## Trig in triangles 09.02 [101 marks]

1. Barry is at the top of a cliff, standing 80 m above sea level, and observes[6 marks] two yachts in the sea.

"Seaview" (S) is at an angle of depression of 25°.

"Nauti Buoy" (N) is at an angle of depression of 35°.

The following three dimensional diagram shows Barry and the two yachts at S and N.

X lies at the foot of the cliff and angle  $\mathrm{SXN}=$  70°.



Find, to 3 significant figures, the distance between the two yachts.

The lengths of two of the sides in a triangle are 4 cm and 5 cm. Let  $\theta$  be the angle between the two given sides. The triangle has an area of  $\frac{5\sqrt{15}}{2}$  cm<sup>2</sup>.

<sup>2a.</sup> Show that 
$$\sin \theta = \frac{\sqrt{15}}{4}$$
. [1 mark]


3. In triangle ABC, AB = 5, BC = 14 and AC = 11.

Find all the interior angles of the triangle. Give your answers in degrees to one decimal place.

. . . . . . . . . .  4. Consider quadrilateral PQRS where [PQ] is parallel to [SR].



In PQRS, PQ = x, SR = y, R $\widehat{S}P = \alpha$  and Q $\widehat{R}S = \beta$ . Find an expression for PS in terms of x, y, sin  $\beta$  and sin  $(\alpha + \beta)$ .

5a. Given that  $\cos 75^\circ = q$ , show that  $\cos 105^\circ = -q$ .

[1 mark]

[5 marks]

In the following diagram, the points A, B, C and D are on the circumference of a circle with centre O and radius r. [AC] is a diameter of the circle. BC = r,

$$AD = CD$$
 and  $A \stackrel{_{ heta}}{B} C = A \stackrel{_{ heta}}{D} C = 90^{\circ}$ .



5b. Show that  $B \overset{\wedge}{A} D = 75^{\circ}.$ 

[3 marks]


5d. By considering triangle CBD, find another expression for  $\mathrm{BD}^2$  in terms *[3 marks]* of r and q.

[3 marks]

5e. Use your answers to part (c) to show that  $\cos 75^\circ = rac{1}{\sqrt{6}+\sqrt{2}}.$ 


The following shape consists of three arcs of a circle, each with centre at the opposite vertex of an equilateral triangle as shown in the diagram.





For this shape, calculate

6a. the perimeter.

[2 marks]

7. A sector of a circle with radius r cm, where r > 0, is shown on the [4 marks] following diagram. The sector has an angle of 1 radian at the centre.



Let the area of the sector be  $A \operatorname{cm}^2$  and the perimeter be  $P \operatorname{cm}$ . Given that A = P, find the value of r.


Consider the rectangle OABC such that AB = OC = 10 and BC = OA = 1, with the points P, Q and R placed on the line OC such that OP = p, OQ = q and OR = r, such that 0 .



Let  $\theta_p$  be the angle APO,  $\theta_q$  be the angle AQO and  $\theta_r$  be the angle ARO.

8a. Find an expression for  $\theta_p$  in terms of p.

[3 marks]

Consider the case when  $\theta_p= heta_q+ heta_r$  and QR = 1.

<sup>8b.</sup> Show that $p=rac{q^2+q-1}{2q+1}.$	[6 marks]

8c. By sketching the graph of p as a function of q, determine the range of [4 marks] values of p for which there are possible values of q.

9. Boat A is situated 10km away from boat B, and each boat has a marine [6 marks] radio transmitter on board. The range of the transmitter on boat A is 7km, and the range of the transmitter on boat B is 5km. The region in which both transmitters can be detected is represented by the shaded region in the following diagram. Find the area of this region.



Consider the following diagram.



The sides of the equilateral triangle ABC have lengths 1 m. The midpoint of [AB] is denoted by P. The circular arc AB has centre, M, the midpoint of [CP].

LOa. Find AM.	[3 marks]

11. This diagram shows a metallic pendant made out of four equal sectors of [4 marks] a larger circle of radius OB = 9 cm and four equal sectors of a smaller circle of radius OA = 3 cm. The angle  $BOC = 20^{\circ}$ .



Find the area of the pendant.

12a. Find the set of values of k that satisfy the inequality  $k^2 - k - 12 < 0$ . [2 marks]

12b. The triangle ABC is shown in the following diagram. Given that [4 marks]  $\cos B < \frac{1}{4}$ , find the range of possible values for AB. С В 2 . . . . . . . . . . . . . 

In a triangle ABC, AB = 4cm, BC = 3cm and  $B\hat{A}C = \frac{\pi}{9}$ .

13a. Use the cosine rule to find the two possible values for AC.

[5 marks]

13b. Find the difference between the areas of the two possible triangles ABC.[3 marks]

The diagram shows two circles with centres at the points A and B and radii 2r and r, respectively. The point B lies on the circle with centre A. The circles intersect at the points C and D.



Let  $\alpha$  be the measure of the angle CAD and  $\theta$  be the measure of the angle CBD in radians.

14a. Find an expression for the shaded area in terms of  $\alpha$ ,  $\theta$  and r. [3 marks]

14b. Show that  $\alpha = 4 \arcsin \frac{1}{4}$ .

[2 marks]

Г

[3 marks]


© International Baccalaureate Organization 2023 International Baccalaureate® - Baccalauréat International® - Bachillerato Internacional®

Printed for 2 SPOLECZNE LICEUM