5 Solve for x:

a $4^x - 6(2^x) + 8 = 0$ **b** $4^x - 2^x - 2 = 0$ **c** $9^x - 12(3^x) + 27 = 0$

d $9^x = 3^x + 6$ **e** $25^x - 23(5^x) - 50 = 0$ **f** $49^x + 1 = 2(7^x)$

g $3^x - 1 = 6(3^{-x})$ **h** $2(4^x) - 5(2^x) + 2 = 0$

i $4(9^x) - 35(3^x) = 9$ **j** $4^{x+1} + 2 = 9(2^x)$

k $3^{2x-1} = 3^x + 18$ **l** $4^x + 2^{x+\frac{1}{2}} = 4$

Check your answers using technology.

GRAPHICS CALCULATOR INSTRUCTIONS

6 Solve simultaneously: $4^x = 8^y$ and $9^y = \dfrac{243}{3^x}$.

D EXPONENTIAL FUNCTIONS

We have already seen how to evaluate a^n for any $n \in \mathbb{Q}$.

But how do we evaluate a^n when $n \in \mathbb{R}$, so n is real but not necessarily rational?

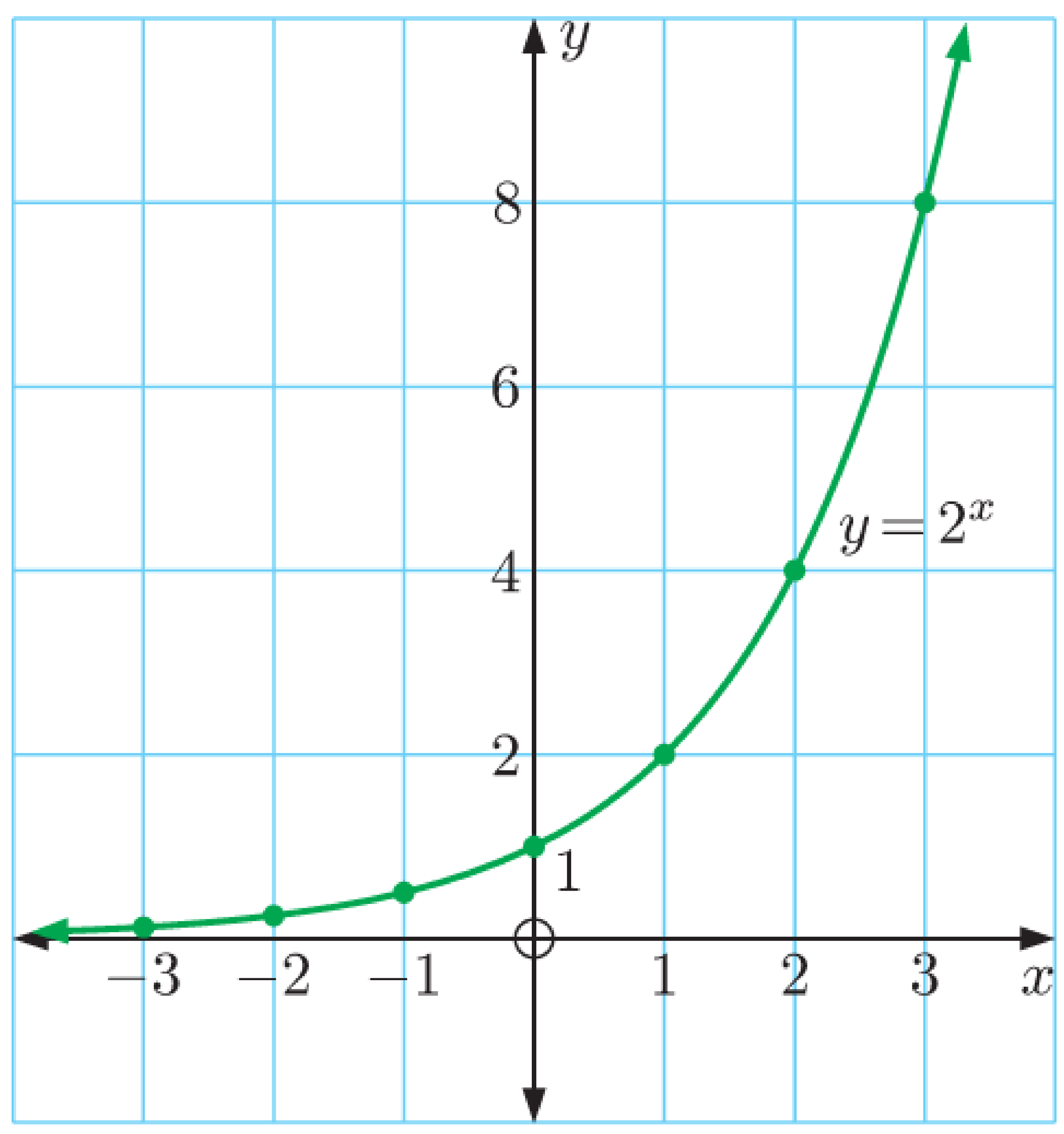
To answer this question, we can study the graphs of exponential functions.

> The most simple **exponential function** has the form $y = a^x$ where $a > 0$, $a \neq 1$.

For example, $y = 2^x$ is an exponential function.

We construct a table of values from which we graph the function:

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



As x becomes large and negative, the graph of $y = 2^x$ approaches the x-axis from above. However, it never touches the x-axis, since 2^x becomes very small but never zero.

So, as $x \to -\infty$, $y \to 0^+$.

$y = 0$ is therefore a **horizontal asymptote**.

Plotting $y = a^x$ for $x \in \mathbb{Q}$ suggests a smooth, continuous curve. This allows us to complete the curve for all $x \in \mathbb{R}$, giving meaning to a^x for irrational values of x.

INVESTIGATION 1 GRAPHS OF EXPONENTIAL FUNCTIONS

In this Investigation we examine the graphs of various families of exponential functions. You can use the **graphing package** or your calculator.

GRAPHING PACKAGE

What to do:

1 **a** State the transformation which maps $y = a^x$ to $y = a^x + k$.

b Predict the effect, if any, this transformation will have on:

i the shape of the graph **ii** the position of the graph

iii the horizontal asymptote.

c Check your predictions by graphing $y = 2^x$, $y = 2^x + 1$, and $y = 2^x - 2$ on the same set of axes.

2 **a** State the transformation which maps $y = a^x$ to $y = a^{x-h}$.

b Predict the effect, if any, this transformation will have on:

i the shape of the graph **ii** the position of the graph

iii the horizontal asymptote.

c Check your predictions by graphing $y = 2^x$, $y = 2^{x-1}$, $y = 2^{x+2}$, and $y = 2^{x-3}$ on the same set of axes.

3 **a** State the transformation which maps $y = a^x$ to $y = p \times a^x$, $p > 0$.

b Predict the effect, if any, this transformation will have on:

i the shape of the graph **ii** the position of the graph

iii the horizontal asymptote.

c Check your predictions by graphing $y = 2^x$, $y = 3 \times 2^x$, and $y = \frac{1}{2} \times 2^x$ on the same set of axes.

4 **a** State the transformation which maps $y = a^x$ to $y = -a^x$.

b Predict what the graph of $y = -2^x$ will look like, and check your answer using technology.

5 **a** State the transformation which maps $y = a^x$ to $y = a^{qx}$, $q > 0$.

b Predict the effect, if any, this transformation will have on:

i the shape of the graph **ii** the position of the graph

iii the horizontal asymptote.

c Notice that $2^{2x} = (2^2)^x = 4^x$ and $2^{3x} = (2^3)^x = 8^x$.
Check your predictions by graphing $y = 2^x$, $y = 4^x$, and $y = 8^x$ on the same set of axes.

6 **a** State the transformation which maps $y = a^x$ to $y = a^{-x}$.

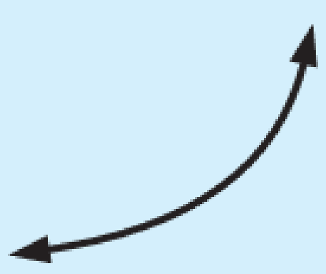
b Notice that $2^{-x} = (2^{-1})^x = \left(\frac{1}{2}\right)^x$.

Predict what the graph of $y = \left(\frac{1}{2}\right)^x$ will look like, and check your answer using technology.

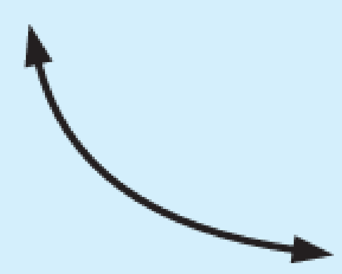
From your **Investigation** you should have discovered that:

For the general exponential function $\boldsymbol{y = p \times a^{x-h} + k}$ where $a > 0$, $a \neq 1$, $p \neq 0$:

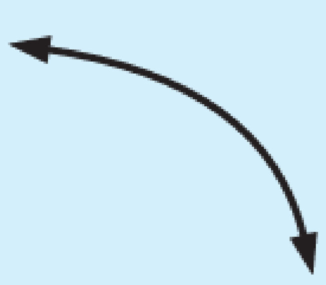
- a controls how steeply the graph increases or decreases.
- h controls horizontal translation.
- k controls vertical translation.
- The equation of the horizontal asymptote is $y = k$.
- If $p > 0$, $a > 1$ the function is increasing.



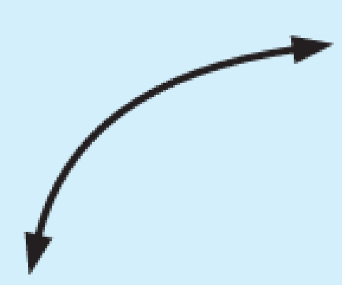
- If $p > 0$, $0 < a < 1$ the function is decreasing.



- If $p < 0$, $a > 1$ the function is decreasing.

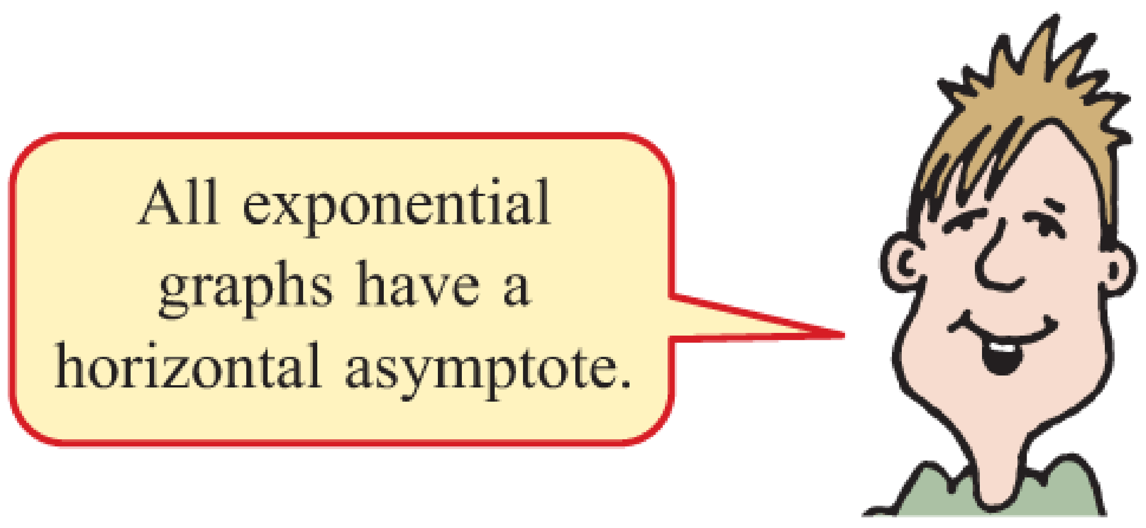


- If $p < 0$, $0 < a < 1$ the function is increasing.



We can sketch the graphs of exponential functions using:

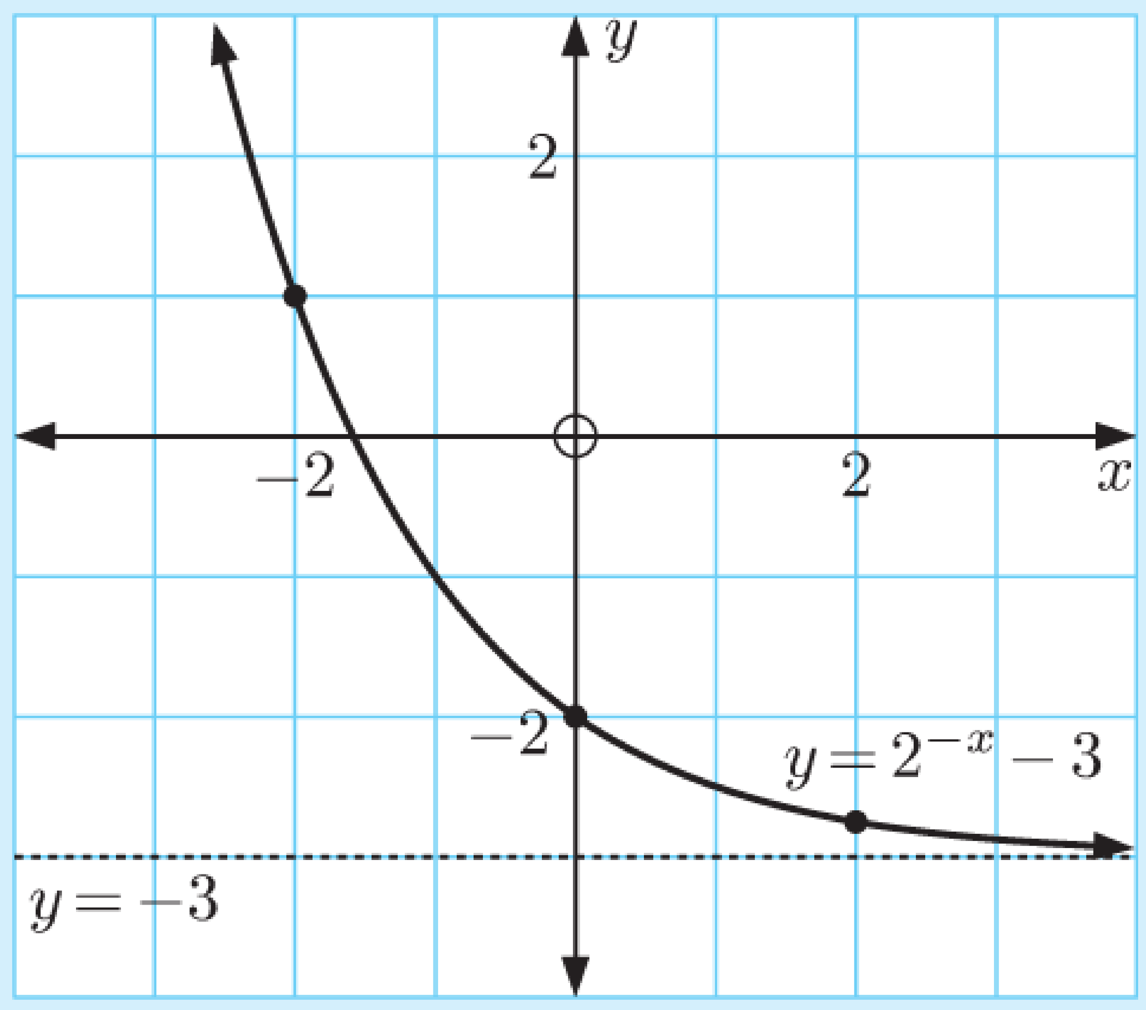
- the horizontal asymptote
- the y-intercept
- two other points.



Example 12 — Self Tutor

Sketch the graph of $y = 2^{-x} - 3$.
Hence state the domain and range of $f(x) = 2^{-x} - 3$.

For $y = 2^{-x} - 3$,
the horizontal asymptote is $y = -3$.



When $x = 0$, $y = 2^0 - 3$
$= 1 - 3$
$= -2$

$\therefore$ the y-intercept is -2.

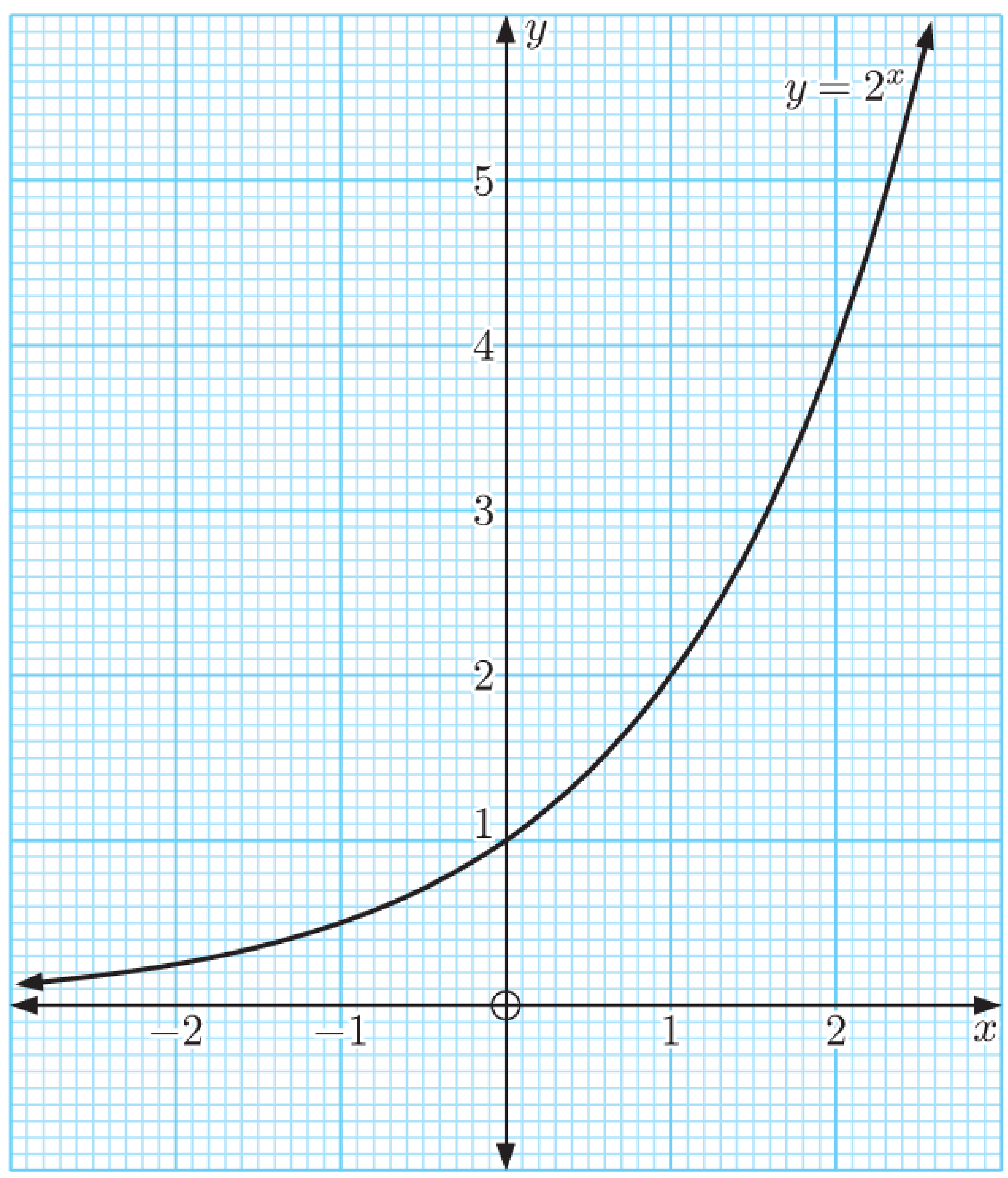
When $x = 2$, $y = 2^{-2} - 3$
$= \frac{1}{4} - 3$
$= -2\frac{3}{4}$

When $x = -2$, $y = 2^2 - 3 = 1$

The domain is $\{x \mid x \in \mathbb{R}\}$. The range is $\{y \mid y > -3\}$.

EXERCISE 2D

1 Consider the graph of $y = 2^x$ alongside.



a Use the graph to estimate the value of:

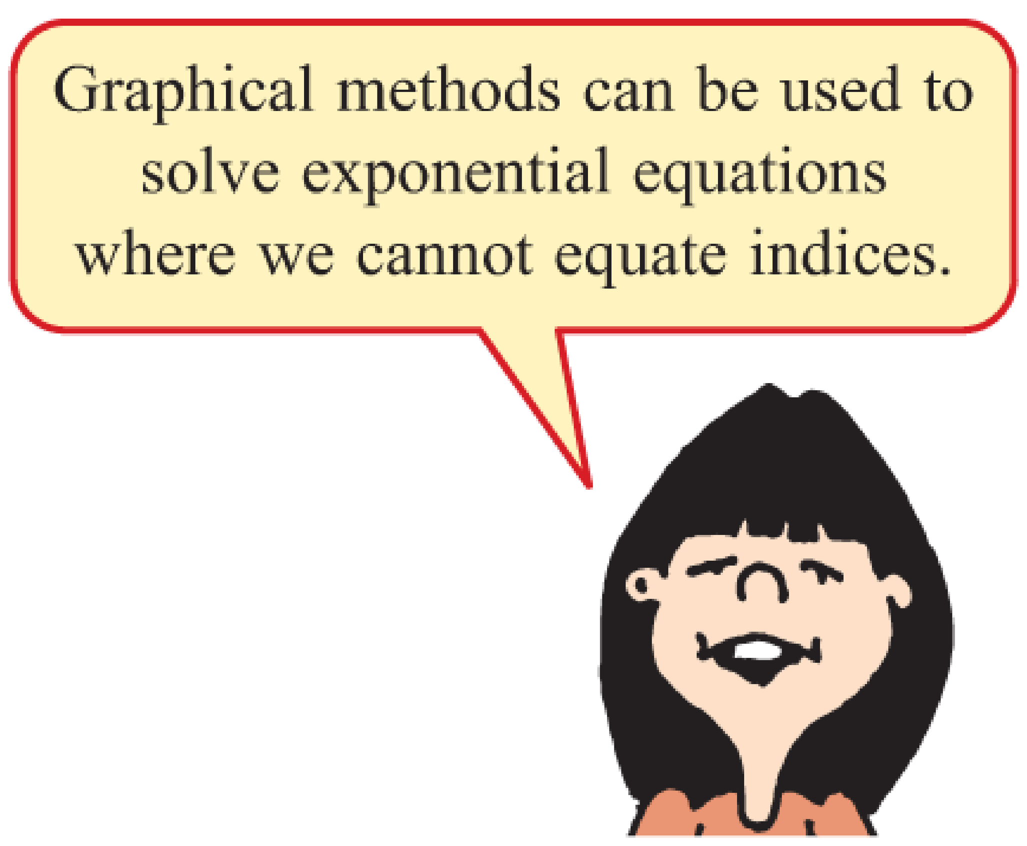
i $2^{\frac{1}{2}}$ or $\sqrt{2}$ **ii** $2^{0.8}$

iii $2^{1.5}$ **iv** $2^{-\sqrt{2}}$

b Use the graph to estimate the solution to:

i $2^x = 3$ **ii** $2^x = 0.6$

c Use the graph to explain why $2^x = 0$ has no solutions.

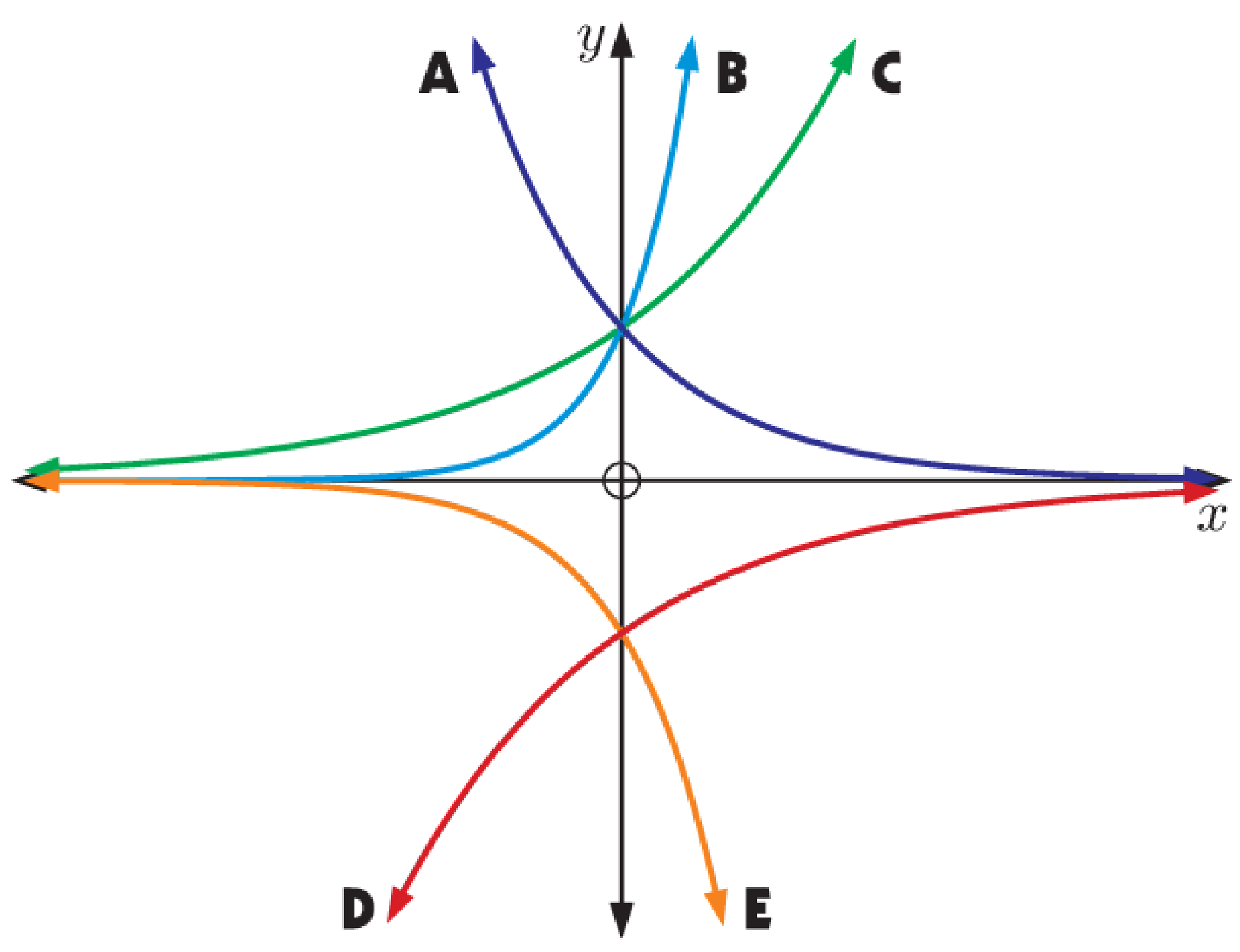


2 Match each function with its graph:

a $y = 2^x$ **b** $y = 10^x$

c $y = -5^x$ **d** $y = \left(\frac{1}{3}\right)^x$

e $y = -\left(\frac{1}{2}\right)^x$



3 Use a transformation to help sketch each pair of functions on the same set of axes:

GRAPHING PACKAGE

a $y = 2^x$ and $y = 2^x - 2$ **b** $y = 2^x$ and $y = 2^{-x}$

c $y = 2^x$ and $y = 2^{x-2}$ **d** $y = 2^x$ and $y = 2(2^x)$

4 Draw freehand sketches of the following pairs of graphs:

a $y = 3^x$ and $y = 3^{-x}$ **b** $y = 3^x$ and $y = 3^x + 1$

c $y = 3^x$ and $y = -3^x$ **d** $y = 3^x$ and $y = 3^{x-1}$

5 State the equation of the horizontal asymptote of:

a $y = 3^{-x}$ **b** $y = 2^x - 1$ **c** $y = 3 - 2^{-x}$

d $y = 4 \times 2^x + 2$ **e** $y = 5 \times 3^{x+2}$ **f** $y = -2 \times 3^{1-x} - 4$

6 Consider the exponential function $f(x) = 3^x - 2$.

a Find: **i** $f(0)$ **ii** $f(2)$ **iii** $f(-2)$

b State the equation of the horizontal asymptote.

c Sketch the graph of the function.

d State the domain and range of the function.

7 Consider the function $g(x) = 3 \times \left(\frac{1}{2}\right)^x + 4$.

a Find: **i** $g(0)$ **ii** $g(2)$ **iii** $g(-2)$

b State the equation of the horizontal asymptote.

c Sketch the graph of the function.

d State the domain and range of the function.

8 Consider the function $h(x) = -2^{x-3} + 1$.

a Find: **i** $h(0)$ **ii** $h(3)$ **iii** $h(6)$

b State the equation of the horizontal asymptote.

c Sketch the graph of the function.

d State the domain and range of the function.

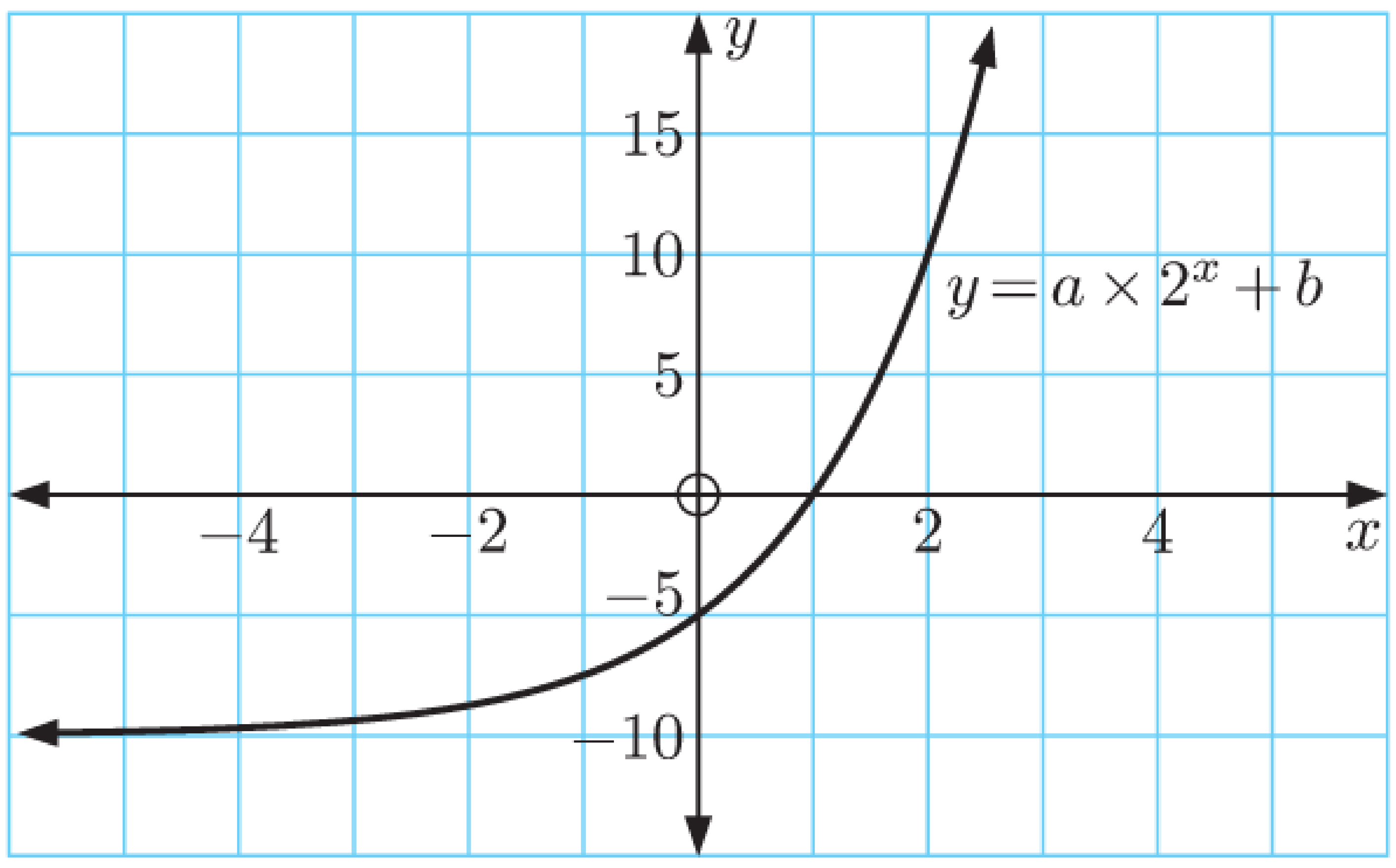
9 For each of the functions below:

  i Sketch the graph of the function.
  ii State the domain and range.
  iii Use your calculator to find the value of y when $x = \sqrt{2}$.
  iv Discuss the behaviour of y as $x \to \pm\infty$.
  v Determine the horizontal asymptote.

a $y = 2^x + 1$ **b** $y = 2 - 2^x$ **c** $y = 2^{-x} + 3$ **d** $y = 3 - 2^{-x}$

10 The graph alongside shows the curve $y = a \times 2^x + b$, where a and b are constants.

  a Find the values of a and b.
  b Find y when $x = 6$.



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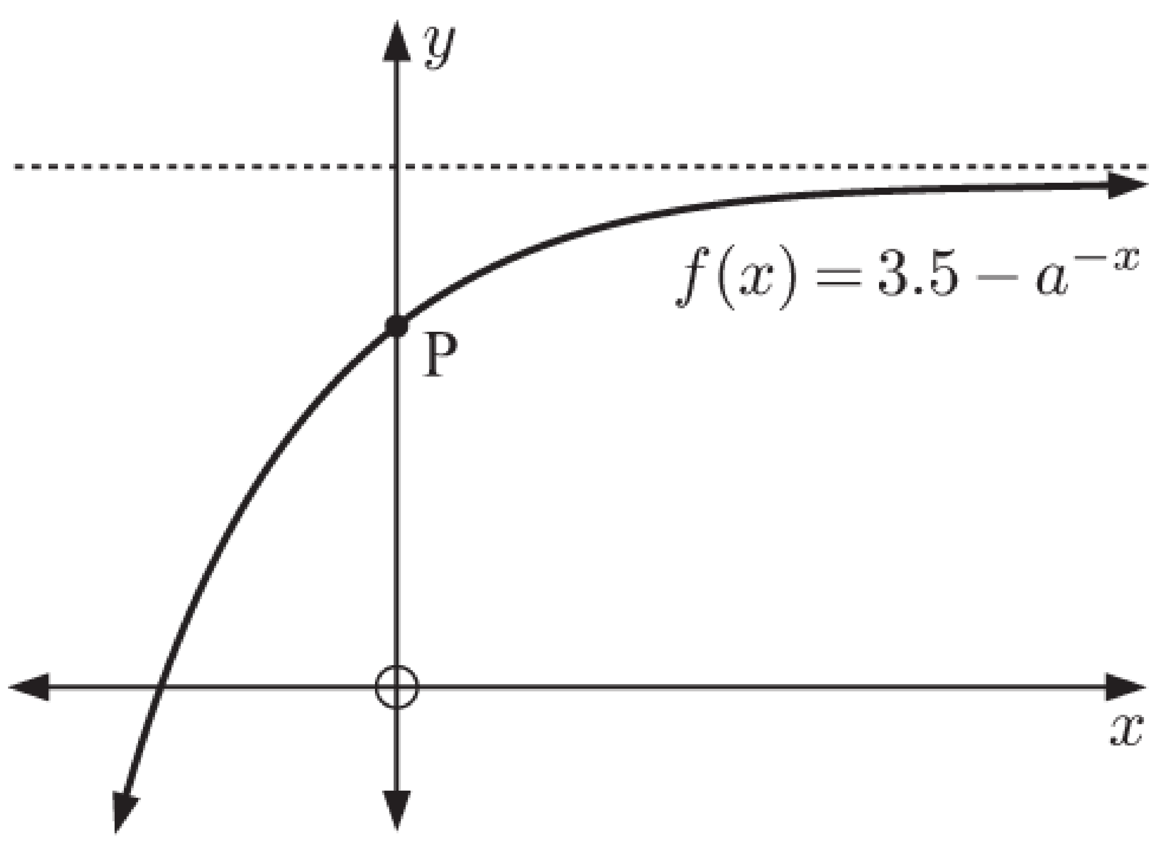


This graph shows the function $f(x) = 3.5 - a^{-x}$, where a is a positive constant.
The point $(-1, 2)$ lies on the graph.

  a Write down the coordinates of P.
  b Find the value of a.
  c Find the equation of the horizontal asymptote.

12 Find the domain and range of:

a $y = 2^{x^2+1}$ **b** $y = \dfrac{1}{3^x - 1}$ **c** $y = \sqrt{5^x - 5}$

13 Let $f(x) = 3^x - 9$ and $g(x) = \sqrt{x}$.

  a Find $(f \circ g)(x)$, and state its domain and range.
  b Find $(g \circ f)(x)$, and state its domain and range.
  c Solve:
    i $(f \circ g)(x) = 0$ **ii** $(g \circ f)(x) = 3\sqrt{2}$

14 Suppose $f(x) = 2^x - 3$ and $g(x) = 1 + 2^{-x}$.

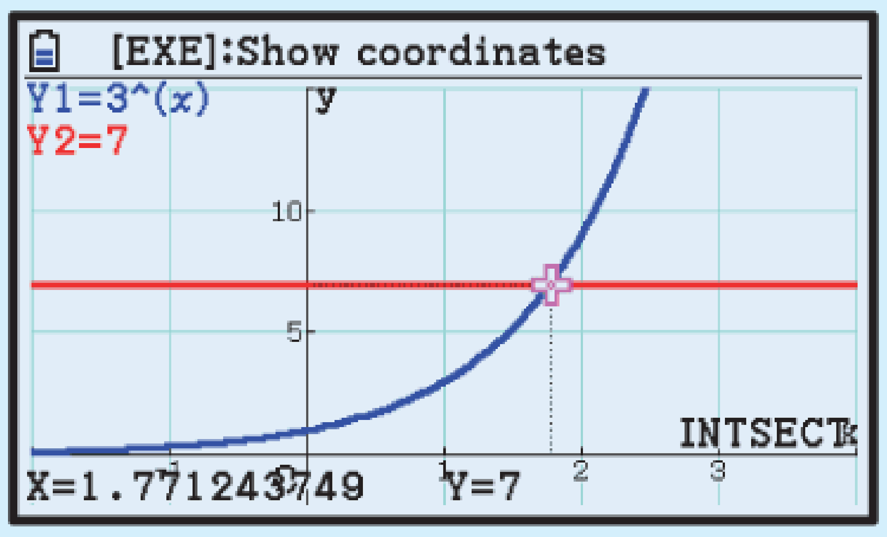
  a For each function, find the:
    i horizontal asymptote **ii** range **iii** y-intercept.
  b Graph the functions on the same set of axes.
  c Find the exact y-coordinate of the point where the graphs intersect.

Example 13 — Self Tutor

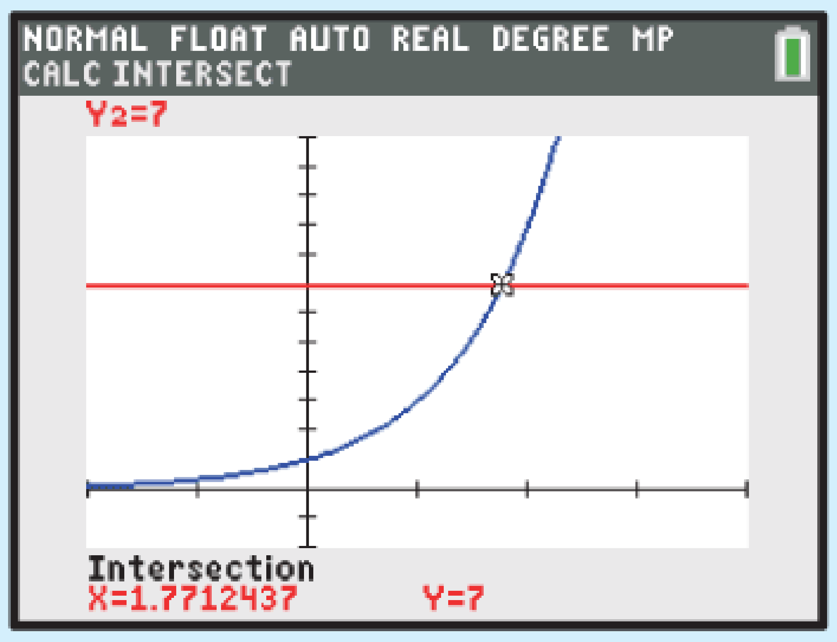
Use technology to solve the equation $3^x = 7$.

We graph $\mathrm{Y}_1 = 3^{\mathrm{X}}$ and $\mathrm{Y}_2 = 7$ on the same set of axes, and find their point of intersection.

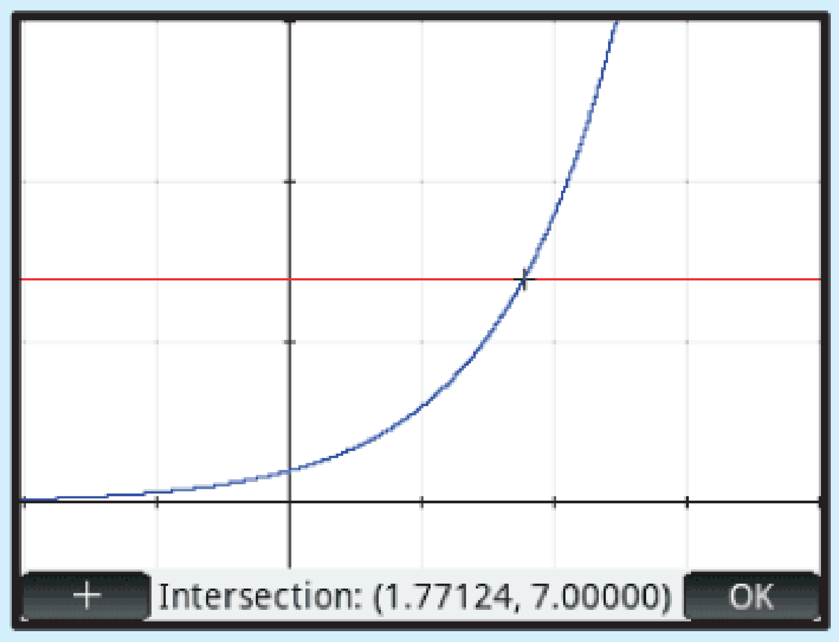
Casio fx-CG50



TI-84 Plus CE



HP Prime



The solution is $x \approx 1.77$.

15 Use technology to solve:

a $2^x = 11$ **b** $3^x = 15$ **c** $4^x + 5 = 10$

d $3^{x+2} = 4$ **e** $5 \times 2^x = 18$ **f** $3^{-x} = 0.9$

g $2 \times 3^{x-2} = 168$ **h** $26 \times (0.95)^x = 2$ **i** $2000 \times (1.03)^x = 5000$

DISCUSSION

For the exponential function $y = a^x$, why do we choose to specify $a > 0$?

What would the graph of $y = (-2)^x$ look like? What is its domain and range?

E GROWTH AND DECAY

In this Section we will examine situations where quantities are either increasing or decreasing exponentially. These situations are known as **growth** and **decay** modelling, and occur frequently in the world around us.

Populations of animals, people, and bacteria usually *grow* in an exponential way.

Radioactive substances, cooling, and items that depreciate in value, usually *decay* exponentially.

For the exponential function $y = p \times a^{x-h} + k$ where $a, p > 0$, $a \neq 1$, we see:

- growth if $a > 1$
- decay if $a < 1$.

GROWTH

Consider a population of 100 mice which under favourable conditions is increasing by 20% each week.

To increase a quantity by 20%, we multiply it by 1.2.

13 $3\sin x - 5\cos x \approx \sqrt{34}\cos(x + 3.68)$

14 **a** $2\sin x + \sqrt{3}\cos x \approx \sqrt{7}\sin(x + 0.714)$

b **i** $A = \sqrt{7}$ **ii** $b \approx 2.43$

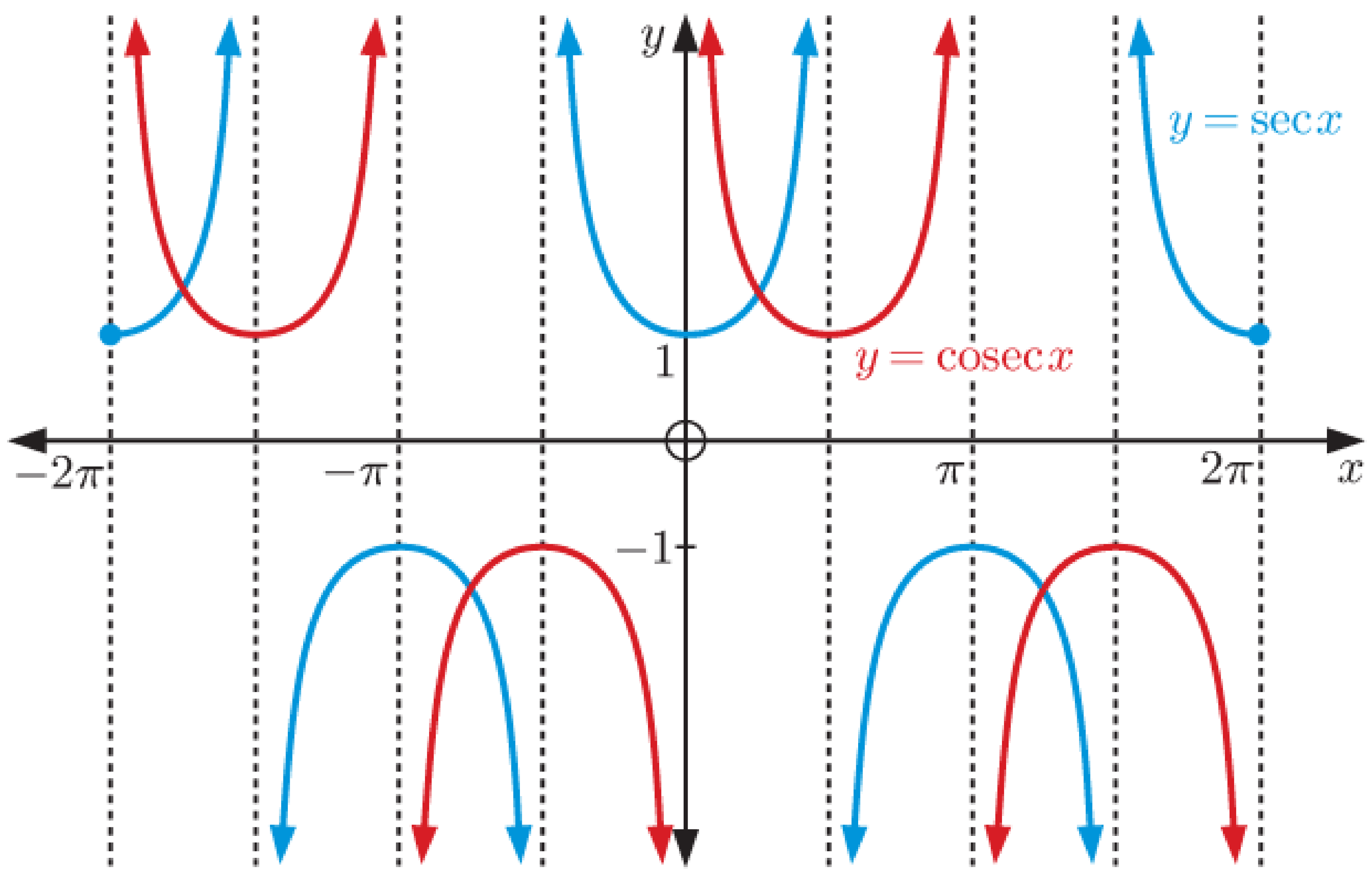
15 $\frac{\pi}{4}$

REVIEW SET 1B

1 $\sin x = -\frac{2\sqrt{2}}{3}$, $\tan x = 2\sqrt{2}$, $\operatorname{cosec} x = -\frac{3}{2\sqrt{2}}$,

$\sec x = -3$, $\cot x = \frac{1}{2\sqrt{2}}$

2 **a**



b translation $\frac{\pi}{2}$ units right

3 $x = -\frac{5\pi}{6}$ or $\frac{\pi}{6}$ **4** **a** $x = \frac{\sqrt{3}}{2}$ **b** $x = 2 + \frac{1}{\sqrt{3}}$

5 **a** $\sec x$ **b** $\sin x$ **c** $\cos x$

6 **a** $\cos\theta$ **b** $-\sin\theta$ **c** $5\cos^2\theta$ **d** $-\cos\theta$

e $\operatorname{cosec}\theta$ **f** $\sin 2\theta$

7 **a** $\frac{120}{169}$ **b** $\frac{119}{169}$ **c** $\frac{120}{119}$

10 **a** $x = -\frac{2\pi}{3}, -\frac{\pi}{2}, -\frac{\pi}{3}$, or $\frac{\pi}{2}$ **b** $\theta = \frac{\pi}{3}$

11 $\sin\left(\theta + \frac{\pi}{6}\right) = \frac{3\sqrt{3}-\sqrt{7}}{8}$ **12** $\tan\theta = \frac{9}{19}$

13 $3\sin x + 4\cos x \approx 5\sin(x + 0.927)$ **14** 1.5 m

15 **b** $y = 2\sec 2x$ has range $\{y \mid y \leqslant -2 \text{ or } y \geqslant 2\}$

$\therefore \quad \frac{1}{1+\sqrt{2}\sin x} + \frac{1}{1-\sqrt{2}\sin x} = 1$ has no solutions.

EXERCISE 2A

1 **a** $2^{\frac{1}{5}}$ **b** $2^{-\frac{1}{5}}$ **c** $2^{\frac{3}{2}}$ **d** $2^{\frac{5}{2}}$ **e** $2^{-\frac{1}{3}}$

f $2^{\frac{4}{3}}$ **g** $2^{\frac{3}{2}}$ **h** $2^{\frac{3}{2}}$ **i** $2^{-\frac{4}{3}}$ **j** $2^{-\frac{3}{2}}$

2 **a** $3^{\frac{1}{3}}$ **b** $3^{-\frac{1}{3}}$ **c** $3^{\frac{1}{4}}$ **d** $3^{\frac{3}{2}}$ **e** $3^{-\frac{5}{2}}$

3 **a** $7^{\frac{1}{3}}$ **b** $3^{\frac{3}{4}}$ **c** $2^{\frac{4}{5}}$ **d** $2^{\frac{5}{3}}$ **e** $7^{\frac{2}{7}}$

f $7^{-\frac{1}{3}}$ **g** $3^{-\frac{3}{4}}$ **h** $2^{-\frac{4}{5}}$ **i** $2^{-\frac{5}{3}}$ **j** $7^{-\frac{2}{7}}$

4 **a** $x^{\frac{1}{2}}$ **b** $x^{\frac{3}{2}}$ **c** $x^{-\frac{1}{2}}$ **d** $x^{\frac{5}{2}}$ **e** $x^{-\frac{3}{2}}$

5 **a** ≈ 2.28 **b** ≈ 0.435 **c** ≈ 1.68 **d** ≈ 1.93

e ≈ 0.523

6 **a** $\sqrt[3]{5}$ **b** $\frac{1}{\sqrt{3}}$ **c** $9\sqrt{3}$ **d** $m\sqrt{m}$ **e** $x^3\sqrt{x}$

7 **a** 8 **b** 32 **c** 8 **d** 125 **e** 4

f $\frac{1}{2}$ **g** $\frac{1}{27}$ **h** $\frac{1}{16}$ **i** $\frac{1}{81}$ **j** $\frac{1}{25}$

EXERCISE 2B

1 **a** 1 **b** x **c** $x^{\frac{1}{2}}$ or $\sqrt{x}$

2 **a** $x^5 + 2x^4 + x^2$ **b** $2^{2x} + 2^x$ **c** $x + 1$

d $7^{2x} + 2(7^x)$ **e** $2(3^x) - 1$ **f** $x^2 + 2x + 3$

g $1 + 5(2^{-x})$ **h** $5^x + 1$ **i** $x^{\frac{3}{2}} + x^{\frac{1}{2}} + 1$

j $3^{2x} + 5(3^x) + 1$ **k** $2x^{\frac{3}{2}} - x^{\frac{1}{2}} + 5$ **l** $2^{3x} - 3(2^{2x}) - 1$

3 **a** $2^{2x} + 2^{x+1} - 3$ **b** $3^{2x} + 7(3^x) + 10$

c $5^{2x} - 6(5^x) + 8$ **d** $2^{2x} + 6(2^x) + 9$

e $3^{2x} - 2(3^x) + 1$ **f** $4^{2x} + 14(4^x) + 49$

g $x - 4$ **h** $4^x - 9$ **i** $x - \frac{1}{x}$ **j** $x^2 + 4 + \frac{4}{x^2}$

k $7^{2x} - 2 + 7^{-2x}$ **l** $25 - 10(2^{-x}) + 2^{-2x}$

4 **a** $5^x(5^x + 1)$ **b** $10(3^n)$ **c** $7^n(1 + 7^{2n})$

d $5(5^n - 1)$ **e** $6(6^{n+1} - 1)$ **f** $16(4^n - 1)$

g $2^n(2^n - 8)$ **h** $\frac{5}{2}(2^n)$ **i** $\frac{9}{2}(2^{2n})$

5 **a** $(3^x + 2)(3^x - 2)$ **b** $(2^x + 5)(2^x - 5)$

c $(4 + 3^x)(4 - 3^x)$ **d** $(5 + 2^x)(5 - 2^x)$

e $(3^x + 2^x)(3^x - 2^x)$ **f** $(2^x + 3)^2$

g $(3^x + 5)^2$ **h** $(2^x - 7)^2$ **i** $(5^x - 2)^2$

6 **a** $(2^x + 1)(2^x - 2)$ **b** $(3^x + 3)(3^x - 2)$

c $(2^x - 3)(2^x - 4)$ **d** $(2^x + 3)(2^x + 6)$

e $(2^x + 4)(2^x - 5)$ **f** $(3^x + 2)(3^x + 7)$

g $(3^x + 5)(3^x - 1)$ **h** $(5^x + 2)(5^x - 1)$

i $(7^x - 4)(7^x - 3)$

7 **a** 2^n **b** 10^a **c** 3^b **d** $\frac{1}{5^n}$ **e** 5^x

f $(\frac{3}{4})^a$ **g** $(\frac{8}{3})^k$ **h** 5 **i** 5^n

8 **a** $3^m + 1$ **b** $1 + 6^n$ **c** $4^n + 2^n$ **d** $4^x - 1$

e 6^n **f** 5^n **g** 4 **h** $2^n - 1$ **i** $\frac{1}{2}$

9 **a** $n\,2^{n+1}$ **b** -3^{n-1}

EXERCISE 2C

1 **a** $x = 5$ **b** $x = 2$ **c** $x = 4$ **d** $x = 0$

e $x = -1$ **f** $x = \frac{1}{2}$ **g** $x = -3$ **h** $x = 2$

i $x = -3$ **j** $x = -4$ **k** $x = 2$ **l** $x = \frac{3}{4}$

2 **a** $x = \frac{5}{3}$ **b** $x = -\frac{3}{2}$ **c** $x = -\frac{3}{2}$ **d** $x = -\frac{1}{2}$

e $x = -\frac{2}{3}$ **f** $x = -\frac{5}{4}$ **g** $x = \frac{3}{2}$ **h** $x = \frac{5}{2}$

i $x = \frac{1}{8}$ **j** $x = \frac{9}{2}$ **k** $x = -4$ **l** $x = -\frac{7}{2}$

m $x = 0$ **n** $x = \frac{7}{2}$ **o** $x = -\frac{2}{3}$ **p** $x = -6$

3 **a** $x = \frac{1}{7}$ **b** no solution **c** $x = \frac{5}{2}$

d $x = \frac{1}{3}$ **e** $x = -\frac{1}{4}$ **f** $x = -1$ or 3

4 **a** $x = 3$ **b** $x = 2$ **c** $x = -1$ **d** $x = 2$

e $x = -2$ **f** $x = -2$

5 **a** $x = 1$ or 2 **b** $x = 1$ **c** $x = 1$ or 2

d $x = 1$ **e** $x = 2$ **f** $x = 0$

g $x = 1$ **h** $x = 1$ or -1 **i** $x = 2$

j $x = -2$ or 1 **k** $x = 2$ **l** $x = \frac{1}{2}$

6 $x = \frac{15}{7}$, $y = \frac{10}{7}$

EXERCISE 2D

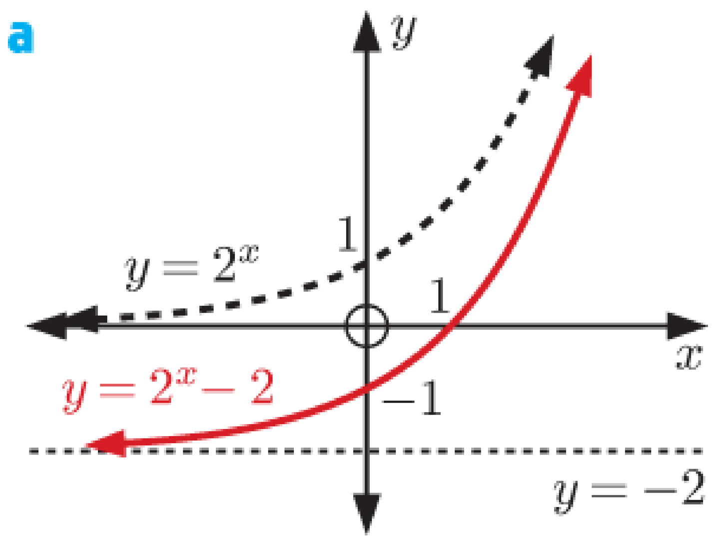
1 **a** **i** ≈ 1.4 **ii** ≈ 1.7 **iii** ≈ 2.8 **iv** ≈ 0.4

b **i** $x \approx 1.6$ **ii** $x \approx -0.7$

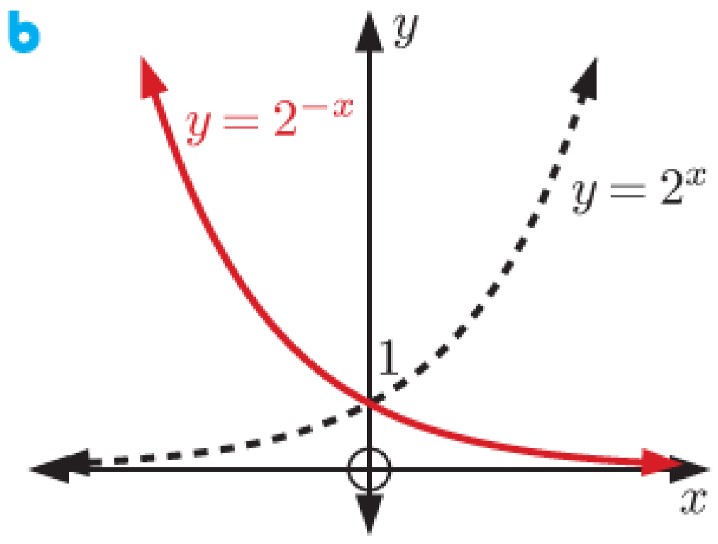
c $y = 2^x$ has a horizontal asymptote of $y = 0$.

2 **a** **C** **b** **B** **c** **E** **d** **A** **e** **D**

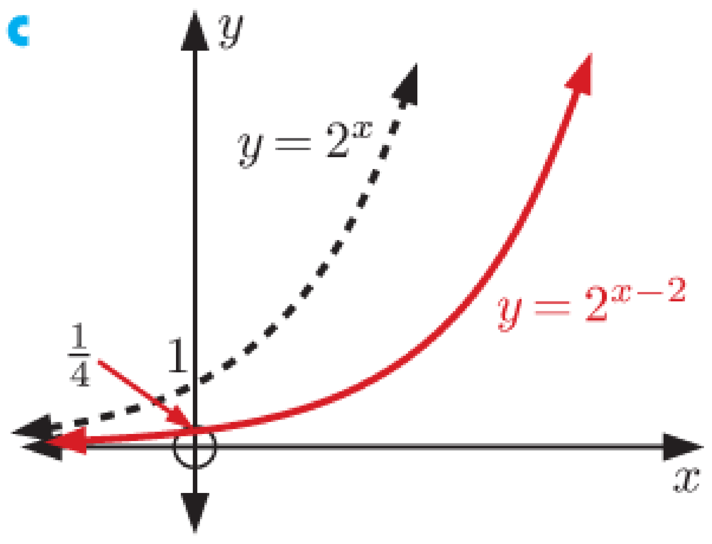
3 **a**



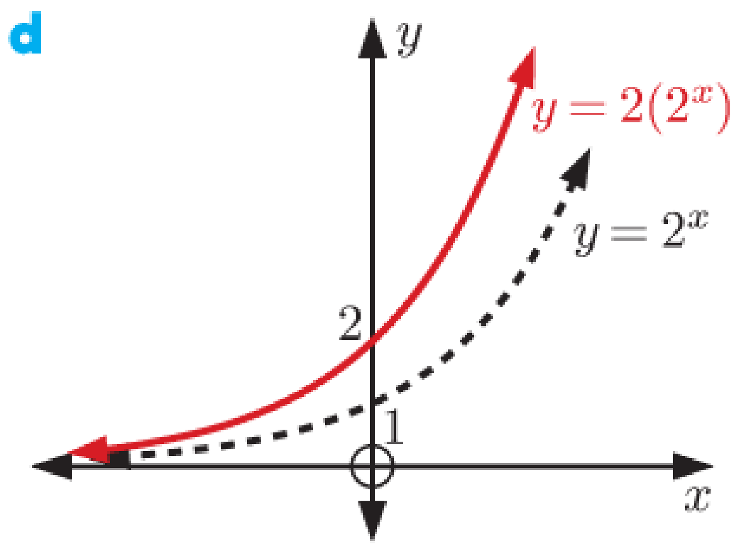
b



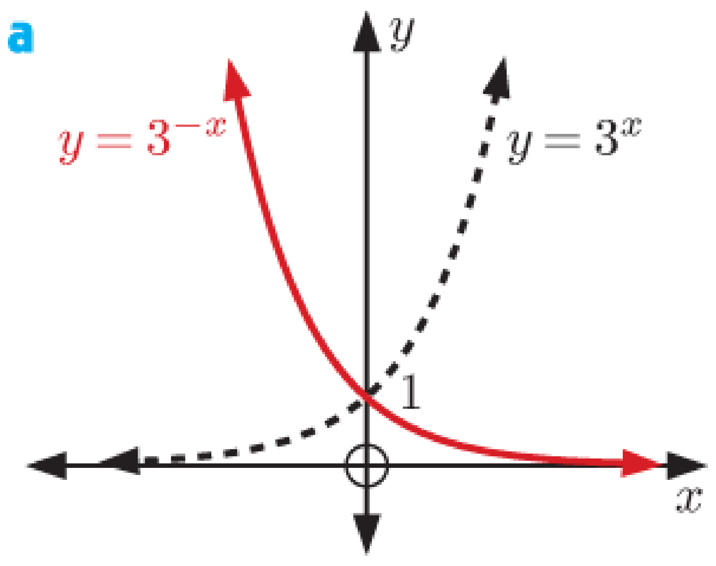
c



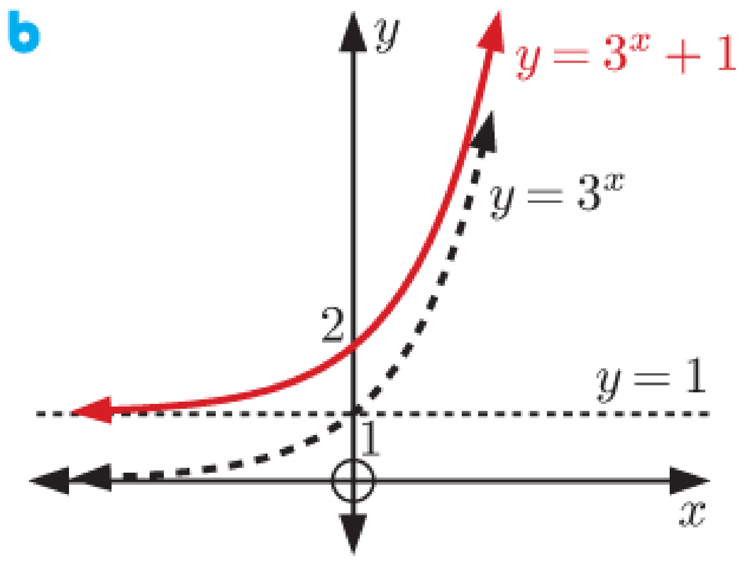
d



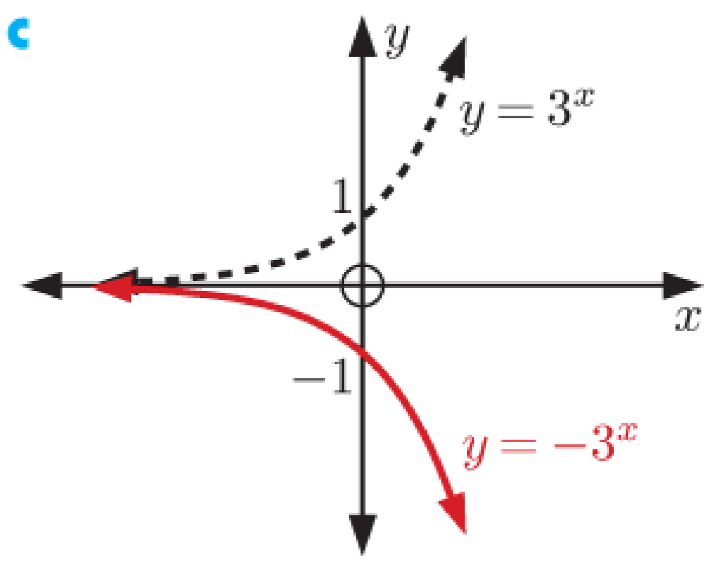
4 **a**



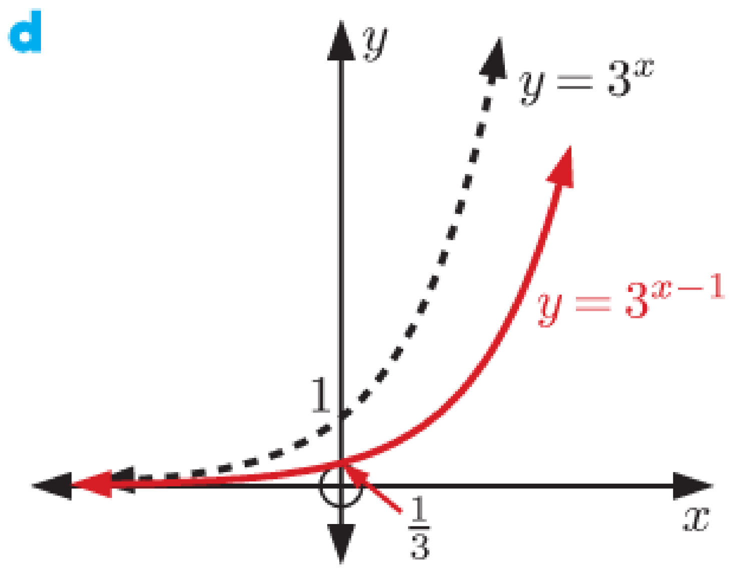
b



c

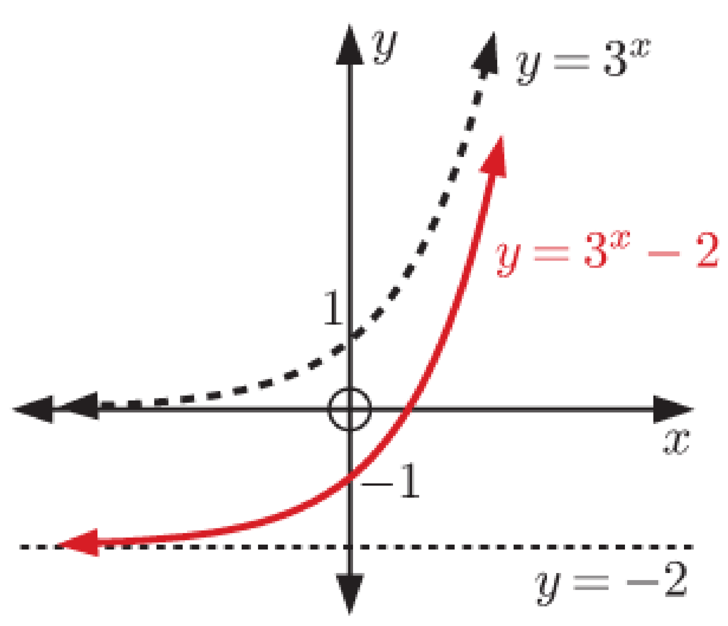


d



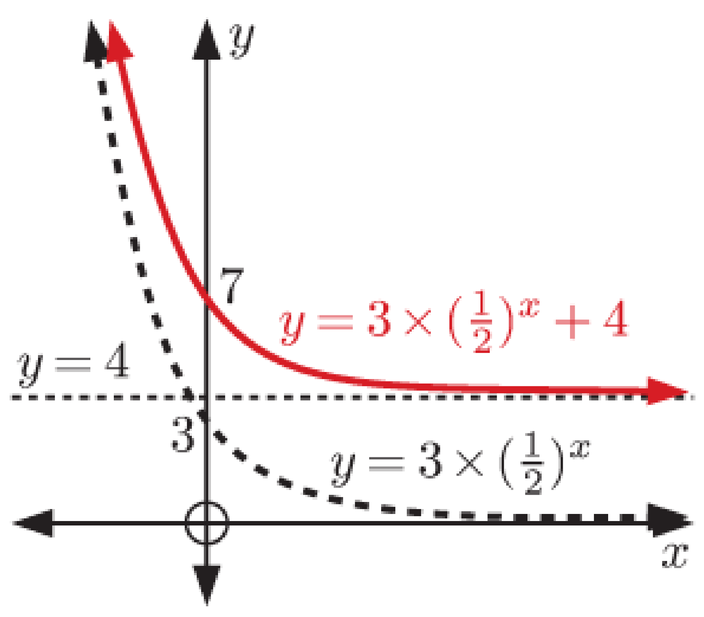
5 **a** $y = 0$ **b** $y = -1$ **c** $y = 3$ **d** $y = 2$
e $y = 0$ **f** $y = -4$

6 **a** **i** -1 **ii** 7
iii $-\frac{17}{9} = -1\frac{8}{9}$
b $y = -2$
c



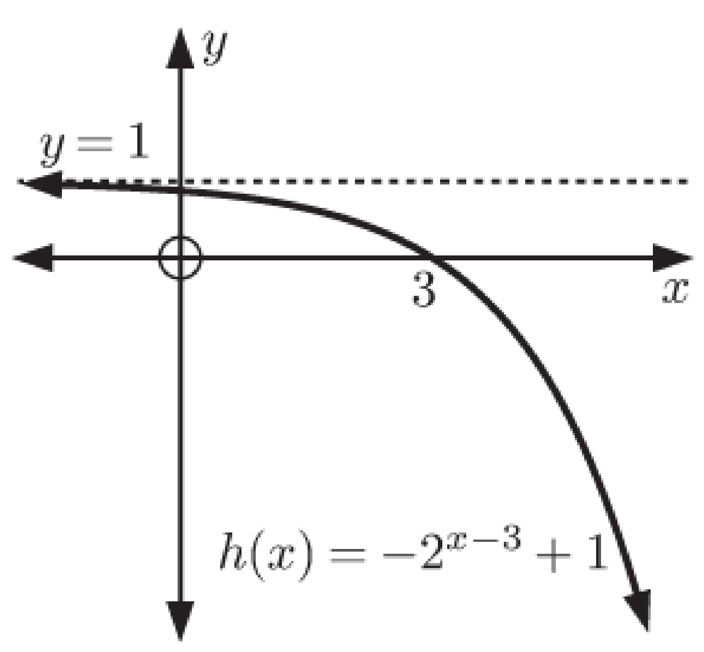
d Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y > -2\}$

7 **a** **i** 7
ii $\frac{19}{4} = 4\frac{3}{4}$
iii 16
b $y = 4$
c



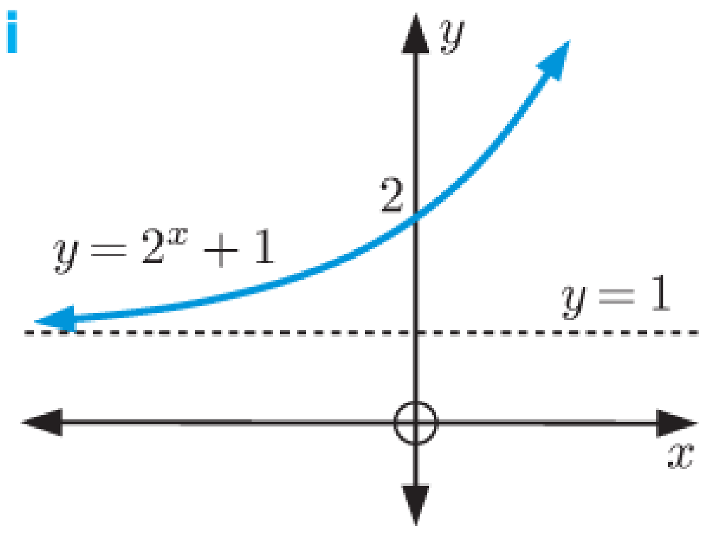
d Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y > 4\}$

8 **a** **i** $\frac{7}{8}$ **ii** 0
iii -7
b $y = 1$
c



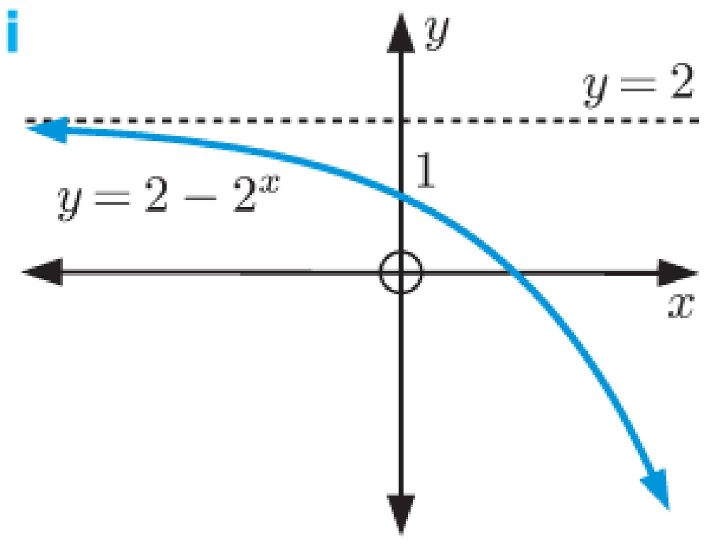
d Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y < 1\}$

9 **a** **i**



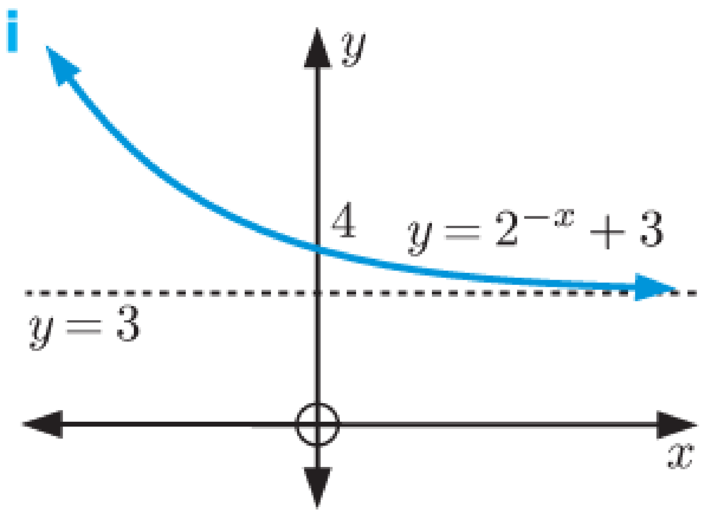
ii Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y > 1\}$
iii $y \approx 3.67$
iv as $x \to \infty,\ y \to \infty$
as $x \to -\infty,\ y \to 1^+$
v $y = 1$

b **i**



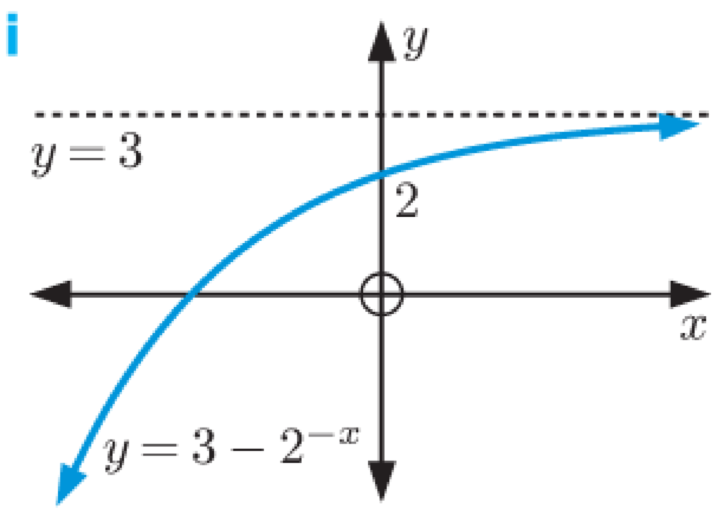
ii Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y < 2\}$
iii $y \approx -0.665$
iv as $x \to \infty,\ y \to -\infty$
as $x \to -\infty,\ y \to 2^-$
v $y = 2$

c **i**



ii Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y > 3\}$
iii $y \approx 3.38$
iv as $x \to \infty,\ y \to 3^+$
as $x \to -\infty,\ y \to \infty$
v $y = 3$

d **i**



ii Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y < 3\}$
iii $y \approx 2.62$
iv as $x \to \infty,\ y \to 3^-$
as $x \to -\infty,\ y \to -\infty$
v $y = 3$

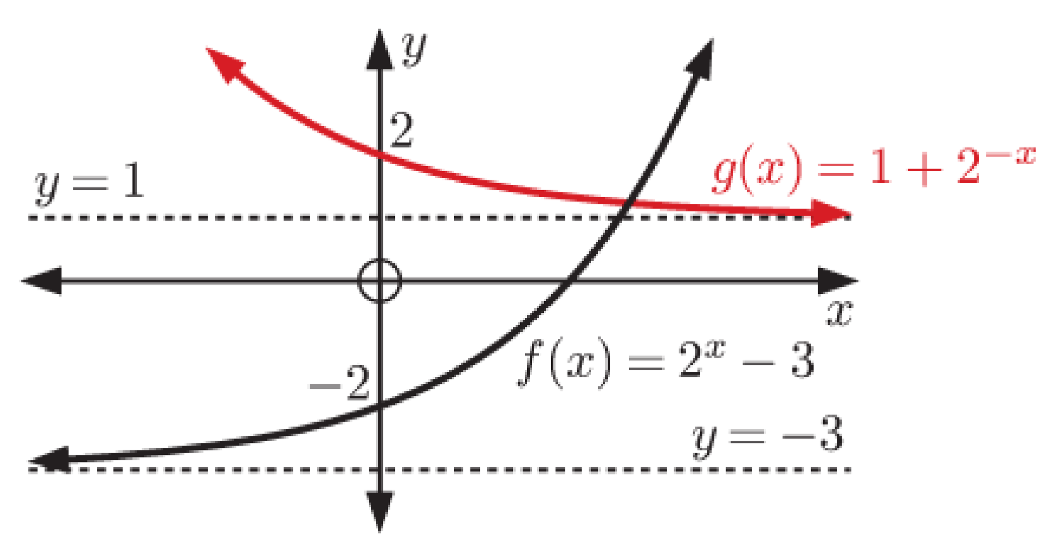
10 **a** $a = 5,\ b = -10$ **b** $y = 310$

11 **a** $P(0, 2.5)$ **b** $a = 1.5$ **c** $y = 3.5$

12 **a** Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y \geqslant 2\}$
b Domain is $\{x \mid x \neq 0\}$, Range is $\{y \mid y > 0,\ y < -1\}$
c Domain is $\{x \mid x \geqslant 1\}$, Range is $\{y \mid y \geqslant 0\}$

13 **a** $(f \circ g)(x) = 3^{\sqrt{x}} - 9$
Domain is $\{x \mid x \geqslant 0\}$, Range is $\{y \mid y \geqslant -8\}$
b $(g \circ f)(x) = \sqrt{3^x - 9}$
Domain is $\{x \mid x \geqslant 2\}$, Range is $\{y \mid y \geqslant 0\}$
c **i** $x = 4$ **ii** $x = 3$

14 **a** **i** $f(x)$: $y = -3$, $g(x)$: $y = 1$
ii $f(x)$: Range is $\{y \mid y > -3\}$
$g(x)$: Range is $\{y \mid y > 1\}$
iii $f(x)$: y-intercept -2, $g(x)$: y-intercept 2
b

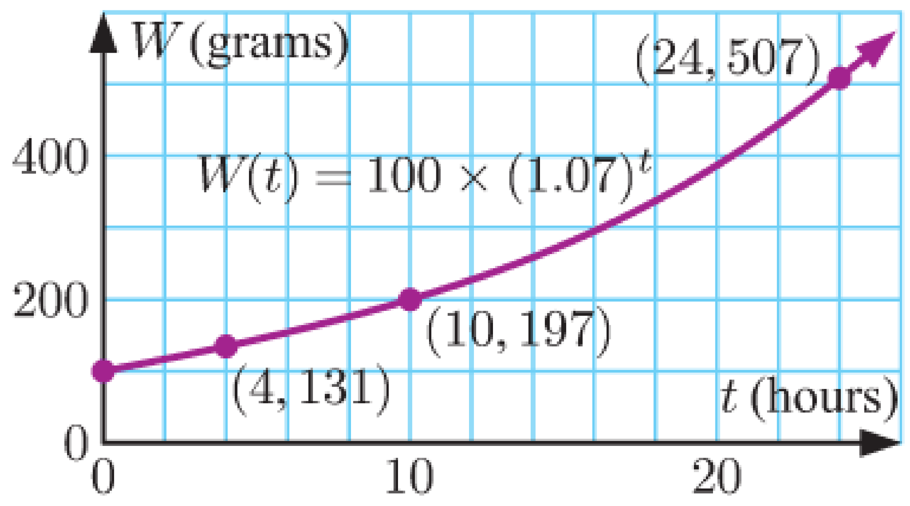


c $-1 + \sqrt{5}$

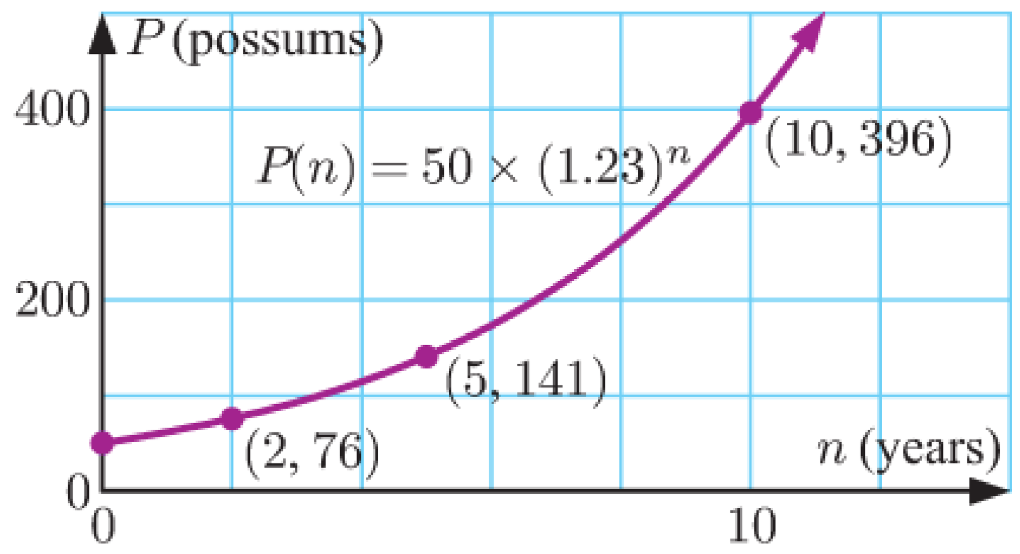
15 **a** $x \approx 3.46$ **b** $x \approx 2.46$ **c** $x \approx 1.16$
d $x \approx -0.738$ **e** $x \approx 1.85$ **f** $x \approx 0.0959$
g $x \approx 6.03$ **h** $x \approx 50.0$ **i** $x \approx 31.0$

EXERCISE 2E.1

1 **a** 100 grams
b **i** ≈ 131 g **ii** ≈ 197 g **iii** ≈ 507 g
c



2 **a** $P_0 = 50$
b **i** ≈ 76 possums **ii** ≈ 141 possums
iii ≈ 396 possums
c



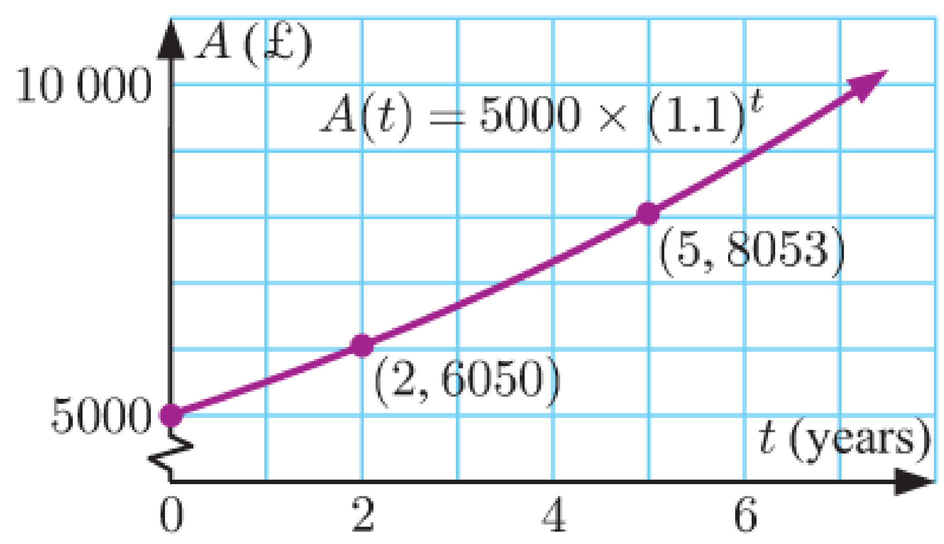
d ≈ 11 years **e** ≈ 11.1 years

3 **a** 4 people **b** ≈ 393 people **c** ≈ 19.9 days

4 **a** $B_0 = 200$
b $a = 1.1$, the bear population is increasing by 10% every year.
c ≈ 1350 bears **d** $\approx 159\%$ increase **e** ≈ 24.2 years

5 **a** **i** V_0 **ii** $2V_0$ **b** 100%
c $\approx 183\%$ increase, it is the percentage increase at 50°C compared with 20°C.

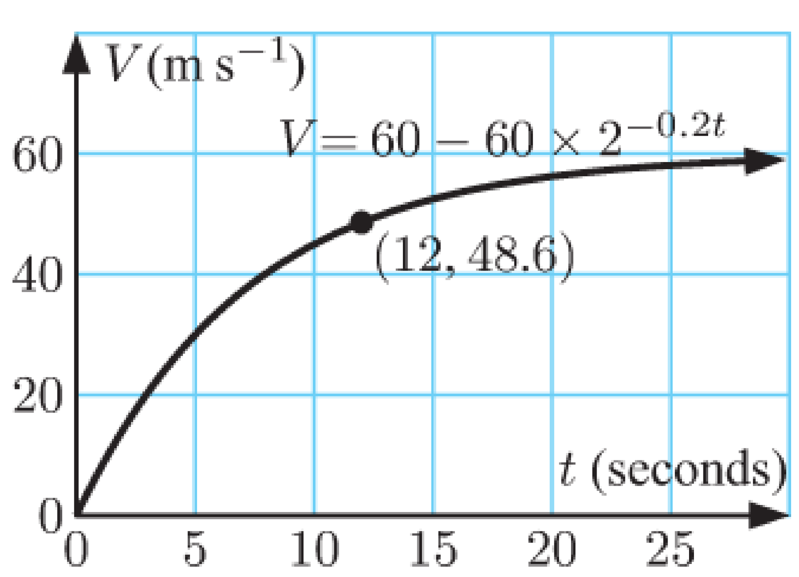
6 **a** $A(t) = 5000 \times (1.1)^t$ **b** **i** £6050 **ii** £8052.55
c



d ≈ 4.93 years

7 **a** $a = 1.08$, the expected value of the house is increasing by 8% per year.
$k = 375\,000$, the original value of the house was \$375 000.
b ≈ 4.98 years

8 **a** When $t = 0$, $V = c - 60 = 0$
$\therefore\ c = 60$
b $k = -\frac{1}{5} = -0.2$
c $\approx 48.6\ \text{m s}^{-1}$ **d**

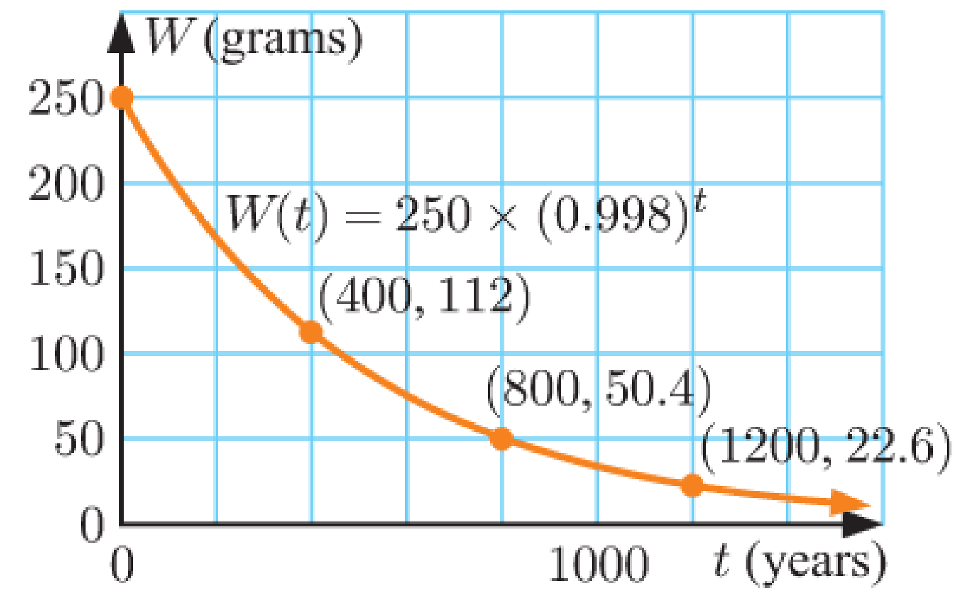


e The parachutist accelerates rapidly until he approaches his terminal velocity of $60\ \text{m s}^{-1}$.

9 ≈ 2.27 hours

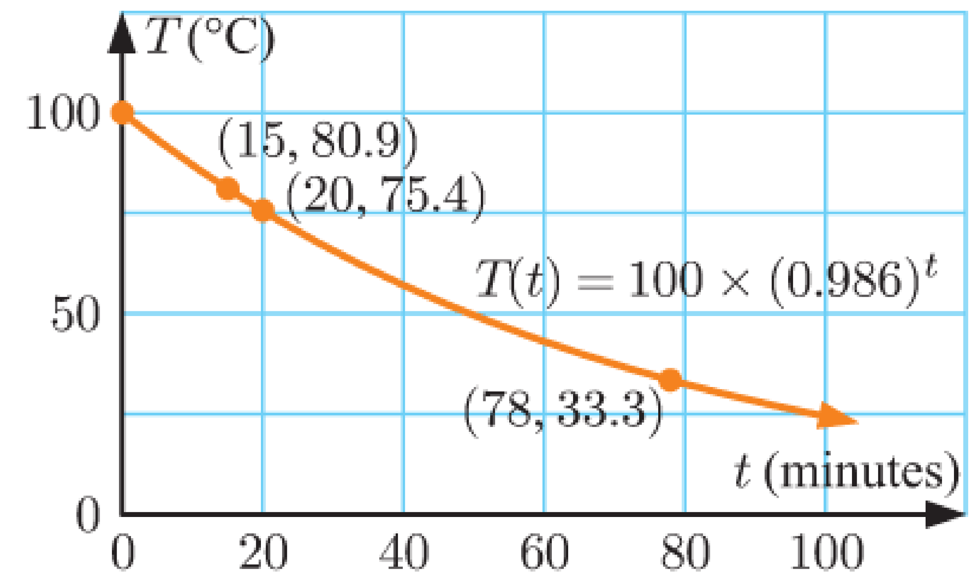
EXERCISE 2E.2

1 **a** 250 g **b** **i** ≈ 112 g **ii** ≈ 50.4 g **iii** ≈ 22.6 g
c

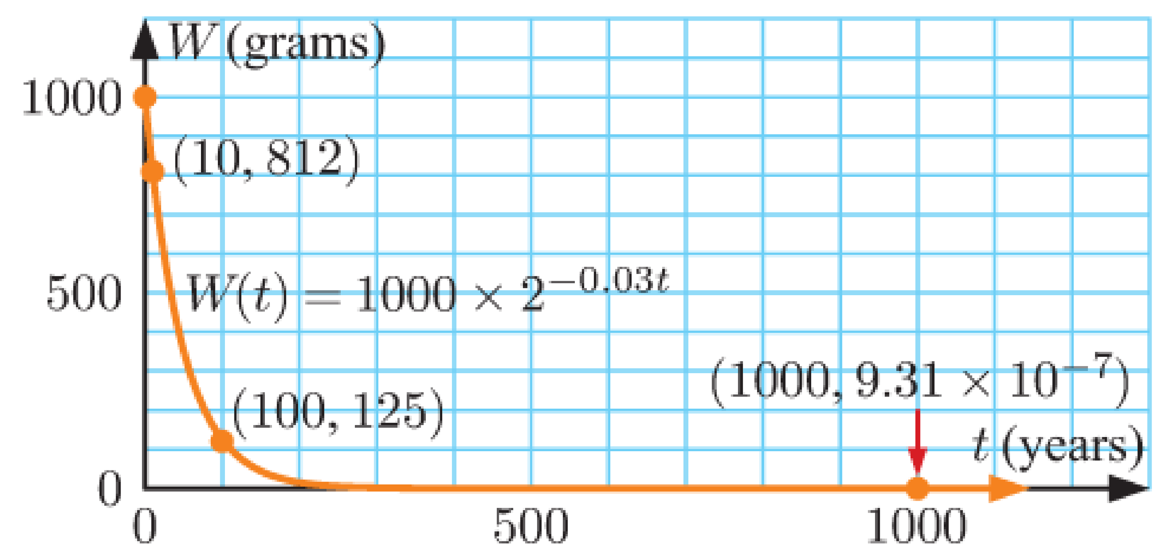


d ≈ 346 years

2 **a** 100°C
b **i** ≈ 80.9°C **ii** ≈ 75.4°C **iii** ≈ 33.3°C
c

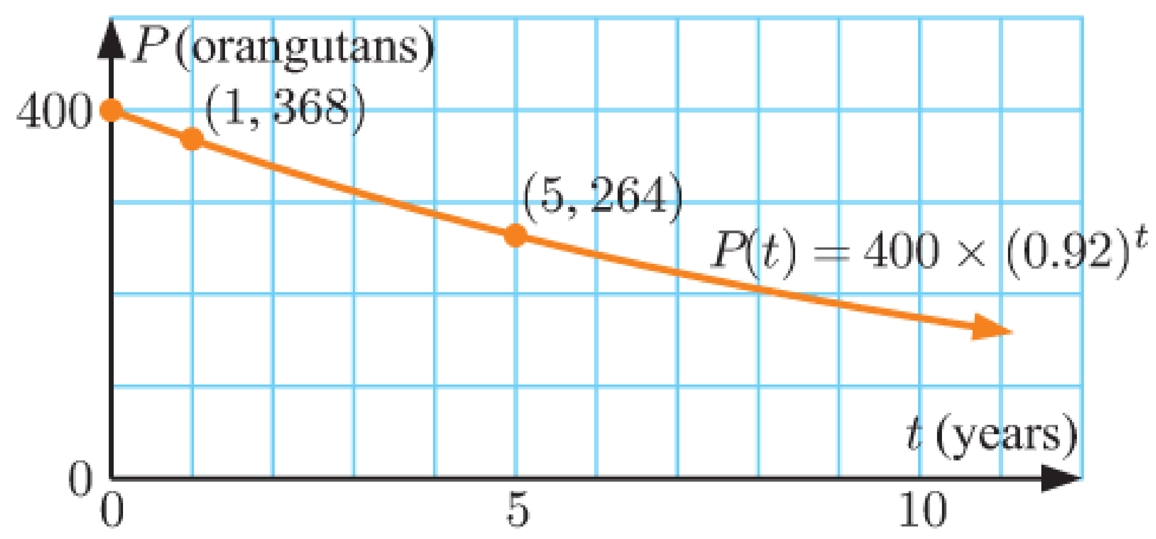


3 **a** 1000 g
b **i** ≈ 812 g **ii** 125 g **iii** $\approx 9.31 \times 10^{-7}$ g
c



d ≈ 221 years **e** $1000(1 - 2^{-0.03t})$ grams

4 **a** $P(t) = 400 \times (0.92)^t$
b **i** 368 orangutans **ii** ≈ 264 orangutans
c

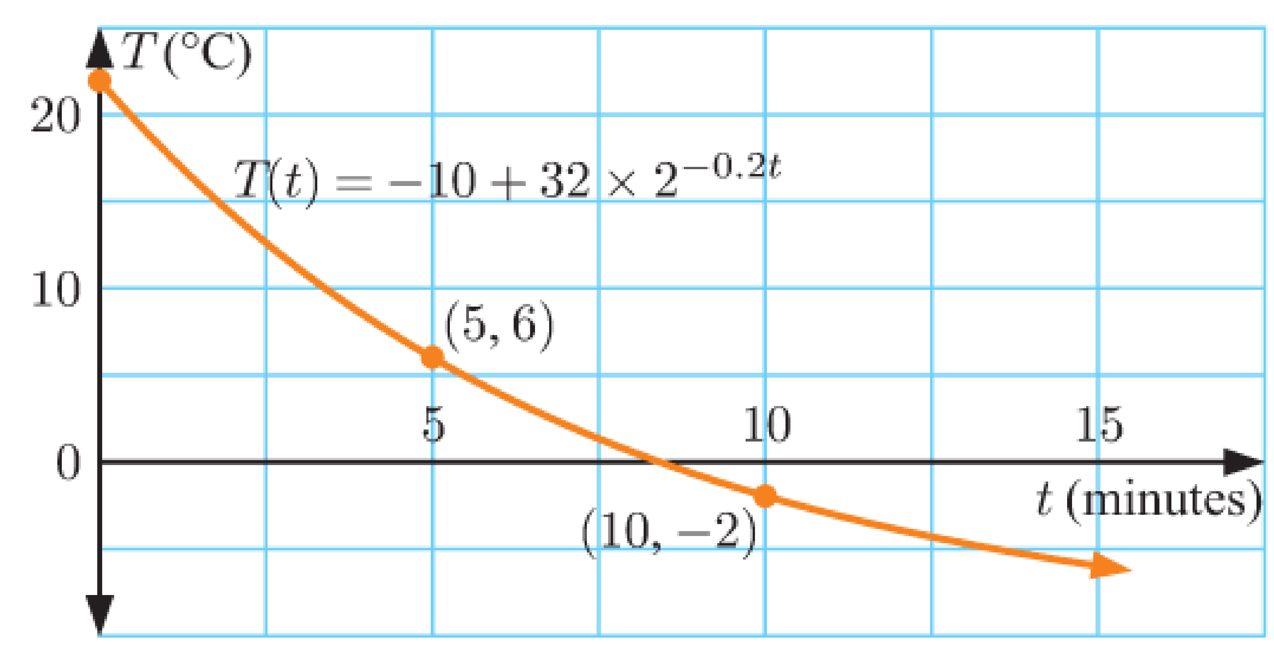


d ≈ 8.31 years, or ≈ 8 years 114 days

5 **a** $L_0 = 10$ units **b** ≈ 2.77 units **c** ≈ 17.9 m
d between ≈ 23.5 m and ≈ 44.9 m

6 **a** \$24 000 **b** $r = 0.85$ **c** 7 years

7 **a** **i** 22°C **ii** 6°C **iii** −2°C
b



c ≈ 8.39 min or ≈ 8 min 23 s
d No, as $32 \times 2^{-0.2t} > 0$ for any value of t.

8 **a** W_0 **b** $\approx 12.9\%$ **c** 45 000 years

9 **a** $A(t) = 150 \times (1.48)^{\frac{t}{3}}$, $B(t) = 400 \times (0.8)^t + 100$
b **i** $t \approx 4.16$ years **ii** $t \approx 3.45$ years **iii** $t \approx 1.69$ years

10 **a** The initial weight of the isotope is 10 mg.