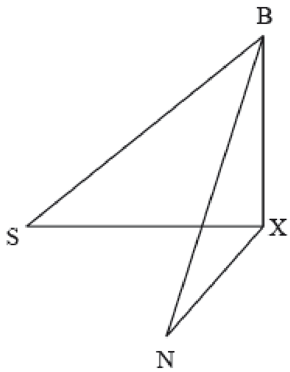
Trig in triangles 09.02 *[101 marks]*

1. Barry is at the top of a cliff, standing 80 m above sea level, and observes two yachts in the sea. *[6 marks]*
   "*Seaview*" (S) is at an angle of depression of 25°.
   "*Nauti Buoy*" (N) is at an angle of depression of 35°.
   The following three dimensional diagram shows Barry and the two yachts at S and N.
   X lies at the foot of the cliff and angle $\mathrm{SXN} = 70°$.



Find, to 3 significant figures, the distance between the two yachts.

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

attempt to use tan, or sine rule, in triangle BXN or BXS ***(M1)***

$\mathrm{NX} = 80\tan 55^\circ \left(= \frac{80}{\tan 35^\circ} = 114.25\right)$ ***(A1)***

$\mathrm{SX} = 80\tan 65^\circ \left(= \frac{80}{\tan 25^\circ} = 171.56\right)$ ***(A1)***

Attempt to use cosine rule ***M1***

$\mathrm{SN}^2 = 171.56^2 + 114.25^2 - 2 \times 171.56 \times 114.25 \cos 70°$ ***(A1)***

$\mathrm{SN} = 171\ (\mathrm{m})$ ***A1***

Note: Award final ***A1*** only if the correct answer has been given to 3 significant figures.

[6 marks]

The lengths of two of the sides in a triangle are 4 cm and 5 cm. Let θ be the angle between the two given sides. The triangle has an area of $\frac{5\sqrt{15}}{2}$ cm^2.

2a. Show that $\sin\theta = \frac{\sqrt{15}}{4}$. *[1 mark]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

EITHER

$\frac{5\sqrt{15}}{2} = \frac{1}{2} \times 4 \times 5\sin\theta$ ***A1***

OR

height of triangle is $\frac{5\sqrt{15}}{4}$ if using 4 as the base or $\sqrt{15}$ if using 5 as the base ***A1***

THEN

$\sin\theta = \frac{\sqrt{15}}{4}$ ***AG***

[1 mark]

2b. Find the two possible values for the length of the third side. *[6 marks]*

Markscheme

let the third side be x

$x^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \times \cos\theta$ ***M1***

valid attempt to find $\cos\theta$ ***(M1)***

Note: Do not accept writing $\cos\left(\arcsin\left(\frac{\sqrt{15}}{4}\right)\right)$ as a valid method.

$\cos\theta = \pm\sqrt{1 - \frac{15}{16}}$

$= \frac{1}{4},\ -\frac{1}{4}$ ***A1A1***

$x^2 = 16 + 25 - 2 \times 4 \times 5 \times \pm\frac{1}{4}$

$x = \sqrt{31}$ or $\sqrt{51}$ ***A1A1***

[6 marks]

3. In triangle ABC, AB = 5, BC = 14 and AC = 11. *[5 marks]*

Find all the interior angles of the triangle. Give your answers in degrees to one decimal place.

Markscheme

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attempt to apply cosine rule ***M1***

$\cos \mathrm{A} = \frac{5^2+11^2-14^2}{2\times5\times11} = -0.4545\ldots$

$\Rightarrow \mathrm{A} = 117.03569\ldots^\circ$

$\Rightarrow \mathrm{A} = 117.0^\circ$ ***A1***

attempt to apply sine rule or cosine rule: ***M1***

$\frac{\sin 117.03569\ldots^\circ}{14} = \frac{\sin \mathrm{B}}{11}$

$\Rightarrow \mathrm{B} = 44.4153\ldots^\circ$

$\Rightarrow \mathrm{B} = 44.4^\circ$ ***A1***

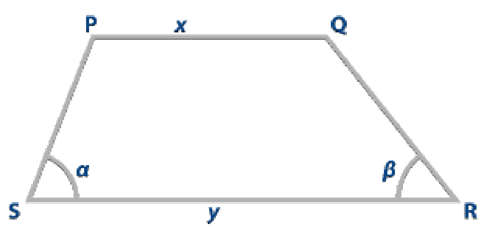
$\mathrm{C} = 180^\circ - \mathrm{A} - \mathrm{B}$

$\mathrm{C} = 18.5^\circ$ ***A1***

Note: Candidates may attempt to find angles in any order of their choosing.

[5 marks]

4. Consider quadrilateral PQRS where $[\mathrm{PQ}]$ is parallel to $[\mathrm{SR}]$. *[5 marks]*



In PQRS, $\mathrm{PQ} = x$, $\mathrm{SR} = y$, $\mathrm{R\hat{S}P} = \alpha$ and $\mathrm{Q\hat{R}S} = \beta$.

Find an expression for PS in terms of $x,\ y,\ \sin\beta$ and $\sin(\alpha+\beta)$.

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

METHOD 1

from vertex P, draws a line parallel to $[\mathrm{QR}]$ that meets $[\mathrm{SR}]$ at a point X **(M1)**

uses the sine rule in $\Delta\mathrm{PSX}$ **M1**

$\frac{\mathrm{PS}}{\sin\beta} = \frac{y-x}{\sin\left(180^\circ - \alpha - \beta\right)}$ **A1**

$\sin\left(180^\circ - \alpha - \beta\right) = \sin\left(\alpha + \beta\right)$ **(A1)**

$\mathrm{PS} = \frac{\left(y-x\right)\sin\beta}{\sin\left(\alpha+\beta\right)}$ **A1**

METHOD 2

let the height of quadrilateral PQRS be h

$h = \mathrm{PS}\sin\alpha$ **A1**

attempts to find a second expression for h **M1**

$h = \left(y - x - \mathrm{PS}\cos\alpha\right)\tan\beta$

$\mathrm{PS}\sin\alpha = \left(y - x - \mathrm{PS}\cos\alpha\right)\tan\beta$

writes $\tan\beta$ as $\frac{\sin\beta}{\cos\beta}$, multiplies through by $\cos\beta$ and expands the RHS **M1**

$\mathrm{PS}\sin\alpha\cos\beta = \left(y - x\right)\sin\beta - \mathrm{PS}\cos\alpha\sin\beta$

$\mathrm{PS} = \frac{\left(y-x\right)\sin\beta}{\sin\alpha\cos\beta+\cos\alpha\sin\beta}$ **A1**

$\mathrm{PS} = \frac{\left(y-x\right)\sin\beta}{\sin\left(\alpha+\beta\right)}$ **A1**

[5 marks]

5a. Given that $\cos 75^\circ = q$, show that $\cos 105^\circ = -q$. *[1 mark]*

Markscheme

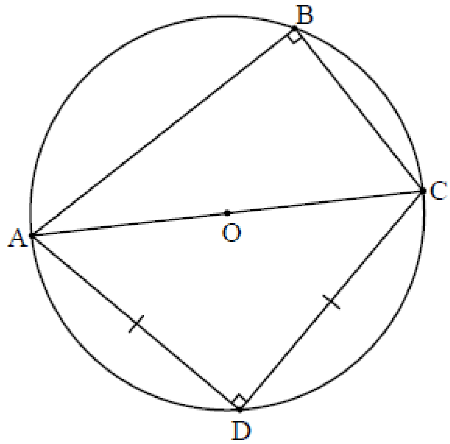
$\cos 105° = \cos(180° - 75°) = -\cos 75°$ ***R1***

$= -q$ ***AG***

Note: Accept arguments using the unit circle or graphical/diagrammatical considerations.

[1 mark]

In the following diagram, the points A, B, C and D are on the circumference of a circle with centre O and radius r. $[\mathrm{AC}]$ is a diameter of the circle. $\mathrm{BC} = r$, $\mathrm{AD} = \mathrm{CD}$ and $\mathrm{A}\hat{\mathrm{B}}\mathrm{C} = \mathrm{A}\hat{\mathrm{D}}\mathrm{C} = 90°$.



5b. *[3 marks]*

Show that $\mathrm{B}\hat{\mathrm{A}}\mathrm{D} = 75°$.

Markscheme

$\mathrm{AD} = \mathrm{CD} \Rightarrow \mathrm{C}\hat{\mathrm{A}}\mathrm{D} = 45°$ ***A1***

valid method to find $\mathrm{B}\hat{\mathrm{A}}\mathrm{C}$ ***(M1)***

for example: $\mathrm{BC} = r \Rightarrow \mathrm{B}\hat{\mathrm{C}}\mathrm{A} = 60°$

$\Rightarrow \mathrm{B}\hat{\mathrm{A}}\mathrm{C} = 30°$ ***A1***

hence $\mathrm{B}\hat{\mathrm{A}}\mathrm{D} = 45° + 30° = 75°$ ***AG***

[3 marks]

5c. By considering triangle ABD, show that $\mathrm{BD}^2 = 5r^2 - 2r^2 q\sqrt{6}$. *[4 marks]*

Markscheme

$\mathrm{AB} = r\sqrt{3}$, $\mathrm{AD} = (\mathrm{CD}) = r\sqrt{2}$ ***A1A1***

applying cosine rule ***(M1)***

$\mathrm{BD}^2 = \left(r\sqrt{3}\right)^2 + \left(r\sqrt{2}\right)^2 - 2\left(r\sqrt{3}\right)\left(r\sqrt{2}\right)\cos 75^\circ$ ***A1***

$= 3r^2 + 2r^2 - 2r^2\sqrt{6}\cos 75^\circ$

$= 5r^2 - 2r^2 q\sqrt{6}$ ***AG***

[4 marks]

5d. By considering triangle CBD, find another expression for BD^2 in terms of r and q. *[3 marks]*

Markscheme

$\mathrm{B}\hat{\mathrm{C}}\mathrm{D} = 105^\circ$ ***(A1)***

attempt to use cosine rule on $\Delta\mathrm{BCD}$ ***(M1)***

$\mathrm{BD}^2 = r^2 + \left(r\sqrt{2}\right)^2 - 2r\left(r\sqrt{2}\right)\cos 105^\circ$

$= 3r^2 + 2r^2 q\sqrt{2}$ ***A1***

[3 marks]

5e. Use your answers to part (c) to show that $\cos 75^\circ = \frac{1}{\sqrt{6}+\sqrt{2}}$. *[3 marks]*

Markscheme

$5r^2 - 2r^2q\sqrt{6} = 3r^2 + 2r^2q\sqrt{2}$ ***(M1)(A1)***

$2r^2 = 2r^2q\left(\sqrt{6} + \sqrt{2}\right)$ ***A1***

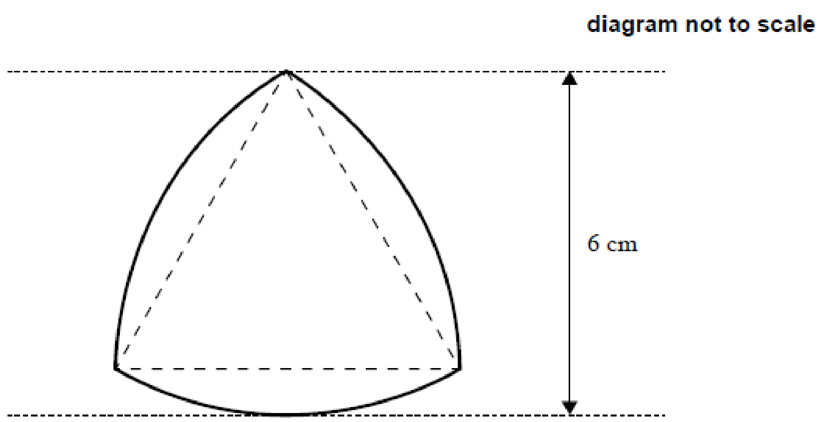
Note: Award ***A1*** for any correct intermediate step seen using only two terms.

$q = \frac{1}{\sqrt{6}+\sqrt{2}}$ ***AG***

Note: Do not award the final ***A1*** if follow through is being applied.

[3 marks]

The following shape consists of three arcs of a circle, each with centre at the opposite vertex of an equilateral triangle as shown in the diagram.



For this shape, calculate

6a. the perimeter. *[2 marks]*

Markscheme

each arc has length $r\theta = 6 \times \frac{\pi}{3} = 2\pi\ (= 6.283\ldots)$ ***(M1)***

perimeter is therefore $6\pi\ (= 18.8)$ (cm) ***A1***

[2 marks]

6b. the area. *[5 marks]*

Markscheme

area of sector, s, is $\frac{1}{2}r^2\theta = 18 \times \frac{\pi}{3} = 6\pi \ (= 18.84\ldots)$ ***(A1)***

area of triangle, t, is $\frac{1}{2} \times 6 \times 3\sqrt{3} = 9\sqrt{3} \ (= 15.58\ldots)$ ***(M1)(A1)***

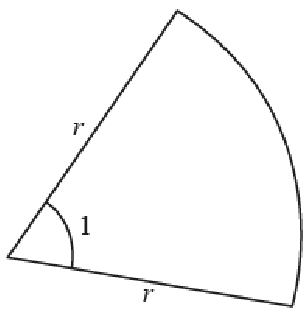
Note: area of segment, k, is 3.261… implies area of triangle

finding $3s - 2t$ or $3k + t$ or similar

area $= 3s - 2t = 18\pi - 18\sqrt{3} \ (= 25.4)$ (cm^2) ***(M1)A1***

[5 marks]

7. A sector of a circle with radius r cm , where r > 0, is shown on the following diagram. *[4 marks]*
The sector has an angle of 1 radian at the centre.



Let the area of the sector be A cm^2 and the perimeter be P cm. Given that $A = P$, find the value of r.

Markscheme

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$A = P$

use of the correct formula for area and arc length ***(M1)***

perimeter is $r\theta + 2r$ ***(A1)***

Note: ***A1*** independent of previous ***M1***.

$\frac{1}{2}r^2(1) = r(1) + 2r$ ***A1***

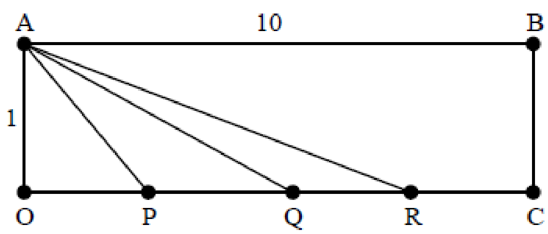
$r^2 - 6r = 0$

$r = 6$ (as r > 0) ***A1***

Note: Do not award final ***A1*** if $r = 0$ is included.

[4 marks]

Consider the rectangle OABC such that AB = OC = 10 and BC = OA = 1 , with the points P , Q and R placed on the line OC such that OP = p, OQ = q and OR = r, such that 0 < p < q < r < 10.



Let θ_p be the angle APO, θ_q be the angle AQO and θ_r be the angle ARO.

8a. Find an expression for θ_p in terms of p. *[3 marks]*

Markscheme

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METHOD 1

use of tan ***(M1)***

$\tan \theta_p = \frac{1}{p}$ ***(A1)***

$\theta_p = \arctan\left(\frac{1}{p}\right)$ ***A1***

METHOD 2

AP $= \sqrt{p^2 + 1}$ ***(A1)***

use of sin, cos, sine rule or cosine rule using the correct length of AP ***(M1)***

$\theta_p = \arcsin\left(\frac{1}{\sqrt{p^2+1}}\right)$ or $\theta_p = \arccos\left(\frac{p}{\sqrt{p^2+1}}\right)$ ***A1***

[3 marks]

Consider the case when $\theta_p = \theta_q + \theta_r$ and QR = 1.

8b. Show that $p = \frac{q^2+q-1}{2q+1}$. *[6 marks]*

Markscheme

QR = 1 ⇒ $r = q + 1$ ***(A1)***

Note: This may be seen anywhere.

$\tan\theta_p = \tan\left(\theta_q + \theta_r\right)$

attempt to use compound angle formula for tan ***M1***

$\tan\theta_p = \frac{\tan\theta_q + \tan\theta_r}{1 - \tan\theta_q \tan\theta_r}$ ***(A1)***

$\frac{1}{p} = \frac{\frac{1}{q} + \frac{1}{r}}{1 - \left(\frac{1}{q}\right)\left(\frac{1}{r}\right)}$ ***(M1)***

$\frac{1}{p} = \frac{\frac{1}{q} + \frac{1}{q+1}}{1 - \left(\frac{1}{q}\right)\left(\frac{1}{q+1}\right)}$ or $p = \frac{1 - \left(\frac{1}{q}\right)\left(\frac{1}{q+1}\right)}{\left(\frac{1}{q}\right) + \left(\frac{1}{q+1}\right)}$ ***A1***

$\frac{1}{p} = \frac{q + q + 1}{q(q+1) - 1}$ ***M1***

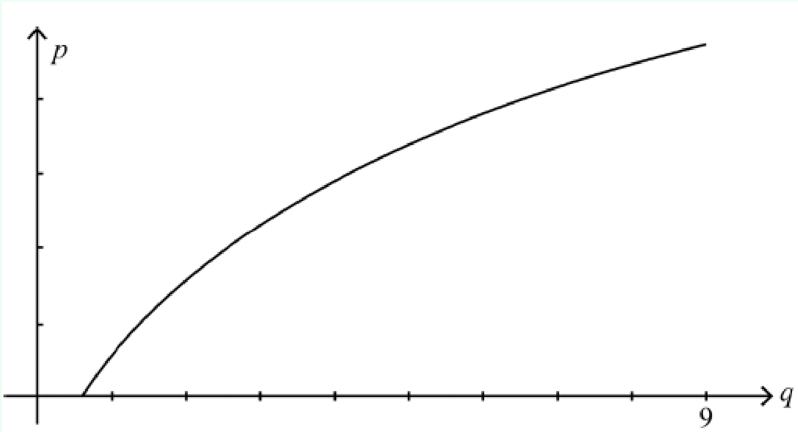
Note: Award ***M1*** for multiplying top and bottom by $q\left(q+1\right)$.

$p = \frac{q^2 + q - 1}{2q + 1}$ ***AG***

[6 marks]

8c. By sketching the graph of p as a function of q, determine the range of values of p for which there are possible values of q. *[4 marks]*

Markscheme



increasing function with positive q-intercept ***A1***

Note: Accept curves which extend beyond the domain shown above.

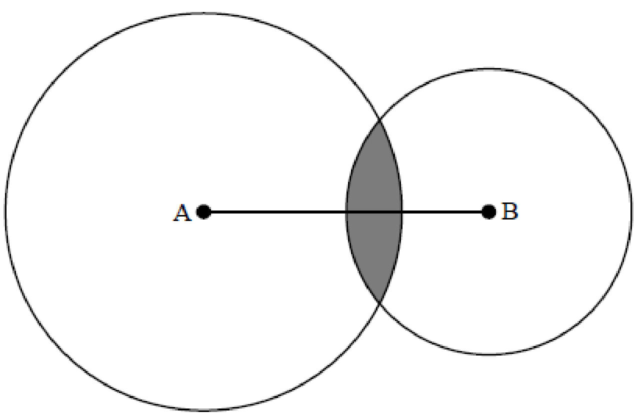
(0.618 <) q < 9 ***(A1)***

⇒ range is (0 <) p < 4.68 **(A1)**

0 < p < 4.68 ***A1***

[4 marks]

9. Boat A is situated 10km away from boat B, and each boat has a marine radio transmitter on board. The range of the transmitter on boat A is 7km, and the range of the transmitter on boat B is 5km. The region in which both transmitters can be detected is represented by the shaded region in the following diagram. Find the area of this region. *[6 marks]*



Markscheme

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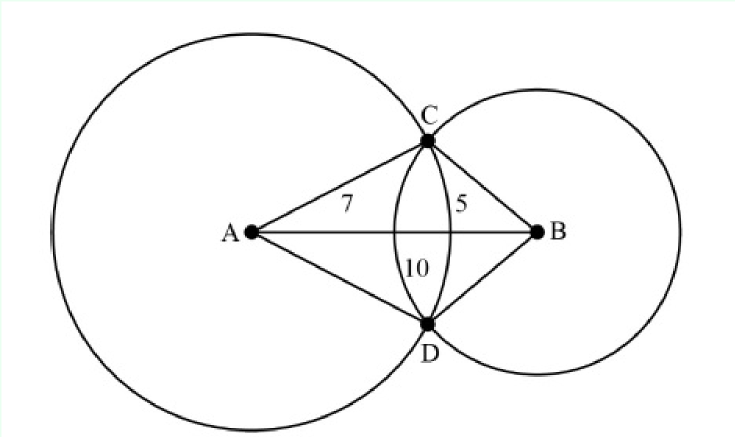


use of cosine rule ***(M1)***

$\text{C}\hat{\text{A}}\text{B} = \arccos\left(\frac{49+100-25}{2\times7\times10}\right) = 0.48276\ldots(= 27.660\ldots°)$ ***(A1)***

$\text{C}\overset{\wedge}{\text{B}}\text{A} = \arccos\left(\frac{25+100-49}{2\times5\times10}\right) = 0.70748\ldots(= 40.535\ldots°)$ ***(A1)***

attempt to subtract triangle area from sector area ***(M1)***

$\text{area} = \frac{1}{2}\times49\left(2\text{C}\overset{\wedge}{\text{A}}\text{B} - \sin 2\text{C}\overset{\wedge}{\text{A}}\text{B}\right) + \frac{1}{2}\times25\left(2\text{C}\overset{\wedge}{\text{B}}\text{A} - \sin 2\text{C}\overset{\wedge}{\text{B}}\text{A}\right)$

= 3.5079… + 5.3385… ***(A1)***

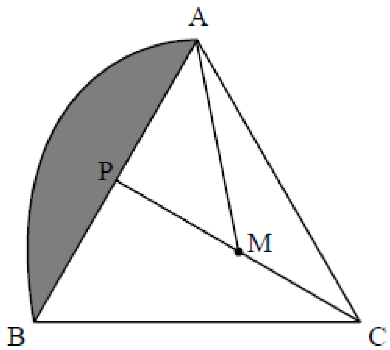
Note: Award this ***A1*** for either of these two values.

= 8.85 (km^2) ***A1***

Note: Accept all answers that round to 8.8 or 8.9.

[6 marks]

Consider the following diagram.



The sides of the equilateral triangle ABC have lengths 1 m. The midpoint of [AB] is denoted by P. The circular arc AB has centre, M, the midpoint of [CP].

10a. Find AM. *[3 marks]*

Markscheme

METHOD 1

PC $= \frac{\sqrt{3}}{2}$ or 0.8660 ***(M1)***

PM $= \frac{1}{2}$PC $= \frac{\sqrt{3}}{4}$ or 0.4330 **(A1)**

AM $= \sqrt{\frac{1}{4} + \frac{3}{16}}$

$= \frac{\sqrt{7}}{4}$ or 0.661 (m) ***A1***

METHOD 2

using the cosine rule

AM² $= 1^2 + \left(\frac{\sqrt{3}}{4}\right)^2 - 2 \times \frac{\sqrt{3}}{4} \times \cos\left(30^\circ\right)$ ***M1A1***

AM $= \frac{\sqrt{7}}{4}$ or 0.661 (m) ***A1***

[3 marks]

10b. Find $\mathrm{A}\hat{\mathrm{M}}\mathrm{P}$ in radians. *[2 marks]*

Markscheme

$\tan(\mathrm{A}\hat{\mathrm{M}}\mathrm{P}) = \frac{2}{\sqrt{3}}$ or equivalent ***(M1)***

= 0.857 ***A1***

[2 marks]

10c. Find the area of the shaded region. *[3 marks]*

Markscheme

EITHER

$\frac{1}{2}\mathrm{AM}^2\left(2\,\mathrm{A}\hat{\mathrm{M}}\mathrm{P} - \sin\left(2\,\mathrm{A}\hat{\mathrm{M}}\mathrm{P}\right)\right)$ ***(M1)A1***

OR

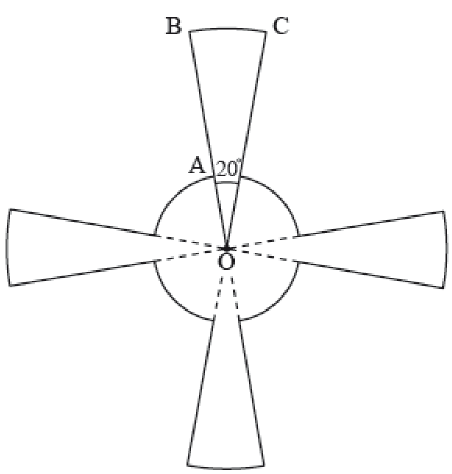
$\frac{1}{2}\mathrm{AM}^2 \times 2\,\mathrm{A}\hat{\mathrm{M}}\mathrm{P} - = \frac{\sqrt{3}}{8}$ ***(M1)A1***

= 0.158(m^2) ***A1***

Note: Award ***M1*** for attempting to calculate area of a sector minus area of a triangle.

[3 marks]

11. This diagram shows a metallic pendant made out of four equal sectors of a larger circle of radius $\mathrm{OB} = 9\ \mathrm{cm}$ and four equal sectors of a smaller circle of radius $\mathrm{OA} = 3\ \mathrm{cm}$. *[4 marks]*
The angle $\mathrm{BOC} =$ 20°.



Find the area of the pendant.

Markscheme

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METHOD 1

area = (four sector areas radius 9) + (four sector areas radius 3) ***(M1)***

$= 4\left(\frac{1}{2}9^2\frac{\pi}{9}\right) + 4\left(\frac{1}{2}3^2\frac{7\pi}{18}\right)$ ***(A1)(A1)***

$= 18\pi + 7\pi$

$= 25\pi \left(= 78.5 \text{ cm}^2\right)$ ***A1***

METHOD 2

area =

(area of circle radius 3) + (four sector areas radius 9) – (four sector areas radius 3) ***(M1)***

$\pi 3^2 + 4\left(\frac{1}{2}9^2\frac{\pi}{9}\right) - 4\left(\frac{1}{2}3^2\frac{\pi}{9}\right)$ ***(A1)(A1)***

Note: Award ***A1*** for the second term and ***A1*** for the third term.

$= 9\pi + 18\pi - 2\pi$

$= 25\pi \left(= 78.5 \text{ cm}^2\right)$ ***A1***

Note: Accept working in degrees.

[4 marks]

12a. Find the set of values of k that satisfy the inequality $k^2 - k - 12 < 0$. *[2 marks]*

Markscheme

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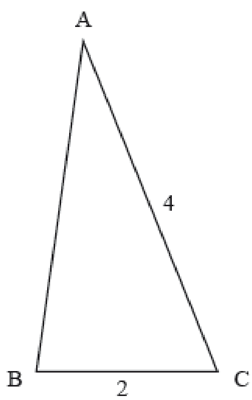
$k^2 - k - 12 < 0$

$(k-4)(k+3) < 0$ ***(M1)***

$-3 < k < 4$ ***A1***

[2 marks]

12b. The triangle ABC is shown in the following diagram. Given that $\cos B < \frac{1}{4}$, find the range of possible values for AB. *[4 marks]*



Markscheme

$\cos B = \frac{2^2 + c^2 - 4^2}{4c}$ $\left(\text{or } 16 = 2^2 + c^2 - 4c\cos B\right)$ ***M1***

$\Rightarrow \frac{c^2 - 12}{4c} < \frac{1}{4}$ ***A1***

$\Rightarrow c^2 - c - 12 < 0$

from result in (a)

$0 < \text{AB} < 4$ or $-3 < \text{AB} < 4$ ***(A1)***

but AB must be at least 2

$\Rightarrow 2 < \text{AB} < 4$ ***A1***

Note: Allow $\leqslant \text{AB}$ for either of the final two ***A*** marks.

[4 marks]

In a triangle ABC, $\text{AB} = 4\text{cm}$, $\text{BC} = 3\text{cm}$ and $\text{B}\hat{\text{A}}\text{C} = \frac{\pi}{9}$.

13a. Use the cosine rule to find the two possible values for AC. *[5 marks]*

Markscheme

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METHOD 1

let $\text{AC} = x$

$3^2 = x^2 + 4^2 - 8x\cos\frac{\pi}{9}$ ***M1A1***

attempting to solve for x ***(M1)***

$x = 1.09,\ 6.43$ ***A1A1***

METHOD 2

let $\text{AC} = x$

using the sine rule to find a value of C ***M1***

$4^2 = x^2 + 3^2 - 6x\cos(152.869\ldots^\circ) \Rightarrow x = 1.09$ ***(M1)A1***

$4^2 = x^2 + 3^2 - 6x\cos(27.131\ldots^\circ) \Rightarrow x = 6.43$ ***(M1)A1***

METHOD 3

let $\text{AC} = x$

using the sine rule to find a value of B and a value of C ***M1***

obtaining $B = 132.869\ldots^\circ,\ 7.131\ldots^\circ$ and $C = 27.131\ldots^\circ,\ 152.869\ldots^\circ$ ***A1***

$(B = 2.319\ldots,\ 0.124\ldots$ and $C = 0.473\ldots,\ 2.668\ldots)$

attempting to find a value of x using the cosine rule ***(M1)***

$x = 1.09,\ 6.43$ ***A1A1***

Note: Award ***M1A0(M1)A1A0*** for one correct value of x

[5 marks]

13b. Find the difference between the areas of the two possible triangles ABC. *[3 marks]*

Markscheme

$\frac{1}{2} \times 4 \times 6.428\ldots \times \sin\frac{\pi}{9}$ and $\frac{1}{2} \times 4 \times 1.088\ldots \times \sin\frac{\pi}{9}$ ***(A1)***

($4.39747\ldots$ and $0.744833\ldots$)

let D be the difference between the two areas

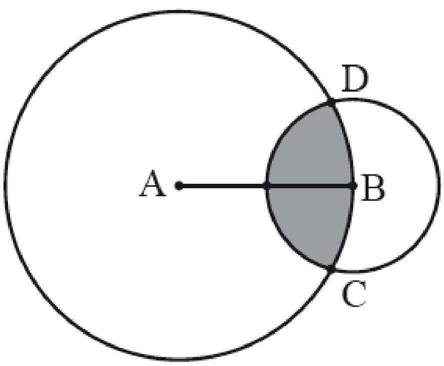
$D = \frac{1}{2} \times 4 \times 6.428\ldots \times \sin\frac{\pi}{9} - \frac{1}{2} \times 4 \times 1.088\ldots \times \sin\frac{\pi}{9}$ ***(M1)***

$(D = 4.39747\ldots - 0.744833\ldots)$

$= 3.65\ (\mathrm{cm}^2)$ ***A1***

[3 marks]

The diagram shows two circles with centres at the points A and B and radii $2r$ and r, respectively. The point B lies on the circle with centre A. The circles intersect at the points C and D.



Let α be the measure of the angle CAD and θ be the measure of the angle CBD in radians.

14a. Find an expression for the shaded area in terms of α, θ and r. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$A = 2(\alpha - \sin\alpha)r^2 + \frac{1}{2}(\theta - \sin\theta)r^2$ ***M1A1A1***

Note: Award ***M1A1A1*** for alternative correct expressions *eg*.
$A = 4\left(\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)r^2 + \frac{1}{2}\theta r^2$.

[3 marks]

14b. Show that $\alpha = 4\arcsin\frac{1}{4}$. *[2 marks]*

Markscheme

METHOD 1

consider for example triangle ADM where M is the midpoint of BD ***M1***

$\sin \frac{\alpha}{4} = \frac{1}{4}$ ***A1***

$\frac{\alpha}{4} = \arcsin \frac{1}{4}$

$\alpha = 4 \arcsin \frac{1}{4}$ ***AG***

METHOD 2

attempting to use the cosine rule (to obtain $1 - \cos \frac{\alpha}{2} = \frac{1}{8}$) ***M1***

$\sin \frac{\alpha}{4} = \frac{1}{4}$ (obtained from $\sin \frac{\alpha}{4} = \sqrt{\frac{1-\cos \frac{\alpha}{2}}{2}}$) ***A1***

$\frac{\alpha}{4} = \arcsin \frac{1}{4}$

$\alpha = 4 \arcsin \frac{1}{4}$ ***AG***

METHOD 3

$\sin\left(\frac{\pi}{2} - \frac{\alpha}{4}\right) = 2 \sin \frac{\alpha}{2}$ where $\frac{\theta}{2} = \frac{\pi}{2} - \frac{\alpha}{4}$

$\cos \frac{\alpha}{4} = 4 \sin \frac{\alpha}{4} \cos \frac{\alpha}{4}$ ***M1***

Note: Award ***M1*** either for use of the double angle formula or the conversion from sine to cosine.

$\frac{1}{4} = \sin \frac{\alpha}{4}$ ***A1***

$\frac{\alpha}{4} = \arcsin \frac{1}{4}$

$\alpha = 4 \arcsin \frac{1}{4}$ ***AG***

[2 marks]

14c. Hence find the value of r given that the shaded area is equal to 4. *[3 marks]*

Markscheme

(from triangle ADM), $\theta = \pi - \frac{\alpha}{2}$ $\left(= \pi - 2\arcsin\frac{1}{4} = 2\arcsin\frac{1}{4} = 2.6362\ldots\right)$
A1

attempting to solve $2(\alpha - \sin\alpha)r^2 + \frac{1}{2}(\theta - \sin\theta)r^2 = 4$

with $\alpha = 4\arcsin\frac{1}{4}$ and $\theta = \pi - \frac{\alpha}{2}$ $\left(= 2\arccos\frac{1}{4}\right)$ for r ***(M1)***

$r = 1.69$ ***A1***

[3 marks]