Logarithms part 2

Introduction

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In general we have $\log_a b = c$ if and only if $a^c = b$, with the restriction that a and b have to be positive real numbers and $a \neq 1$.

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By definition we get that:

$$3^4 = x - 2$$

So $x = 83$.

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We get that:

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So $x = 4$ or $x = -4$, but the base of the logarithm cannot be negative, so finally we have only one solution $x = 4$.

Simple equations - practice

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$4^{x+2}=\frac{1}{8}$, so $x=-\frac{7}{2}$.

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$(2x+1)^4=625$, so $x=2$.

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b) $\log_{2x+1}81=625$

$(2x+1)^4=625$, so $x=2$.

c) $\log_9\sqrt{3}=2x-3$

$9^{2x-3}=\sqrt{3}$, so $x=\frac{13}{8}$.

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More complicated equations require the use of the properties of logarithms. The three basic rules we will be using are:

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3. $k \log_a b = \log_a(b^k)$

   Example $\log_2(2^{10}) = 10 \log_2 2 = 10$

Equations - example 1

Consider the equation:

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Now we can use the definition of logarithm to get:

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Which gives $x+1 = 8x-8$, so we get that $x = \frac{9}{7}$.

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Now consider a slightly different equation:

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And using the definition of logarithm to get:

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Which gives $x - 1 = 8x + 8$ and we get that $x = -\frac{9}{7}$, but this would mean that we have a negative number inside the logarithm, which is not allowed, so the equation has no solutions.

Equations - example 3

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Using the definition of logarithm to get:

$$x^2 + 2x = 2^3$$

Which gives $(x+4)(x-2) = 0$ and we get two solutions $x = -4$ and $x = 2$, but $x = -4$ would give us a negative number inside the logarithm, so in the end we have one solution $x = 2$.

Equations - example 4

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We use the third property to get (remember $\sqrt{x} = x^{\frac{1}{2}}$ and $\sqrt[3]{x} = x^{\frac{1}{3}}$):

$$\frac{1}{2}\log_3 x + \frac{1}{3}\log_3 x = \frac{5}{3}$$

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$$\frac{1}{2} \log_3 x + \frac{1}{3} \log_3 x = \frac{5}{3}$$

Multiplying both sides by 6, adding the logarithms and then dividing by 5 we get:

$$\log_3 x = 2$$

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Multiplying both sides by 6, adding the logarithms and then dividing by 5 we get:

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Which gives $x = 3^2$, so $x = 9$.

Practice questions

Next slide contains practice questions. Try them yourselves first, before checking the answers.

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We get the equation $\frac{4x}{1-x} = \frac{1}{5}$, which gives $x = \frac{1}{21}$.

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c) $\log_2 x^3 - \log_2 \sqrt[3]{x^2} = 14$

Practice questions

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c) $\log_2 x^3 - \log_2 \sqrt[3]{x^2} = 14$

We get $x = 2^6 = 64$.

Change of base

Another rule for logarithms we need to know is the change of base formula:

$$\log_a b = \frac{\log_c b}{\log_c a}$$

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For example:

$$\log_4 8 = \frac{\log_2 8}{\log_2 4} = \frac{3}{2}$$

Change of base - practice

Calculate the following using the change of base formula:

a) $\log_9 \frac{1}{3} =$

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b) $\log_{\sqrt{5}} \frac{1}{125} = \frac{\log_5 \frac{1}{125}}{\log_5 \sqrt{5}} = \frac{-3}{\frac{1}{2}} = -6$

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c) $\log_1 6 \frac{1}{\sqrt[3]{2}} =$

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c) $\log_1 6 \frac{1}{\sqrt[3]{2}} = \frac{\log_2 \frac{1}{\sqrt[3]{2}}}{\log_2 16} = \frac{-\frac{1}{3}}{4} = -\frac{1}{12}$

In case of any questions, you can email me at T.J.Lechowski@gmail.com.