

# Trigonometry 8.02 [106 marks]

1. Solve the equation  $2 \cos^2 x + 5 \sin x = 4$ ,  $0 \leq x \leq 2\pi$ . [7 marks]

2. Let  $a = \sin b$ ,  $0 < b < \frac{\pi}{2}$ .

[5 marks]

Find, in terms of  $b$ , the solutions of  $\sin 2x = -a$ ,  $0 \leq x \leq \pi$ .

Consider the function  $f(x) = 2\sin^2 x + 7\sin 2x + \tan x - 9$ ,  $0 \leq x < \frac{\pi}{2}$ .

3a. Determine an expression for  $f'(x)$  in terms of  $x$ .

[2 marks]

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3b. Sketch a graph of  $y = f'(x)$  for  $0 \leq x < \frac{\pi}{2}$ .

[4 marks]

3c. Find the  $x$ -coordinate(s) of the point(s) of inflexion of the graph of  $y = f(x)$ , labelling these clearly on the graph of  $y = f'(x)$ .

[2 marks]

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Let  $u = \tan x$ .

3d. Express  $\sin x$  in terms of  $u$ .

[2 marks]

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3e. Express  $\sin 2x$  in terms of  $u$ .

[3 marks]

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3f. Hence show that  $f(x) = 0$  can be expressed as  $u^3 - 7u^2 + 15u - 9 = 0$  [2 marks]

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3g. Solve the equation  $f(x) = 0$ , giving your answers in the form  $\arctan k$  [3 marks] where  $k \in \mathbb{Z}$ .

4a. Show that  $\cot 2\theta = \frac{1-\tan^2 \theta}{2\tan \theta}$ . [1 mark]

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4b. Verify that  $x = \tan \theta$  and  $x = -\cot \theta$  satisfy the equation  $x^2 + (2 \cot 2\theta)x - 1 = 0$ .

[7 marks]

4c. Hence, or otherwise, show that the exact value of  $\tan \frac{\pi}{12} = 2 - \sqrt{3}$ . [5 marks]

4d. Using the results from parts (b) and (c) find the exact value of  $\tan \frac{\pi}{24} - \cot \frac{\pi}{24}$ .

[6 marks]

Give your answer in the form  $a + b\sqrt{3}$  where  $a, b \in \mathbb{Z}$ .

5. It is given that  $\operatorname{cosec} \theta = \frac{3}{2}$ , where  $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$ . Find the exact value of  $\cot \theta$ . [4 marks]

6.  $A$  and  $B$  are acute angles such that  $\cos A = \frac{2}{3}$  and  $\sin B = \frac{1}{3}$ . [7 marks]

Show that  $\cos(2A + B) = -\frac{2\sqrt{2}}{27} - \frac{4\sqrt{5}}{27}$ .

7a. Show that  $(\sin x + \cos x)^2 = 1 + \sin 2x$ .

[2 marks]

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7b. Show that  $\sec 2x + \tan 2x = \frac{\cos x + \sin x}{\cos x - \sin x}$ .

[4 marks]

- 7c. Hence or otherwise find  $\int_0^{\frac{\pi}{6}} (\sec 2x + \tan 2x) dx$  in the form  $\ln(a + \sqrt{b})$  where  $a, b \in \mathbb{Z}$ .

[9 marks]

Consider the function  $g(x) = 4 \cos x + 1$ ,  $a \leq x \leq \frac{\pi}{2}$  where  $a < \frac{\pi}{2}$ .

- 8a. For  $a = -\frac{\pi}{2}$ , sketch the graph of  $y = g(x)$ . Indicate clearly the maximum and minimum values of the function. [3 marks]

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- 8b. Write down the least value of  $a$  such that  $g$  has an inverse. [1 mark]

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- 8c. For the value of  $a$  found in part (b), write down the domain of  $g^{-1}$ . [1 mark]

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8d. For the value of  $a$  found in part (b), find an expression for  $g^{-1}(x)$ . [2 marks]

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9. Solve the equation  $\sec^2 x + 2 \tan x = 0$ ,  $0 \leq x \leq 2\pi$ . [5 marks]

10a. Find the value of  $\sin \frac{\pi}{4} + \sin \frac{3\pi}{4} + \sin \frac{5\pi}{4} + \sin \frac{7\pi}{4} + \sin \frac{9\pi}{4}$ .

[2 marks]

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10b. Show that  $\frac{1-\cos 2x}{2 \sin x} \equiv \sin x$ ,  $x \neq k\pi$  where  $k \in \mathbb{Z}$ .

[2 marks]

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10c. Use the principle of mathematical induction to prove that

[9 marks]

$$\sin x + \sin 3x + \dots + \sin(2n-1)x = \frac{1-\cos 2nx}{2\sin x}, \quad n \in \mathbb{Z}^+, \quad x \neq k\pi \text{ where } k \in \mathbb{Z}.$$

10d. Hence or otherwise solve the equation  $\sin x + \sin 3x = \cos x$  in the interval  $0 < x < \pi$ . [6 marks]