Normal distribution 28.02 *[47 marks]*

A factory produces bags of sugar with a labelled weight of 500 g. The weights of the bags are normally distributed with a mean of 500 g and a standard deviation of 3 g.

1a. Write down the percentage of bags that weigh more than 500 g. *[1 mark]*

Markscheme

50% ***A1***

Note: Do not accept 0.5 or $\frac{1}{2}$.

[1 mark]

A bag that weighs less than 495 g is rejected by the factory for being underweight.

1b. Find the probability that a randomly chosen bag is rejected for being underweight. *[2 marks]*

Markscheme

0.0478 $(0.0477903\ldots, 4.78\%)$ ***A2***

[2 marks]

1c. A bag that weighs more than k grams is rejected by the factory for being overweight. The factory rejects 2% of bags for being overweight. *[3 marks]*

Find the value of k.

Markscheme

$\mathrm{P}(X < k) = 0.98$ **OR** $\mathrm{P}(X > k) = 0.02$ ***(M1)***

Note: Award ***(M1)*** for a sketch with correct region identified.

$506\ \mathrm{g}$ $(506.161\ldots)$ ***A2***

[3 marks]

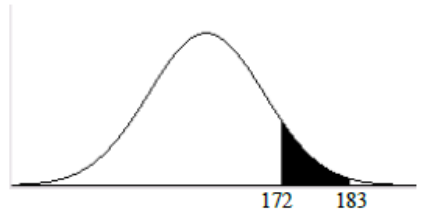
The masses of Fuji apples are normally distributed with a mean of $163\ \mathrm{g}$ and a standard deviation of $6.83\ \mathrm{g}$.

When Fuji apples are picked, they are classified as small, medium, large or extra large depending on their mass. Large apples have a mass of between $172\ \mathrm{g}$ and $183\ \mathrm{g}$.

2a. Determine the probability that a Fuji apple selected at random will be a large apple. *[2 marks]*

Markscheme

sketch of normal curve with shaded region to the right of the mean and correct values ***(M1)***



0.0921 $(0.0920950\ldots)$ ***A1***

[2 marks]

Approximately 68% of Fuji apples have a mass within the medium-sized category, which is between k and $172\ \mathrm{g}$.

2b. Find the value of k. *[3 marks]*

Markscheme

EITHER

$(\mathrm{P}(x < 172))$

$0.906200\ldots$ ***(A1)***

$(0.906200\ldots - 0.68)$

$0.226200\ldots$ ***(A1)***

OR

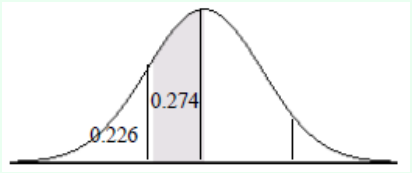
$(\mathrm{P}(163 < x < 172))$

$0.406200\ldots$ ***(A1)***

$0.5 - (0.68 - 0.406200\ldots)$ **OR** $0.5 + (0.68 - 0.406200\ldots)$

$0.226200\ldots$ **OR** $0.773799\ldots$ ***(A1)***

OR



(A1)(A1)

Note: Award ***A1*** for a normal distribution curve with a vertical line on each side of the mean and a correct probability of either 0.406 or 0.274 or 0.906 shown, ***A1*** for a probability of 0.226 seen.

THEN

$(k =)\ 158\ \mathrm{g}\ \ (157.867\ldots\ \mathrm{g})$ ***A1***

[3 marks]

The Malthouse Charity Run is a 5 kilometre race. The time taken for each runner to complete the race was recorded. The data was found to be normally distributed with a mean time of 28 minutes and a standard deviation of 5 minutes.

A runner who completed the race is chosen at random.

3a. Write down the probability that the runner completed the race in more than 28 minutes. *[1 mark]*

Markscheme

$0.5\ \left(\frac{1}{2},\ 50\%\right)$ ***(A1) (C1)***

[1 mark]

3b. Calculate the probability that the runner completed the race in less than 26 minutes. *[2 marks]*

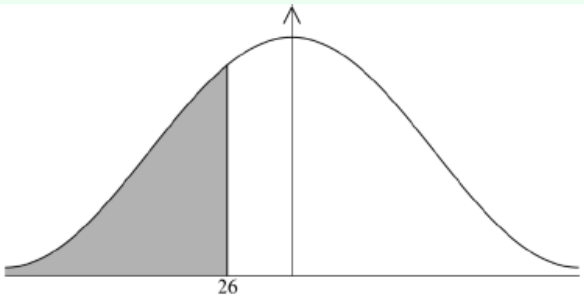
Markscheme

$\mathrm{P}(X \leq 26)$ ***(M1)***

Note: Award ***(M1)*** for a correct mathematical statement.

OR

Award ***(M1)*** for a diagram that shows the value 26 labelled to the left of the mean and the correct shaded region.



$3.45\ (0.344578\ldots,\ 34.5\%)$ ***(A1) (C2)***

[2 marks]

3c. It is known that 20% of the runners took more than 28 minutes and less than k minutes to complete the race. *[3 marks]*

Find the value of k.

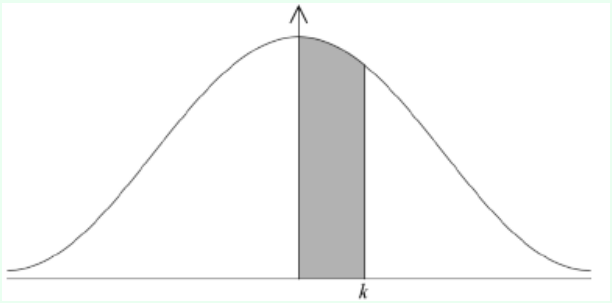
Markscheme

0.7 **OR** 0.3 (seen) ***(A1)***

Note: Award ***(A1)*** for 0.7 or 0.3 seen.

$\mathrm{P}(\text{time} < 7) = 0.7$ **OR** $\mathrm{P}(\text{time} > k) = 0.3$ ***(M1)***

Note: Award ***(M1)*** for a correct mathematical statement.
OR
Award ***(M1)*** for a diagram that shows k greater than the mean and shading in the region below k, above k, or between k and the mean.



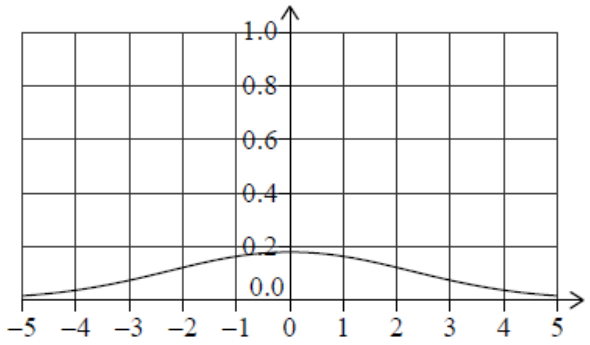
$(k =)\ 30.6\ (30.6220\ldots)$ (minutes) ***(A1)*** ***(C3)***

Note: Accept "30 minutes and 37 seconds" or (from 3 sf k value) "30 minutes and 36 seconds".

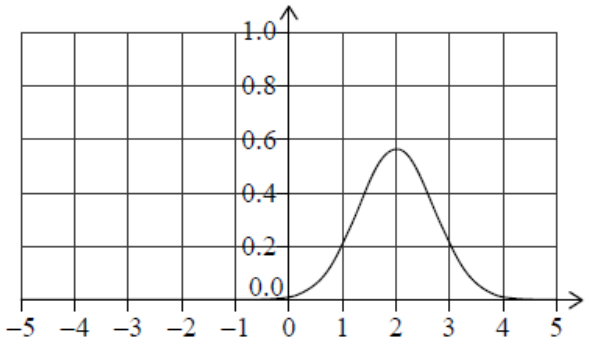
[3 marks]

Consider the following graphs of normal distributions.

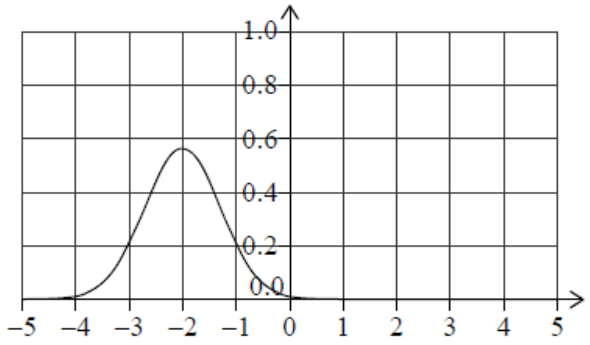
Graph A



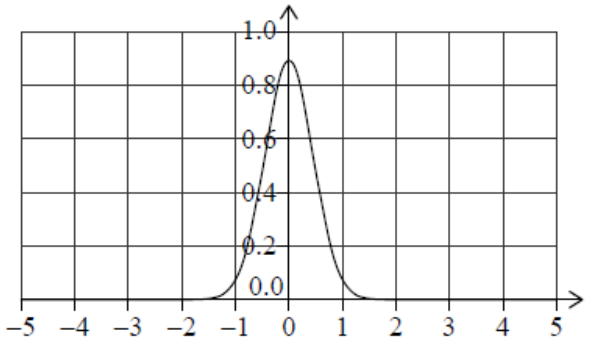
Graph B



Graph C



Graph D



4a. In the following table, write down the letter of the corresponding graph next to the given mean and standard deviation. *[2 marks]*

Mean and standard deviation	Graph
Mean = −2; standard deviation = 0.707	
Mean = 0; standard deviation = 0.447	

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

Mean and standard deviation	Graph
Mean = −2; standard deviation = 0.707	C
Mean = 0; standard deviation = 0.447	D

(A1)(A1)

(C2)

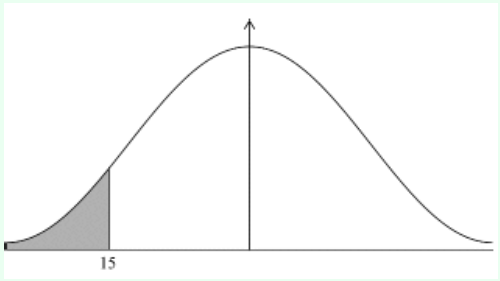
Note: Award ***(A1)*** for each correct entry.

[2 marks]

At an airport, the weights of suitcases (in kg) were measured. The weights are normally distributed with a mean of 20 kg and standard deviation of 3.5 kg.

4b. Find the probability that a suitcase weighs less than 15 kg. *[2 marks]*

Markscheme



(M1)

Note: Award ***(M1)*** for sketch with 15 labelled and left tail shaded **OR** for a correct probability statement, P($X < 15$).

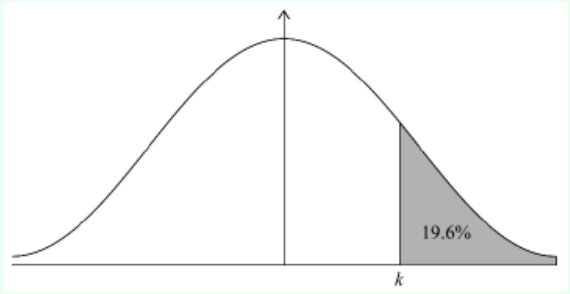
0.0766 (0.0765637..., 7.66%) ***(A1)*** ***(C2)***

[2 marks]

4c. Any suitcase that weighs more than k kg is identified as excess baggage. *[2 marks]*
19.6 % of the suitcases at this airport are identified as excess baggage.

Find the value of k.

Markscheme



(M1)

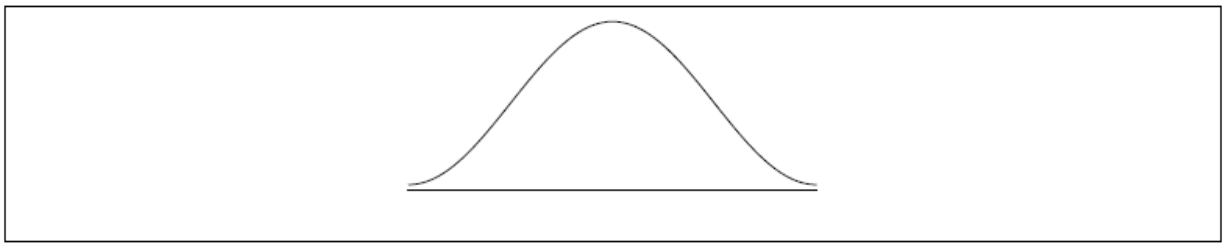
Note: Award ***(M1)*** for a sketch showing correctly shaded region to the right of the mean with 19.6% labelled (accept shading of the complement with 80.4% labelled) **OR** for a correct probability statement, P($X > k$) = 0.196 or P($X \leq k$) = 0.804.

23.0 (kg) (22.9959... (kg)) ***(A1)*** ***(C2)***

[2 marks]

The price per kilogram of tomatoes, in euro, sold in various markets in a city is found to be normally distributed with a mean of 3.22 and a standard deviation of 0.84.

5a. On the following diagram, shade the region representing the probability that the price of a kilogram of tomatoes, chosen at random, will be higher than 3.22 euro. *[1 mark]*



Markscheme

(A1) ***(C1)***

Note: Award ***(A1)*** for vertical line drawn at the mean (3.22 does not have to be seen) and correct region shaded.

[1 mark]

5b. Find the price that is two standard deviations above the mean price. *[1 mark]*

Markscheme

4.90 ***(A1)*** ***(C1)***

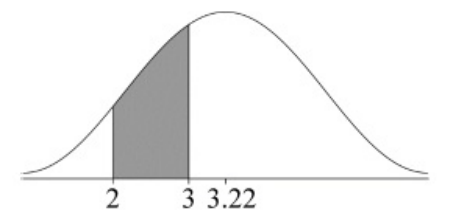
[1 mark]

5c. Find the probability that the price of a kilogram of tomatoes, chosen at random, will be between 2.00 and 3.00 euro. *[2 marks]*

Markscheme

0.323 (0.323499...; 32.3 %) ***(A2)*** ***(C2)***

Note: If final answer is incorrect, ***(M1)(A0)*** may be awarded for correct shaded area shown on a sketch, below, or for a correct probability statement "P($2 \leq X \leq 3$)" (accept other variables for X or "price" and strict inequalities).



[2 marks]

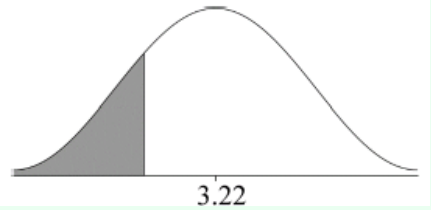
5d. To stimulate reasonable pricing, the city offers a free permit to the sellers whose price of a kilogram of tomatoes is in the lowest 20 %. *[2 marks]*

Find the highest price that a seller can charge and still receive a free permit.

Markscheme

2.51 (2.51303...) ***(A2)*** ***(C2)***

Note: If final answer is incorrect, ***(M1)(A0)*** may be awarded for correct shaded area shown on a sketch, below, or for a correct probability statement "P($X \leq a$) = 0.2" (accept other variables and strict inequalities).



[2 marks]

Malthouse school opens at 08:00 every morning.

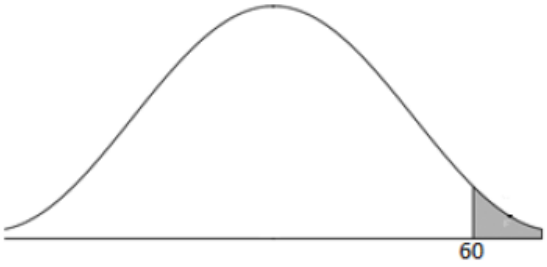
The daily arrival times of the 500 students at Malthouse school follow a normal distribution. The mean arrival time is 52 minutes after the school opens and the standard deviation is 5 minutes.

6a. Find the probability that a student, chosen at random arrives at least 60 minutes after the school opens. *[2 marks]*

Markscheme

0.0548 (0.054799..., 5.48%) ***(A2) (C2)***

Note: Award ***(M1)(A0)*** for a correct probability statement, $\mathrm{P}(X > 60)$, or normal distribution graph with correctly shaded region, leading to incorrect or no answer.



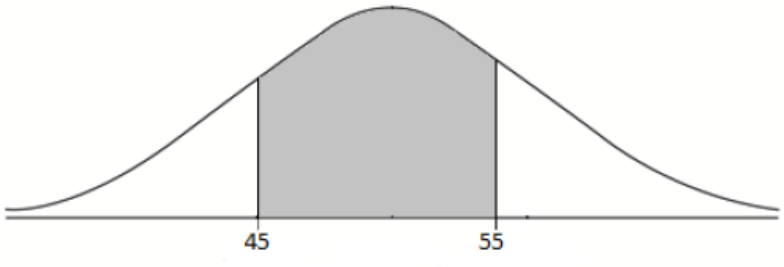
[2 marks]

6b. Find the probability that a student, chosen at random arrives between 45 minutes and 55 minutes after the school opens. *[2 marks]*

Markscheme

0.645 (0.6449900..., 64.5%) ***(A2) (C2)***

Note: Award ***(M1)(A0)*** for a correct probability statement, $\mathrm{P}(45 < X < 55)$, or normal distribution graph with correctly shaded region, leading to incorrect or no answer.



[2 marks]

6c. A second school, Mulberry Park, also opens at 08:00 every morning. The arrival times of the students at this school follows exactly the same distribution as Malthouse school. *[2 marks]*

Given that, on one morning, 15 students arrive at least 60 minutes after the school opens, estimate the number of students at Mulberry Park school.

Markscheme

$\frac{15}{0.0548}$ **(M1)**

Note: Award ***(M1)*** for dividing 15 by their part (a)(i).

Accept an equation of the form $15 = x \times 0.0548$ for ***(M1)***.

274 (273.722...) ***(A1)*****(ft)** ***(C2)***

Note: Follow through from part (a)(i). Accept 273.

[2 marks]

Applicants for a job had to complete a mathematics test. The time they took to complete the test is normally distributed with a mean of 53 minutes and a standard deviation of 16.3. One of the applicants is chosen at random.

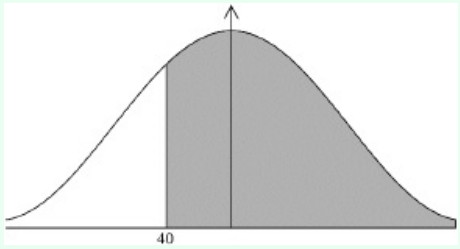
7a. Find the probability that this applicant took at least 40 minutes to complete the test. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

0.787 (0.787433..., 78.7%) ***(M1)(A1)*** ***(C2)***

Note: Award ***(M1)*** for a correct probability statement, $\mathrm{P}(X > 40)$, or a correctly shaded normal distribution graph.



[2 marks]

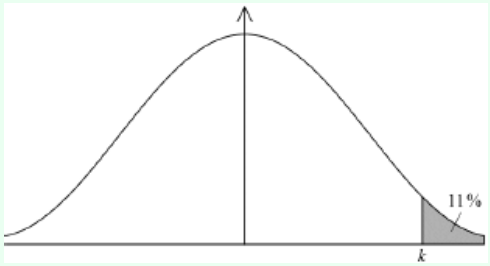
For 11% of the applicants it took longer than k minutes to complete the test.

7b. Find the value of k. *[2 marks]*

Markscheme

73.0 (minutes) (72.9924...) ***(M1)(A1)*** ***(C2)***

Note: Award ***(M1)*** for a correct probability statement, $\mathrm{P}(X > k) = 0.11$, or a correctly shaded normal distribution graph.



[2 marks]

There were 400 applicants for the job.

7c. Estimate the number of applicants who completed the test in less than 25 minutes. *[2 marks]*

Markscheme

$0.0423433\ldots \times 400$ ***(M1)***

Note: Award ***(M1)*** for multiplying a probability by 400. Do not award ***(M1)*** for 0.11×400.

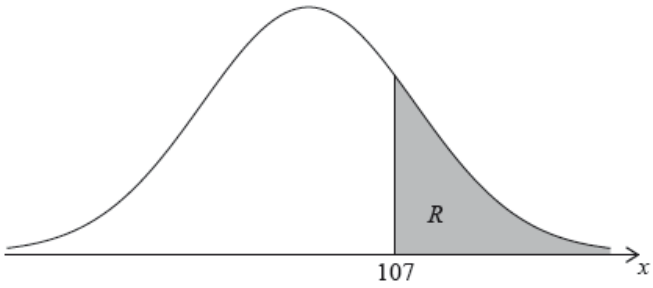
Use of a lower bound less than zero gives a probability of 0.0429172....

$= 16$ ***(A1)*** ***(C2)***

Notes: Accept a final answer of 17. Do not accept a final answer of 18. Accept a non-integer final answer either 16.9 (16.9373...) from use of lower bound zero or 17.2 (17.1669...) from use of the default lower bound of -10^{99}.

[2 marks]

The random variable X is normally distributed with a mean of 100. The following diagram shows the normal curve for X.



Let R be the shaded region under the curve, to the right of 107. The area of R is 0.24.

8a. Write down $\mathrm{P}(X > 107)$. *[1 mark]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\mathrm{P}(X > 107) = 0.24 \quad \left(= \frac{6}{25},\ 24\%\right)$ ***A1*** ***N1***

[1 mark]

8b. Find $\mathrm{P}(100 < X < 107)$. *[3 marks]*

Markscheme

valid approach ***(M1)***

eg $\mathrm{P}(X > 100) = 0.5,\ \mathrm{P}(X > 100) - \mathrm{P}(X > 107)$

correct working ***(A1)***

eg $0.5 - 0.24,\ 0.76 - 0.5$

$\mathrm{P}(100 < X < 107) = 0.26 \quad \left(= \frac{13}{50},\ 26\%\right)$ ***A1*** ***N2***

[3 marks]

8c. Find $\mathrm{P}(93 < X < 107)$. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $2 \times 0.26,\ 1 - 2(0.24),\ \mathrm{P}(93 < X < 100) = \mathrm{P}(100 < X < 107)$

$\mathrm{P}(93 < X < 107) = 0.52\ \ \left(= \frac{13}{25},\ 52\%\right)$ ***A1 N2***

[2 marks]