Vectors 10.02 *[60 marks]*

A line L_1 passes through the points $\mathrm{A}(0, 1, 8)$ and $\mathrm{B}(3, 5, 2)$.

1a. Find $\overrightarrow{AB}$. *[2 marks]*

1b. Hence, write down a vector equation for L_1. *[2 marks]*

1c. *[3 marks]*

A second line L_2, has equation $\boldsymbol{r} = \begin{pmatrix} 1 \\ 13 \\ -14 \end{pmatrix} + s\begin{pmatrix} p \\ 0 \\ 1 \end{pmatrix}$.

Given that L_1 and L_2 are perpendicular, show that $p = 2$.

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1d. The lines L_1 and L_1 intersect at $C(9,\ 13,\ z)$. Find z. *[5 marks]*

1e. Find a unit vector in the direction of L_2. *[2 marks]*

1f. Hence or otherwise, find one point on L_2 which is $\sqrt{5}$ units from C. *[3 marks]*

2. Find the Cartesian equation of plane *Π* containing the points $\mathrm{A}\,(6, 2, 1)$ and $\mathrm{B}\,(3,\ -1, 1)$ and perpendicular to the plane $x + 2y - z - 6 = 0$. *[6 marks]*

3. Given that $\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{b} \times \boldsymbol{c} \neq \boldsymbol{0}$ prove that $\boldsymbol{a} + \boldsymbol{c} = s\boldsymbol{b}$ where s is a scalar. *[4 marks]*

The points A, B and C have the following position vectors with respect to an origin O.

$\overrightarrow{\mathrm{OA}} = 2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$

$\overrightarrow{\mathrm{OB}} = 2\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$

$\overrightarrow{\mathrm{OC}} = \boldsymbol{i} + 3\boldsymbol{j} + 3\boldsymbol{k}$

4a. Find the vector equation of the line (BC). *[3 marks]*

4b. Determine whether or not the lines (OA) and (BC) intersect. *[6 marks]*

4c. Find the Cartesian equation of the plane Π_1, which passes through C and is perpendicular to $\overrightarrow{\mathrm{OA}}$. *[3 marks]*

4d. Show that the line (BC) lies in the plane Π_1. *[2 marks]*

The plane Π_2 contains the points O, A and B and the plane Π_3 contains the points O, A and C.

4e. Verify that $2\boldsymbol{j} + \boldsymbol{k}$ is perpendicular to the plane Π_2. *[3 marks]*

4f. Find a vector perpendicular to the plane Π_3. *[1 mark]*

4g. Find the acute angle between the planes Π_2 and Π_3. *[4 marks]*

5. Find the coordinates of the point of intersection of the planes defined by the equations $x + y + z = 3, x - y + z = 5$ and $x + y + 2z = 6$. *[5 marks]*

Consider the lines l_1 and l_2 defined by

$l_1 : \boldsymbol{r} = \begin{pmatrix} -3 \\ -2 \\ a \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ and $l_2 : \frac{6-x}{3} = \frac{y-2}{4} = 1 - z$ where a is a constant.

Given that the lines l_1 and l_2 intersect at a point P,

6a. find the value of a; *[4 marks]*

6b. determine the coordinates of the point of intersection P. *[2 marks]*