

Normal distribution 28.02 [47 marks]

A factory produces bags of sugar with a labelled weight of 500 g. The weights of the bags are normally distributed with a mean of 500 g and a standard deviation of 3 g.

1a. Write down the percentage of bags that weigh more than 500 g. [1 mark]

A bag that weighs less than 495 g is rejected by the factory for being underweight.

1b. Find the probability that a randomly chosen bag is rejected for being underweight. [2 marks]

1c. A bag that weighs more than k grams is rejected by the factory for being overweight. The factory rejects 2% of bags for being overweight. Find the value of k . [3 marks]

The masses of Fuji apples are normally distributed with a mean of 163 g and a standard deviation of 6.83 g.

When Fuji apples are picked, they are classified as small, medium, large or extra large depending on their mass. Large apples have a mass of between 172 g and 183 g.

2a. Determine the probability that a Fuji apple selected at random will be a large apple. [2 marks]

Approximately 68% of Fuji apples have a mass within the medium-sized category, which is between k and 172 g.

2b. Find the value of k . [3 marks]

The Malthouse Charity Run is a 5 kilometre race. The time taken for each runner to complete the race was recorded. The data was found to be normally distributed with a mean time of 28 minutes and a standard deviation of 5 minutes.

A runner who completed the race is chosen at random.

3a. Write down the probability that the runner completed the race in more than 28 minutes. [1 mark]

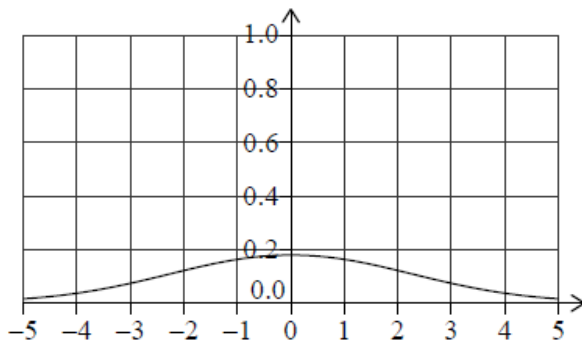
3b. Calculate the probability that the runner completed the race in less than 26 minutes. [2 marks]

3c. It is known that 20% of the runners took more than 28 minutes and less than k minutes to complete the race. [3 marks]

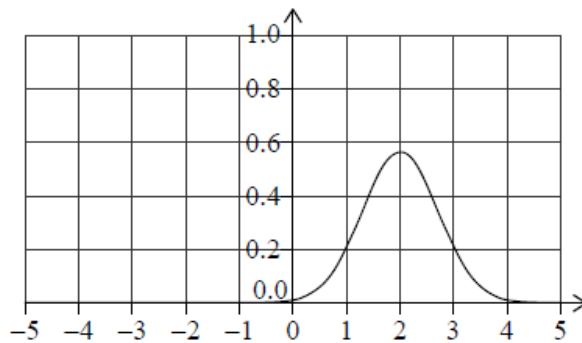
Find the value of k .

Consider the following graphs of normal distributions.

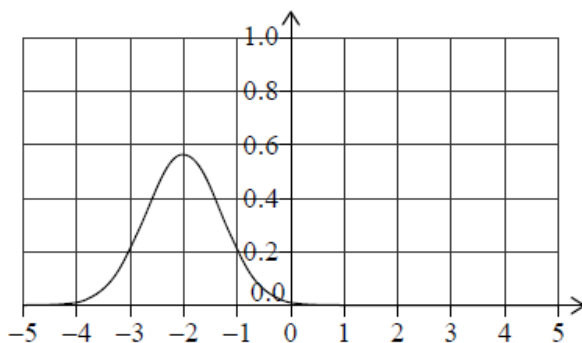
Graph A



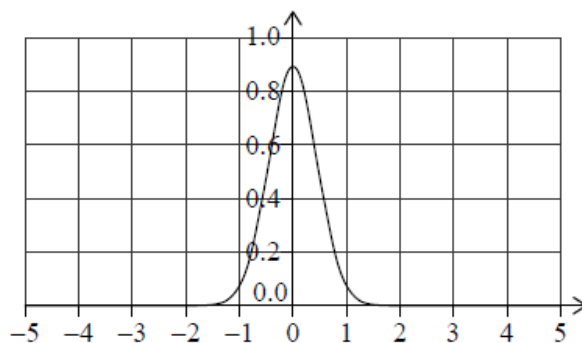
Graph B



Graph C



Graph D



4a. In the following table, write down the letter of the corresponding graph next to the given mean and standard deviation. [2 marks]

Mean and standard deviation	Graph
Mean = -2; standard deviation = 0.707	
Mean = 0; standard deviation = 0.447	

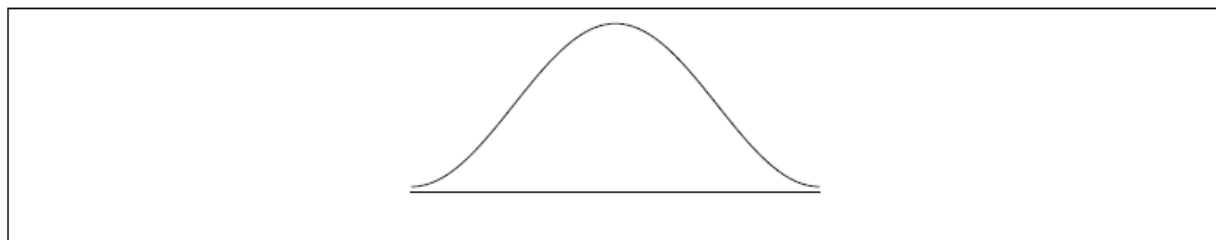
At an airport, the weights of suitcases (in kg) were measured. The weights are normally distributed with a mean of 20 kg and standard deviation of 3.5 kg.

4b. Find the probability that a suitcase weighs less than 15 kg. [2 marks]

4c. Any suitcase that weighs more than k kg is identified as excess baggage. [2 marks]
19.6 % of the suitcases at this airport are identified as excess baggage.
Find the value of k .

The price per kilogram of tomatoes, in euro, sold in various markets in a city is found to be normally distributed with a mean of 3.22 and a standard deviation of 0.84.

5a. On the following diagram, shade the region representing the probability [1 mark] that the price of a kilogram of tomatoes, chosen at random, will be higher than 3.22 euro.



5b. Find the price that is two standard deviations above the mean price. [1 mark]

5c. Find the probability that the price of a kilogram of tomatoes, chosen at random, will be between 2.00 and 3.00 euro. [2 marks]

5d. To stimulate reasonable pricing, the city offers a free permit to the sellers whose price of a kilogram of tomatoes is in the lowest 20%. [2 marks]
Find the highest price that a seller can charge and still receive a free permit.

Malthouse school opens at 08:00 every morning.

The daily arrival times of the 500 students at Malthouse school follow a normal distribution. The mean arrival time is 52 minutes after the school opens and the standard deviation is 5 minutes.

6a. Find the probability that a student, chosen at random arrives at least 60 minutes after the school opens. [2 marks]

6b. Find the probability that a student, chosen at random arrives between 45 minutes and 55 minutes after the school opens. [2 marks]

6c. A second school, Mulberry Park, also opens at 08:00 every morning. The arrival times of the students at this school follows exactly the same distribution as Malthouse school. [2 marks]

Given that, on one morning, 15 students arrive at least 60 minutes after the school opens, estimate the number of students at Mulberry Park school.

Applicants for a job had to complete a mathematics test. The time they took to complete the test is normally distributed with a mean of 53 minutes and a standard deviation of 16.3. One of the applicants is chosen at random.

7a. Find the probability that this applicant took at least 40 minutes to complete the test. [2 marks]

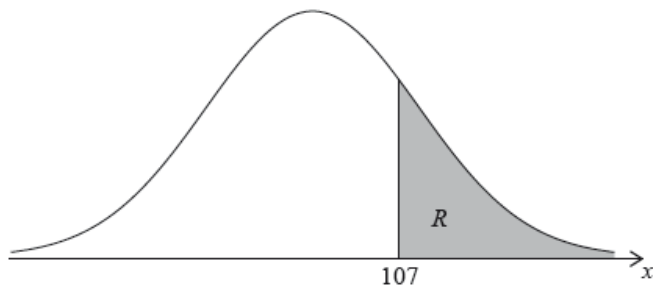
For 11% of the applicants it took longer than k minutes to complete the test.

7b. Find the value of k . [2 marks]

There were 400 applicants for the job.

7c. Estimate the number of applicants who completed the test in less than 25 minutes. [2 marks]

The random variable X is normally distributed with a mean of 100. The following diagram shows the normal curve for X .



Let R be the shaded region under the curve, to the right of 107. The area of R is 0.24.

8a. Write down $P(X > 107)$. [1 mark]

8b. Find $P(100 < X < 107)$. [3 marks]

8c. Find $P(93 < X < 107)$.

[2 marks]

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