Trigonometry 8.02 *[106 marks]*

1. Solve the equation $2\cos^2 x + 5\sin x = 4, 0 \le x \le 2\pi$. *[7 marks]*

Markscheme

attempt to use $\cos^2 x = 1 - \sin^2 x$ ***M1***

$2\sin^2 x - 5\sin x + 2 = 0$ ***A1***

EITHER

attempting to factorise ***M1***

$(2\sin x - 1)(\sin x - 2)$ ***A1***

OR

attempting to use the quadratic formula ***M1***

$\sin x = \frac{5 \pm \sqrt{5^2 - 4\times 2\times 2}}{4} \left(= \frac{5\pm 3}{4}\right)$ ***A1***

THEN

$\sin x = \frac{1}{2}$ ***(A1)***

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ ***A1A1***

[7 marks]

2. Let $a = \sin b,\ 0 < b < \frac{\pi}{2}$. *[5 marks]*

Find, in terms of *b*, the solutions of $\sin 2x = -a,\ 0 \leqslant x \leqslant \pi$.

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

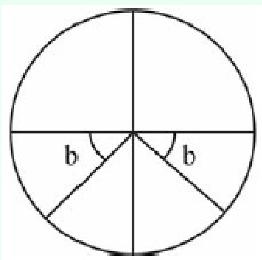
$\sin 2x = -\sin b$

EITHER

$\sin 2x = \sin(-b)$ or $\sin 2x = \sin(\pi + b)$ or $\sin 2x = \sin(2\pi - b)$ … ***(M1)(A1)***

Note: Award ***M1*** for any one of the above, ***A1*** for having final two.

OR



(M1)(A1)

Note: Award ***M1*** for one of the angles shown with b clearly labelled, ***A1*** for both angles shown. Do not award ***A1*** if an angle is shown in the second quadrant and subsequent ***A1*** marks not awarded.

THEN

$2x = \pi + b$ or $2x = 2\pi - b$ ***(A1)(A1)***

$x = \frac{\pi}{2} + \frac{b}{2},\ x = \pi - \frac{b}{2}$ ***A1***

[5 marks]

Consider the function $f(x) = 2\sin^2 x + 7\sin 2x + \tan x - 9, 0 \leqslant x < \frac{\pi}{2}$.

3a. Determine an expression for $f'(x)$ in terms of x. *[2 marks]*

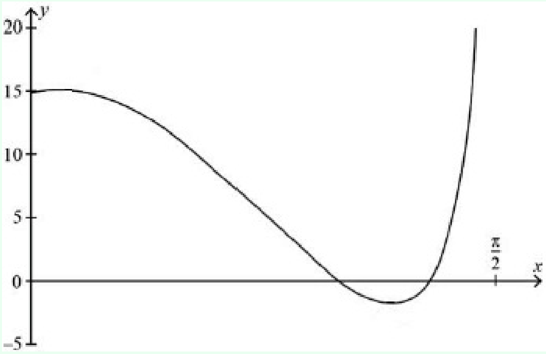
Markscheme

$f'(x) = 4\sin x \cos x + 14\cos 2x + \sec^2 x$ (or equivalent) ***(M1)A1***

[2 marks]

3b. Sketch a graph of $y = f'(x)$ for $0 \leqslant x < \frac{\pi}{2}$. *[4 marks]*

Markscheme



A1A1A1A1

Note: Award ***A1*** for correct behaviour at $x = 0$, ***A1*** for correct domain and correct behaviour for $x \to \frac{\pi}{2}$, ***A1*** for two clear intersections with x-axis and minimum point, ***A1*** for clear maximum point.

[4 marks]

3c. Find the x-coordinate(s) of the point(s) of inflexion of the graph of $y = f(x)$, labelling these clearly on the graph of $y = f'(x)$. *[2 marks]*

Markscheme

$x = 0.0736$ ***A1***

$x = 1.13$ ***A1***

[2 marks]

Let $u = \tan x$.

3d. Express $\sin x$ in terms of u. *[2 marks]*

Markscheme

attempt to write $\sin x$ in terms of u only ***(M1)***

$\sin x = \frac{u}{\sqrt{1+u^2}}$ ***A1***

[2 marks]

3e. Express $\sin 2x$ in terms of u. *[3 marks]*

Markscheme

$\cos x = \frac{1}{\sqrt{1+u^2}}$ ***(A1)***

attempt to use $\sin 2x = 2\sin x\cos x$ $\left(= 2\frac{u}{\sqrt{1+u^2}}\frac{1}{\sqrt{1+u^2}}\right)$ ***(M1)***

$\sin 2x = \frac{2u}{1+u^2}$ ***A1***

[3 marks]

3f. Hence show that $f(x) = 0$ can be expressed as $u^3 - 7u^2 + 15u - 9 = 0$. *[2 marks]*

Markscheme

$2\sin^2 x + 7\sin 2x + \tan x - 9 = 0$

$\frac{2u^2}{1+u^2} + \frac{14u}{1+u^2} + u - 9\ (= 0)$ ***M1***

$\frac{2u^2+14u+u(1+u^2)-9(1+u^2)}{1+u^2} = 0$ (or equivalent) ***A1***

$u^3 - 7u^2 + 15u - 9 = 0$ ***AG***

[2 marks]

3g. Solve the equation $f(x) = 0$, giving your answers in the form $\arctan k$ where $k \in \mathbb{Z}$. *[3 marks]*

Markscheme

$u = 1$ or $u = 3$ ***(M1)***

$x = \arctan(1)$ ***A1***

$x = \arctan(3)$ ***A1***

Note: Only accept answers given the required form.

[3 marks]

4a. Show that $\cot 2\theta = \frac{1-\tan^2\theta}{2\tan\theta}$. *[1 mark]*

Markscheme

stating the relationship between $\cot$ and $\tan$ and stating the identity for $\tan 2\theta$ ***M1***

$\cot 2\theta = \frac{1}{\tan 2\theta}$ and $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\Rightarrow \cot 2\theta = \frac{1-\tan^2\theta}{2\tan\theta}$ ***AG***

[1 mark]

4b. Verify that $x = \tan\theta$ and $x = -\cot\theta$ satisfy the equation $x^2 + (2\cot 2\theta)\,x - 1 = 0$. *[7 marks]*

Markscheme

METHOD 1

attempting to substitute $\tan\theta$ for x and using the result from (a) ***M1***

LHS = $\tan^2\theta + 2\tan\theta\left(\frac{1-\tan^2\theta}{2\tan\theta}\right) - 1$ ***A1***

$\tan^2\theta + 1 - \tan^2\theta - 1 = 0$(= RHS) ***A1***

so $x = \tan\theta$ satisfies the equation ***AG***

attempting to substitute $-\cot\theta$ for x and using the result from (a) ***M1***

LHS = $\cot^2\theta - 2\cot\theta\left(\frac{1-\tan^2\theta}{2\tan\theta}\right) - 1$ ***A1***

$= \frac{1}{\tan^2\theta} - \left(\frac{1-\tan^2\theta}{\tan^2\theta}\right) - 1$ ***A1***

$\frac{1}{\tan^2\theta} - \frac{1}{\tan^2\theta} + 1 - 1 = 0$(= RHS) ***A1***

so $x = -\cot\theta$ satisfies the equation ***AG***

METHOD 2

let $\alpha = \tan\theta$ and $\beta = -\cot\theta$

attempting to find the sum of roots ***M1***

$\alpha + \beta = \tan\theta - \frac{1}{\tan\theta}$

$= \frac{\tan^2\theta - 1}{\tan\theta}$ ***A1***

$= -2\cot 2\theta$ (from part (a)) ***A1***

attempting to find the product of roots ***M1***

$\alpha\beta = \tan\theta \times (-\cot\theta)$ ***A1***

= −1 ***A1***

the coefficient of x and the constant term in the quadratic are $2\cot 2\theta$ and −1 respectively ***R1***

hence the two roots are $\alpha = \tan\theta$ and $\beta = -\cot\theta$ ***AG***

[7 marks]

4c. Hence, or otherwise, show that the exact value of $\tan\frac{\pi}{12} = 2 - \sqrt{3}$. *[5 marks]*

Markscheme

METHOD 1

$x = \tan\frac{\pi}{12}$ and $x = -\cot\frac{\pi}{12}$ are roots of $x^2 + \left(2\cot\frac{\pi}{6}\right)x - 1 = 0$ ***R1***

Note: Award ***R1*** if only $x = \tan\frac{\pi}{12}$ is stated as a root of $x^2 + \left(2\cot\frac{\pi}{6}\right)x - 1 = 0$.

$x^2 + 2\sqrt{3}x - 1 = 0$ ***A1***

attempting to solve **their** quadratic equation ***M1***

$x = -\sqrt{3} \pm 2$ ***A1***

$\tan\frac{\pi}{12} > 0$ $\left(-\cot\frac{\pi}{12} < 0\right)$ ***R1***

so $\tan\frac{\pi}{12} = 2 - \sqrt{3}$ ***AG***

METHOD 2

attempting to substitute $\theta = \frac{\pi}{12}$ into the identity for $\tan 2\theta$ ***M1***

$\tan\frac{\pi}{6} = \frac{2\tan\frac{\pi}{12}}{1-\tan^2\frac{\pi}{12}}$

$\tan^2\frac{\pi}{12} + 2\sqrt{3}\tan\frac{\pi}{12} - 1 = 0$ ***A1***

attempting to solve **their** quadratic equation ***M1***

$\tan\frac{\pi}{12} = -\sqrt{3} \pm 2$ ***A1***

$\tan\frac{\pi}{12} > 0$ ***R1***

so $\tan\frac{\pi}{12} = 2 - \sqrt{3}$ ***AG***

[5 marks]

4d. Using the results from parts (b) and (c) find the exact value of $\tan\frac{\pi}{24} - \cot\frac{\pi}{24}$. *[6 marks]*

Give your answer in the form $a + b\sqrt{3}$ where a, $b \in \mathbb{Z}$.

Markscheme

$\tan\frac{\pi}{24} - \cot\frac{\pi}{24}$ is the sum of the roots of $x^2 + \left(2\cot\frac{\pi}{12}\right)x - 1 = 0$ ***R1***

$\tan\frac{\pi}{24} - \cot\frac{\pi}{24} = -2\cot\frac{\pi}{12}$ ***A1***

$= \frac{-2}{2-\sqrt{3}}$ ***A1***

attempting to rationalise **their** denominator ***(M1)***

$= -4 - 2\sqrt{3}$ ***A1A1***

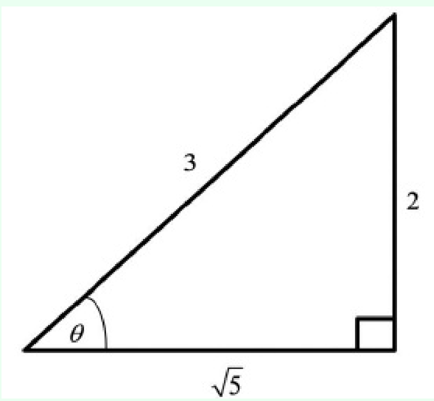
[6 marks]

5. It is given that $\operatorname{cosec}\theta = \frac{3}{2}$, where $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$. Find the exact value of $\cot\theta$. *[4 marks]*

Markscheme

METHOD 1

attempt to use a right angled triangle ***M1***



correct placement of all three values and θ seen in the triangle ***(A1)***

$\cot\theta < 0$ (since $\operatorname{cosec}\theta > 0$ puts θ in the second quadrant) ***R1***

$\cot\theta = -\frac{\sqrt{5}}{2}$ ***A1***

Note: Award ***M1A1R0A0*** for $\cot\theta = \frac{\sqrt{5}}{2}$ seen as the final answer
The ***R1*** should be awarded independently for a negative value only given as a final answer.

METHOD 2

Attempt to use $1 + \cot^2\theta = \operatorname{cosec}^2\theta$ ***M1***

$1 + \cot^2 \theta = \frac{9}{4}$

$\cot^2 \theta = \frac{5}{4}$ ***(A1)***

$\cot \theta = \pm \frac{\sqrt{5}}{2}$

$\cot \theta < 0$ (since $\operatorname{cosec} \theta > 0$ puts θ in the second quadrant) ***R1***

$\cot \theta = -\frac{\sqrt{5}}{2}$ ***A1***

Note: Award ***M1A1R0A0*** for $\cot \theta = \frac{\sqrt{5}}{2}$ seen as the final answer
The ***R1*** should be awarded independently for a negative value only given as a final answer.

METHOD 3

$\sin \theta = \frac{2}{3}$

attempt to use $\sin^2 \theta + \cos^2 \theta = 1$ ***M1***

$\frac{4}{9} + \cos^2 \theta = 1$

$\cos^2 \theta = \frac{5}{9}$ ***(A1)***

$\cos \theta = \pm \frac{\sqrt{5}}{3}$

$\cos \theta < 0$ (since $\operatorname{cosec} \theta > 0$ puts θ in the second quadrant) ***R1***

$\cos \theta = -\frac{\sqrt{5}}{3}$

$\cot \theta = -\frac{\sqrt{5}}{2}$ ***A1***

Note: Award ***M1A1R0A0*** for $\cot \theta = \frac{\sqrt{5}}{2}$ seen as the final answer
The ***R1*** should be awarded independently for a negative value only given as a final answer.

[4 marks]

6. A and B are acute angles such that $\cos A = \frac{2}{3}$ and $\sin B = \frac{1}{3}$. *[7 marks]*

Show that $\cos(2A + B) = -\frac{2\sqrt{2}}{27} - \frac{4\sqrt{5}}{27}$.

Markscheme

attempt to use $\cos(2A+B) = \cos 2A \cos B - \sin 2A \sin B$ (may be seen later) ***M1***

attempt to use any double angle formulae (seen anywhere) ***M1***

attempt to find either $\sin A$ or $\cos B$ (seen anywhere) ***M1***

$\cos A = \frac{2}{3} \Rightarrow \sin A \left(= \sqrt{1 - \frac{4}{9}}\right) = \frac{\sqrt{5}}{3}$ ***(A1)***

$\sin B = \frac{1}{3} \Rightarrow \cos B \left(= \sqrt{1 - \frac{1}{9}} = \frac{\sqrt{8}}{3}\right) = \frac{2\sqrt{2}}{3}$ ***A1***

$\cos 2A \left(= 2\cos^2 A - 1\right) = -\frac{1}{9}$ ***A1***

$\sin 2A (= 2 \sin A \cos A) = \frac{4\sqrt{5}}{9}$ ***A1***

So $\cos(2A+B) = \left(-\frac{1}{9}\right)\left(\frac{2\sqrt{2}}{3}\right) - \left(\frac{4\sqrt{5}}{9}\right)\left(\frac{1}{3}\right)$

$= -\frac{2\sqrt{2}}{27} - \frac{4\sqrt{5}}{27}$ ***AG***

[7 marks]

7a. Show that $(\sin x + \cos x)^2 = 1 + \sin 2x$. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$(\sin x + \cos x)^2 = \sin^2 x + 2\sin x \cos x + \cos^2 x$ ***M1A1***

Note: Do not award the ***M1*** for just $\sin^2 x + \cos^2 x$.

Note: Do not award ***A1*** if correct expression is followed by incorrect working.

$= 1 + \sin 2x$ ***AG***

[2 marks]

7b. Show that $\sec 2x + \tan 2x = \frac{\cos x + \sin x}{\cos x - \sin x}$. *[4 marks]*

Markscheme

$\sec 2x + \tan 2x = \frac{1}{\cos 2x} + \frac{\sin 2x}{\cos 2x}$ ***M1***

Note: ***M1*** is for an attempt to change both terms into sine and cosine forms (with the same argument) or both terms into functions of $\tan x$.

$= \frac{1+\sin 2x}{\cos 2x}$

$= \frac{(\sin x+\cos x)^2}{\cos^2 x-\sin^2 x}$ ***A1A1***

Note: Award ***A1*** for numerator, ***A1*** for denominator.

$= \frac{(\sin x+\cos x)^2}{(\cos x-\sin x)(\cos x+\sin x)}$ ***M1***

$= \frac{\cos x+\sin x}{\cos x-\sin x}$ ***AG***

Note: Apply MS in reverse if candidates have worked from RHS to LHS.

Note: Alternative method using $\tan 2x$ and $\sec 2x$ in terms of $\tan x$.

[4 marks]

7c. Hence or otherwise find $\int_0^{\frac{\pi}{6}} (\sec 2x + \tan 2x)\,\mathrm{d}x$ in the form $\ln\left(a+\sqrt{b}\right)$ where $a, b \in \mathbb{Z}$. *[9 marks]*

Markscheme

METHOD 1

$\int_0^{\frac{\pi}{6}} \left(\frac{\cos x+\sin x}{\cos x-\sin x}\right) \mathrm{d}x$ ***A1***

Note: Award ***A1*** for correct expression with or without limits.

EITHER

$= [-\ln(\cos x - \sin x)]_0^{\frac{\pi}{6}}$ or $[\ln(\cos x - \sin x)]_{\frac{\pi}{6}}^0$ ***(M1)A1A1***

Note: Award ***M1*** for integration by inspection or substitution, ***A1*** for $\ln(\cos x - \sin x)$, ***A1*** for completely correct expression including limits.

$= -\ln\left(\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\right) + \ln(\cos 0 - \sin 0)$ ***M1***

Note: Award ***M1*** for substitution of limits into their integral and subtraction.

$= -\ln\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$ ***(A1)***

OR

let $u = \cos x - \sin x$ ***M1***

$\frac{\mathrm{d}u}{\mathrm{d}x} = -\sin x - \cos x = -(\sin x + \cos x)$

$-\int_1^{\frac{\sqrt{3}}{2}-\frac{1}{2}} \left(\frac{1}{u}\right) \mathrm{d}u$ ***A1A1***

Note: Award ***A1*** for correct limits even if seen later, ***A1*** for integral.

$= [-\ln u]_1^{\frac{\sqrt{3}}{2}-\frac{1}{2}}$ or $[\ln u]_{\frac{\sqrt{3}}{2}-\frac{1}{2}}^1$ ***A1***

$= -\ln\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) (+\ln 1)$ ***M1***

THEN

$= \ln\left(\frac{2}{\sqrt{3}-1}\right)$

Note: Award ***M1*** for both putting the expression over a common denominator and for correct use of law of logarithms.

$= \ln\left(1 + \sqrt{3}\right)$ ***(M1)A1***

METHOD 2

$\left[\frac{1}{2}\ln(\tan 2x + \sec 2x) - \frac{1}{2}\ln(\cos 2x)\right]_0^{\frac{\pi}{6}}$ ***A1A1***

$= \frac{1}{2}\ln\left(\sqrt{3} + 2\right) - \frac{1}{2}\ln\left(\frac{1}{2}\right) - 0$ ***A1A1(A1)***

$= \frac{1}{2}\ln\left(4 + 2\sqrt{3}\right)$ ***M1***

$= \frac{1}{2}\ln\left(\left(1 + \sqrt{3}\right)^2\right)$ ***M1A1***

$= \ln\left(1 + \sqrt{3}\right)$ ***A1***

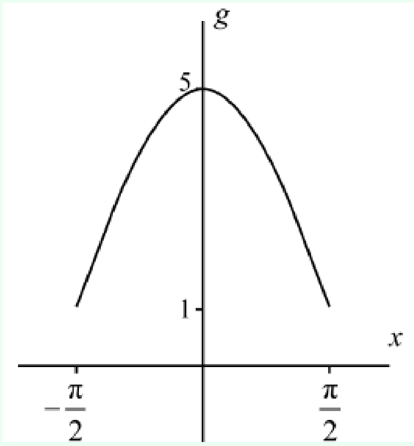
[9 marks]

Consider the function $g(x) = 4\cos x + 1$, $a \leqslant x \leqslant \frac{\pi}{2}$ where $a < \frac{\pi}{2}$.

8a. For $a = -\frac{\pi}{2}$, sketch the graph of $y = g(x)$. Indicate clearly the maximum and minimum values of the function. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.



concave down and symmetrical over correct domain ***A1***

indication of maximum and minimum values of the function (correct range) ***A1A1***

[3 marks]

8b. Write down the least value of a such that g has an inverse. *[1 mark]*

Markscheme

$a = 0$ ***A1***

Note: Award ***A1*** for $a = 0$ only if consistent with their graph.

[1 mark]

8c. For the value of a found in part (b), write down the domain of g^{-1}. *[1 mark]*

Markscheme

$1 \leqslant x \leqslant 5$ ***A1***

Note: Allow FT from their graph.

[1 mark]

8d. For the value of a found in part (b), find an expression for $g^{-1}(x)$. *[2 marks]*

Markscheme

$y = 4\cos x + 1$

$x = 4\cos y + 1$

$\frac{x-1}{4} = \cos y$ ***(M1)***

$\Rightarrow y = \arccos\left(\frac{x-1}{4}\right)$

$\Rightarrow g^{-1}(x) = \arccos\left(\frac{x-1}{4}\right)$ ***A1***

[2 marks]

9. Solve the equation $\sec^2 x + 2\tan x = 0,\ 0 \leqslant x \leqslant 2\pi$. *[5 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

METHOD 1

use of $\sec^2 x = \tan^2 x + 1$ ***M1***

$\tan^2 x + 2\tan x + 1 = 0$

$(\tan x + 1)^2 = 0$ ***(M1)***

$\tan x = -1$ ***A1***

$x = \frac{3\pi}{4},\ \frac{7\pi}{4}$ ***A1A1***

METHOD 2

$\frac{1}{\cos^2 x} + \frac{2\sin x}{\cos x} = 0$ ***M1***

$1 + 2\sin x\cos x = 0$

$\sin 2x = -1$ ***M1A1***

$2x = \frac{3\pi}{2},\ \frac{7\pi}{2}$

$x = \frac{3\pi}{4},\ \frac{7\pi}{4}$ ***A1A1***

Note: Award ***A1A0*** if extra solutions given or if solutions given in degrees (or both).

[5 marks]

10a. Find the value of $\sin\frac{\pi}{4} + \sin\frac{3\pi}{4} + \sin\frac{5\pi}{4} + \sin\frac{7\pi}{4} + \sin\frac{9\pi}{4}$. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\sin\frac{\pi}{4} + \sin\frac{3\pi}{4} + \sin\frac{5\pi}{4} + \sin\frac{7\pi}{4} + \sin\frac{9\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$ ***(M1)A1***

Note: Award ***M1*** for 5 equal terms with \) + \) or $-$ signs.

[2 marks]

10b. Show that $\frac{1-\cos 2x}{2\sin x} \equiv \sin x,\ x \neq k\pi$ where $k \in \mathbb{Z}$. *[2 marks]*

Markscheme

$\frac{1-\cos 2x}{2\sin x} \equiv \frac{1-(1-2\sin^2 x)}{2\sin x}$ ***M1***

$\equiv \frac{2\sin^2 x}{2\sin x}$ ***A1***

$\equiv \sin x$ ***AG***

[2 marks]

10c. Use the principle of mathematical induction to prove that *[9 marks]*

$$\sin x + \sin 3x + \ldots + \sin(2n-1)x = \frac{1-\cos 2nx}{2\sin x},\ n \in \mathbb{Z}^+,\ x \neq k\pi \text{ where } k \in \mathbb{Z}.$$

Markscheme

let $\mathrm{P}(n)$: $\sin x + \sin 3x + \ldots + \sin(2n-1)x \equiv \frac{1-\cos 2nx}{2\sin x}$

if $n = 1$

$\mathrm{P}(1)$: $\frac{1-\cos 2x}{2\sin x} \equiv \sin x$ which is true (as proved in part (b)) ***R1***

assume $\mathrm{P}(k)$ true, $\sin x + \sin 3x + \ldots + \sin(2k-1)x \equiv \frac{1-\cos 2kx}{2\sin x}$ ***M1***

Notes: Only award ***M1*** if the words "assume" and "true" appear. Do not award ***M1*** for "let $n = k$" only. Subsequent marks are independent of this ***M1***.

consider $\mathrm{P}(k+1)$:

$\mathrm{P}(k+1)$: $\sin x + \sin 3x + \ldots + \sin(2k-1)x + \sin(2k+1)x \equiv \frac{1-\cos 2(k+1)x}{2\sin x}$

$LHS = \sin x + \sin 3x + \ldots + \sin(2k-1)x + \sin(2k+1)x$ ***M1***

$\equiv \frac{1-\cos 2kx}{2\sin x} + \sin(2k+1)x$ ***A1***

$\equiv \frac{1-\cos 2kx + 2\sin x \sin(2k+1)x}{2\sin x}$

$\equiv \frac{1-\cos 2kx + 2\sin x \cos x \sin 2kx + 2\sin^2 x \cos 2kx}{2\sin x}$ ***M1***

$\equiv \frac{1-\left((1-2\sin^2 x)\cos 2kx - \sin 2x \sin 2kx\right)}{2\sin x}$ ***M1***

$\equiv \frac{1-(\cos 2x \cos 2kx - \sin 2x \sin 2kx)}{2\sin x}$ ***A1***

$\equiv \frac{1-\cos(2kx+2x)}{2\sin x}$ ***A1***

$\equiv \frac{1-\cos 2(k+1)x}{2\sin x}$

so if true for $n = k$, then also true for $n = k+1$

as true for $n = 1$ then true for all $n \in \mathbb{Z}^+$ ***R1***

Note: Accept answers using transformation formula for product of sines if steps are shown clearly.

Note: Award ***R1*** only if candidate is awarded at least 5 marks in the previous steps.

[9 marks]

10d. Hence or otherwise solve the equation $\sin x + \sin 3x = \cos x$ in the interval $0 < x < \pi$. *[6 marks]*

Markscheme

EITHER

$\sin x + \sin 3x = \cos x \Rightarrow \frac{1-\cos 4x}{2\sin x} = \cos x$ ***M1***

$\Rightarrow 1 - \cos 4x = 2\sin x \cos x,\ (\sin x \neq 0)$ ***A1***

$\Rightarrow 1 - (1 - 2\sin^2 2x) = \sin 2x$ ***M1***

$\Rightarrow \sin 2x(2\sin 2x - 1) = 0$ ***M1***

$\Rightarrow \sin 2x = 0$ or $\sin 2x = \frac{1}{2}$ ***A1***

$2x = \pi,\ 2x = \frac{\pi}{6}$ and $2x = \frac{5\pi}{6}$

OR

$\sin x + \sin 3x = \cos x \Rightarrow 2\sin 2x \cos x = \cos x$ ***M1A1***

$\Rightarrow (2\sin 2x - 1)\cos x = 0,\ (\sin x \neq 0)$ ***M1A1***

$\Rightarrow \sin 2x = \frac{1}{2}$ of $\cos x = 0$ ***A1***

$2x = \frac{\pi}{6},\ 2x = \frac{5\pi}{6}$ and $x = \frac{\pi}{2}$

THEN

$\therefore x = \frac{\pi}{2},\ x = \frac{\pi}{12}$ and $x = \frac{5\pi}{12}$ ***A1***

Note: Do not award the final ***A1*** if extra solutions are seen.

[6 marks]