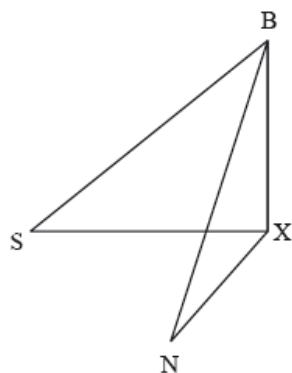


Trig in triangles 09.02 [101 marks]

1. Barry is at the top of a cliff, standing 80 m above sea level, and observes [6 marks] two yachts in the sea.
"Seaview" (S) is at an angle of depression of 25° .
"Nauti Buoy" (N) is at an angle of depression of 35° .
The following three dimensional diagram shows Barry and the two yachts at S and N .
 X lies at the foot of the cliff and angle $SXN = 70^\circ$.



Find, to 3 significant figures, the distance between the two yachts.

The lengths of two of the sides in a triangle are 4 cm and 5 cm. Let θ be the angle between the two given sides. The triangle has an area of $\frac{5\sqrt{15}}{2}$ cm².

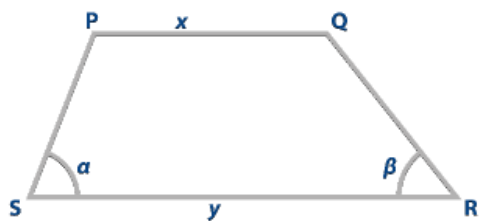
2a. Show that $\sin \theta = \frac{\sqrt{15}}{4}$. [1 mark]

2b. Find the two possible values for the length of the third side. [6 marks]

3. In triangle ABC, $AB = 5$, $BC = 14$ and $AC = 11$. [5 marks]

Find all the interior angles of the triangle. Give your answers in degrees to one decimal place.

4. Consider quadrilateral PQRS where [PQ] is parallel to [SR]. [5 marks]



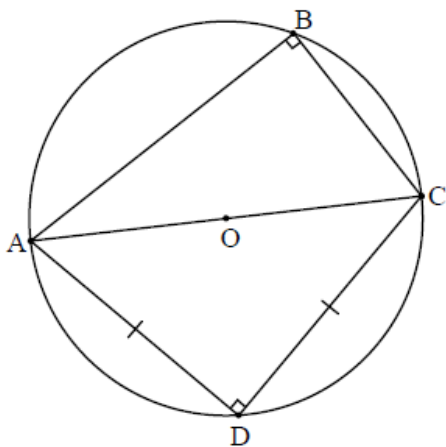
In PQRS, $PQ = x$, $SR = y$, $\widehat{RSP} = \alpha$ and $\widehat{QRS} = \beta$.

Find an expression for PS in terms of x , y , $\sin \beta$ and $\sin(\alpha + \beta)$.

- 5a. Given that $\cos 75^\circ = q$, show that $\cos 105^\circ = -q$. [1 mark]

In the following diagram, the points A, B, C and D are on the circumference of a circle with centre O and radius r . [AC] is a diameter of the circle. $BC = r$,

$AD = CD$ and $\widehat{ABC} = \widehat{ADC} = 90^\circ$.



- 5b. Show that $\widehat{BAD} = 75^\circ$. [3 marks]

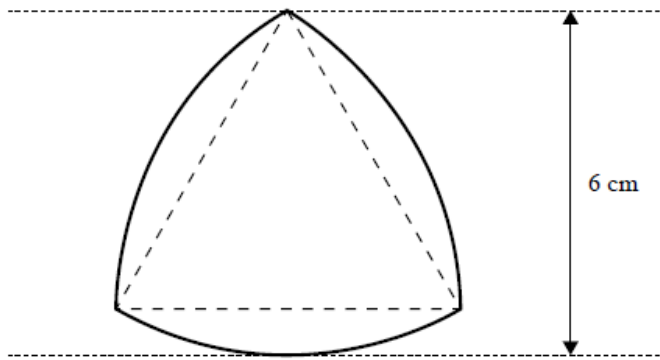
- 5c. By considering triangle ABD, show that $BD^2 = 5r^2 - 2r^2q\sqrt{6}$. [4 marks]

- 5d. By considering triangle CBD, find another expression for BD^2 in terms of r and q . [3 marks]

- 5e. Use your answers to part (c) to show that $\cos 75^\circ = \frac{1}{\sqrt{6} + \sqrt{2}}$. [3 marks]

The following shape consists of three arcs of a circle, each with centre at the opposite vertex of an equilateral triangle as shown in the diagram.

diagram not to scale

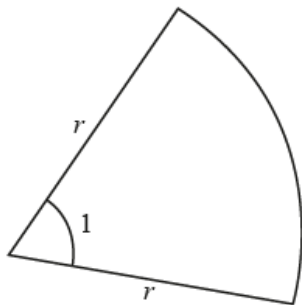


For this shape, calculate

6a. the perimeter. [2 marks]

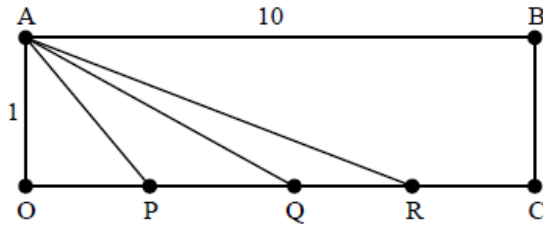
6b. the area. [5 marks]

7. A sector of a circle with radius r cm, where $r > 0$, is shown on the following diagram. [4 marks]
The sector has an angle of 1 radian at the centre.



Let the area of the sector be A cm² and the perimeter be P cm. Given that $A = P$, find the value of r .

Consider the rectangle OABC such that $AB = OC = 10$ and $BC = OA = 1$, with the points P, Q and R placed on the line OC such that $OP = p$, $OQ = q$ and $OR = r$, such that $0 < p < q < r < 10$.



Let θ_p be the angle APO, θ_q be the angle AQO and θ_r be the angle ARO.

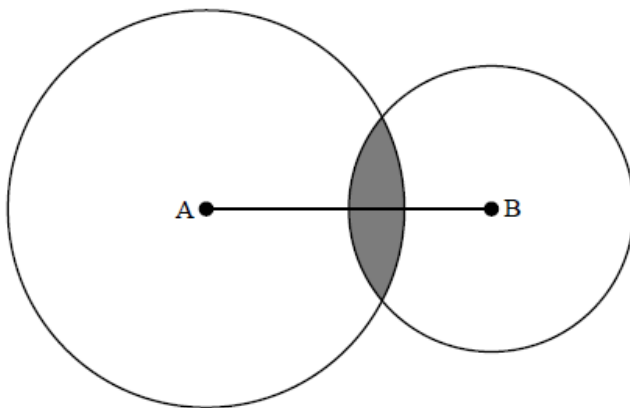
8a. Find an expression for θ_p in terms of p . [3 marks]

Consider the case when $\theta_p = \theta_q + \theta_r$ and $QR = 1$.

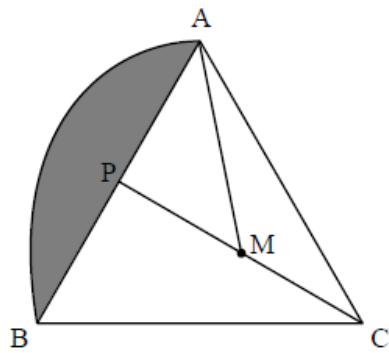
8b. Show that $p = \frac{q^2 + q - 1}{2q + 1}$. [6 marks]

8c. By sketching the graph of p as a function of q , determine the range of values of p for which there are possible values of q . [4 marks]

9. Boat A is situated 10km away from boat B, and each boat has a marine radio transmitter on board. The range of the transmitter on boat A is 7km, and the range of the transmitter on boat B is 5km. The region in which both transmitters can be detected is represented by the shaded region in the following diagram. Find the area of this region. [6 marks]



Consider the following diagram.



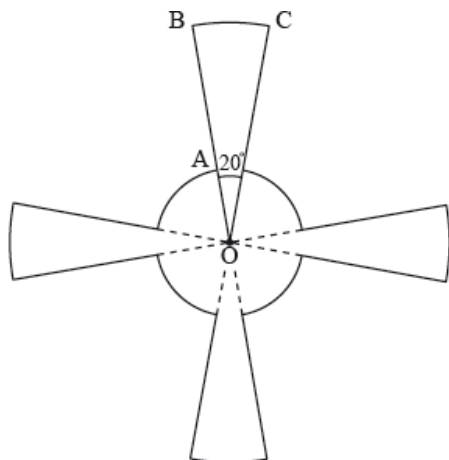
The sides of the equilateral triangle ABC have lengths 1 m. The midpoint of $[AB]$ is denoted by P . The circular arc AB has centre, M , the midpoint of $[BC]$.

10a. Find AM . [3 marks]

10b. Find \hat{AMP} in radians. [2 marks]

10c. Find the area of the shaded region. [3 marks]

11. This diagram shows a metallic pendant made out of four equal sectors of a larger circle of radius $OB = 9$ cm and four equal sectors of a smaller circle of radius $OA = 3$ cm. The angle $BOC = 20^\circ$. [4 marks]

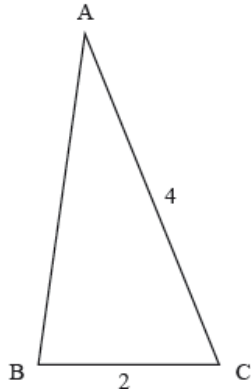


Find the area of the pendant.

12a. Find the set of values of k that satisfy the inequality $k^2 - k - 12 < 0$. [2 marks]

12b. The triangle ABC is shown in the following diagram. Given that $\cos B < \frac{1}{4}$, find the range of possible values for AB.

[4 marks]



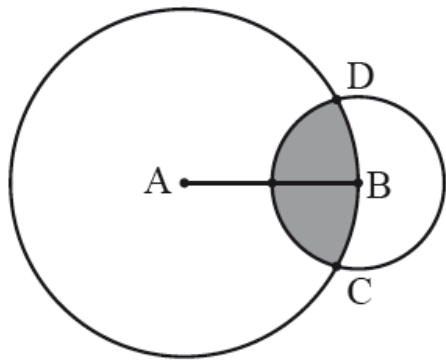
In a triangle ABC, $AB = 4\text{cm}$, $BC = 3\text{cm}$ and $\hat{BAC} = \frac{\pi}{9}$.

13a. Use the cosine rule to find the two possible values for AC.

[5 marks]

13b. Find the difference between the areas of the two possible triangles ABC. [3 marks]

The diagram shows two circles with centres at the points A and B and radii $2r$ and r , respectively. The point B lies on the circle with centre A. The circles intersect at the points C and D.



Let α be the measure of the angle CAD and θ be the measure of the angle CBD in radians.

14a. Find an expression for the shaded area in terms of α , θ and r .

[3 marks]

14b. Show that $\alpha = 4 \arcsin \frac{1}{4}$.

[2 marks]

14c. Hence find the value of r given that the shaded area is equal to 4.

[3 marks]

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