of sines:

$$\sin\left(\frac{\pi}{12}+x\right)-\sin 2x=0$$

So:

$$2\sin\left(\frac{\frac{\pi}{12}-x}{2}\right)\cos\left(\frac{\frac{\pi}{12}+3x}{2}\right)=0$$

IB exam - problem 3 (b)

$$2\sin\left(\frac{\frac{\pi}{12}-x}{2}\right)\cos\left(\frac{\frac{\pi}{12}+3x}{2}\right)=0$$

IB exam - problem 3 (b)

$$2\sin\left(\frac{\frac{\pi}{12} - x}{2}\right)\cos\left(\frac{\frac{\pi}{12} + 3x}{2}\right) = 0$$

Solving the first part (sine) in the required interval gives:

$$\frac{\frac{\pi}{12} - x}{2} = 0$$

which gives $x = \frac{\pi}{12}$ an answer we already had.

IB exam - problem 3 (b)

$$2\sin\left(\frac{\frac{\pi}{12}-x}{2}\right)\cos\left(\frac{\frac{\pi}{12}+3x}{2}\right)=0$$

Solving the first part (sine) in the required interval gives:

$$\frac{\frac{\pi}{12}-x}{2}=0$$

which gives $x=\dfrac{\pi}{12}$ an answer we already had. Solving the second part (cosine) gives:

$$\frac{\frac{\pi}{12}+3x}{2}=\frac{\pi}{2}$$

IB exam - problem 3 (b)

$$2\sin\left(\frac{\frac{\pi}{12}-x}{2}\right)\cos\left(\frac{\frac{\pi}{12}+3x}{2}\right)=0$$

Solving the first part (sine) in the required interval gives:

$$\frac{\frac{\pi}{12}-x}{2}=0$$

which gives $x=\frac{\pi}{12}$ an answer we already had. Solving the second part (cosine) gives:

$$\frac{\frac{\pi}{12}+3x}{2}=\frac{\pi}{2}$$

So $x=\frac{11\pi}{36}$ and this is our second solution.

IB exam - problem 3 (b)

Let's go back to:

$$\sin\left(\frac{\pi}{12}+x\right)=\sin 2x$$

and let's discuss another approach.

IB exam - problem 3 (b)

Let's go back to:

$$\sin\left(\frac{\pi}{12} + x\right) = \sin 2x$$

and let's discuss another approach.

We have sine function on both sides. Of course if the arguments are the same then the values will also be the same, so we can have:

IB exam - problem 3 (b)

Let's go back to:

$$\sin\left(\frac{\pi}{12}+x\right)=\sin 2x$$

and let's discuss another approach.

We have sine function on both sides. Of course if the arguments are the same then the values will also be the same, so we can have:

$$\frac{\pi}{12}+x=2x$$

and this gives $x=\frac{\pi}{12}$.

IB exam - problem 3 (b)

Let's go back to:

$$\sin\left(\frac{\pi}{12} + x\right) = \sin 2x$$

and let's discuss another approach.
We have sine function on both sides. Of course if the arguments are the same then the values will also be the same, so we can have:

$$\frac{\pi}{12} + x = 2x$$

and this gives $x = \frac{\pi}{12}$.
But two sines are also equal if one argument is π minus the other ($\sin\alpha = \sin(\pi - \alpha)$), so we could also have:

IB exam - problem 3 (b)

Let's go back to:

$$\sin\left(\frac{\pi}{12}+x\right)=\sin 2x$$

and let's discuss another approach.
We have sine function on both sides. Of course if the arguments are the same then the values will also be the same, so we can have:

$$\frac{\pi}{12}+x=2x$$

and this gives $x=\frac{\pi}{12}$.
But two sines are also equal if one argument is π minus the other ($\sin\alpha=\sin(\pi-\alpha)$), so we could also have:

$$\frac{\pi}{12}+x=\pi-2x$$

and this gives the solution $x=\frac{11\pi}{36}$.

IB exam - problem 3 (b)

Note that in general if we have:

$$\sin\alpha = \sin\beta$$

Then:

$$\alpha = \beta \quad \text{or} \quad \alpha = \pi - \beta \quad \text{or} \quad \alpha = 2\pi + \beta \quad \text{or} \quad \alpha = 3\pi - \beta \quad \text{or} \quad \ldots$$

IB exam - problem 3 (b)

Note that in general if we have:

$$\sin\alpha = \sin\beta$$

Then:

$$\alpha = \beta \quad \text{or} \quad \alpha = \pi - \beta \quad \text{or} \quad \alpha = 2\pi + \beta \quad \text{or} \quad \alpha = 3\pi - \beta \quad \text{or} \quad \dots$$

If we have:

$$\cos\alpha = \cos\beta$$

Then

$$\alpha = \beta \quad \text{or} \quad \alpha = 2\pi - \beta \quad \text{or} \quad \alpha = 2\pi + \beta \quad \text{or} \quad \alpha = 4\pi - \beta \quad \text{or} \quad \dots$$

IB exam - problem 4

[Maximum mark: 22]

(a) Solve $2\sin(x+60^\circ) = \cos(x+30^\circ)$, $0^\circ \le x \le 180^\circ$. [5]

(b) Show that $\sin 105^\circ + \cos 105^\circ = \dfrac{1}{\sqrt{2}}$. [3]

IB exam - problem 4

[Maximum mark: 22]

(a) Solve $2\sin(x+60^\circ) = \cos(x+30^\circ)$, $0^\circ \leq x \leq 180^\circ$. [5]

(b) Show that $\sin 105^\circ + \cos 105^\circ = \frac{1}{\sqrt{2}}$. [3]

Note that this is part of a longer question which involved topics we haven't covered yet.

IB exam - problem 4

[Maximum mark: 22]

(a) Solve $2\sin(x+60^\circ) = \cos(x+30^\circ)$, $0^\circ \leq x \leq 180^\circ$. [5]

(b) Show that $\sin 105^\circ + \cos 105^\circ = \frac{1}{\sqrt{2}}$. [3]

Note that this is part of a longer question which involved topics we haven't covered yet.
For part (a) it makes sense to apply the formula for the sine and cosine of sums.

IB exam - problem 4 (a)

We get:

$$2(\sin x \cos 60^\circ + \sin 60^\circ \cos x) = \cos x \cos 30^\circ - \sin x \sin 30^\circ$$

IB exam - problem 4 (a)

We get:

$$2(\sin x \cos 60^\circ + \sin 60^\circ \cos x) = \cos x \cos 30^\circ - \sin x \sin 30^\circ$$

Now this becomes:

IB exam - problem 4 (a)

We get:

$$2(\sin x \cos 60^\circ + \sin 60^\circ \cos x) = \cos x \cos 30^\circ - \sin x \sin 30^\circ$$

Now this becomes:

$$\sin x + \sqrt{3}\cos x = \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$$

IB exam - problem 4 (a)

We get:

$$2(\sin x \cos 60^\circ + \sin 60^\circ \cos x) = \cos x \cos 30^\circ - \sin x \sin 30^\circ$$

Now this becomes:

$$\sin x + \sqrt{3} \cos x = \frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x$$

This gives:

$$3 \sin x = -\sqrt{3} \cos x$$

IB exam - problem 4 (a)

We get:

$$2(\sin x \cos 60^\circ + \sin 60^\circ \cos x) = \cos x \cos 30^\circ - \sin x \sin 30^\circ$$

Now this becomes:

$$\sin x + \sqrt{3} \cos x = \frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x$$

This gives:

$$3 \sin x = -\sqrt{3} \cos x$$

which is equivalent to:

$$\tan x = -\frac{\sqrt{3}}{3}$$

IB exam - problem 4 (a)

We get:

$$2(\sin x \cos 60^\circ + \sin 60^\circ \cos x) = \cos x \cos 30^\circ - \sin x \sin 30^\circ$$

Now this becomes:

$$\sin x + \sqrt{3} \cos x = \frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x$$

This gives:

$$3 \sin x = -\sqrt{3} \cos x$$

which is equivalent to:

$$\tan x = -\frac{\sqrt{3}}{3}$$

In the given interval we have only one solution, namely $x = 150^\circ$.

IB exam - problem 4 (b)

For part (b) we can calculate both sine and cosine separately using angles 60° and 45°, so that we have:

IB exam - problem 4 (b)

For part (b) we can calculate both sine and cosine separately using angles 60° and 45°, so that we have:

$$\begin{aligned} LHS &= \sin 105^\circ + \cos 105^\circ = \\ &= \sin(60^\circ + 45^\circ) + \cos(60^\circ + 45^\circ) = \\ &= \sin 60^\circ \cos 45^\circ + \sin 45^\circ \cos 60^\circ + \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ = \\ &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} = RHS \end{aligned}$$

□

IB exam - problem 4 (b)

Alternatively we can change $\cos 105^\circ$ into $-\sin 15^\circ$ and apply the formula for difference of sines:

IB exam - problem 4 (b)

Alternatively we can change $\cos 105^\circ$ into $-\sin 15^\circ$ and apply the formula for difference of sines:

$$\begin{aligned} LHS &= \sin 105^\circ + \cos 105^\circ = \\ &= \sin 105^\circ - \sin 15^\circ = \\ &= 2\sin 45^\circ \cos 60^\circ = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} = RHS \end{aligned}$$

□

That's it. That's all the basics. Note that all IB questions will require you to solve trigonometric equations in a specific interval, but it's still useful to be aware of the general solution. Make sure you understand all examples discussed in the presentation. If you have any questions you can email me at t.j.lechowski@gmail.com.