1. Solve $\log_2 x + \log_2(x - 2) = 3$, for $x > 2$.

(Total 7 marks)

recognizing $\log a + \log b = \log ab$ (seen anywhere)	(A1)	
e.g. $\log_2(x(x-2))$, $x^2 - 2x$		
recognizing $\log_a b = x \Leftrightarrow a^x = b$ (seen anywhere)	(A1)	
e.g. $2^3 = 8$		
correct simplification	A1	
e.g. $x(x-2) = 2^3$, $x^2 - 2x - 8$		
evidence of correct approach to solve	(M1)	
e.g. factorizing, quadratic formula		
correct working	A1	
e.g. $(x-4)(x+2)$, $\frac{2 \pm \sqrt{36}}{2}$		
$x = 4$	A2	N3

[7]

2. (a) Find $\log_2 32$.

(1)

(b) Given that $\log_2\left(\frac{32^x}{8^y}\right)$ can be written as $px + qy$, find the value of p and of q.

(4)
(Total 5 marks)

(a) 5	A1	N1
(b) **METHOD 1**		
$\log_2\left(\frac{32^x}{8^y}\right) = \log_2 32^x - \log_2 8^y$	(A1)	
$= x\log_2 32 - y\log_2 8$	(A1)	
$\log_2 8 = 3$	(A1)	
$p = 5$, $q = -3$ (accept $5x - 3y$)	A1	N3
METHOD 2		
$\frac{32^x}{8^y} = \frac{(2^5)^x}{(2^3)^y}$	(A1)	
$= \frac{2^{5x}}{2^{3y}}$	(A1)	
$= 2^{5x-3y}$	(A1)	
$\log_2(2^{5x-3y}) = 5x - 3y$		
$p = 5$, $q = -3$ (accept $5x - 3y$)	A1	N3

[5]

3. Given that $p = \log_a 5$, $q = \log_a 2$, express the following in terms of p and/or q.

(a) $\log_a 10$

(b) $\log_a 8$

(c) $\log_a 2.5$

(Total 6 marks)

(a) $\log_a 10 = \log_a (5 \times 2)$ (M1)

$= \log_a 5 + \log_a 2$

$= p + q$ A1 N2

(b) $\log_a 8 = \log_a 2^3$ (M1)

$= 3 \log_a 2$

$= 3q$ A1 N2

(c) $\log_a 2.5 = \log_a \frac{5}{2}$ (M1)

$= \log_a 5 - \log_a 2$

$= p - q$ A1 N2

[6]

4. (a) Let $\log_c 3 = p$ and $\log_c 5 = q$. Find an expression in terms of p and q for

(i) $\log_c 15$;

(ii) $\log_c 25$.

(b) Find the value of d if $\log_d 6 = \frac{1}{2}$.

(Total 6 marks)

(a) (i) $\log_c 15 = \log_c 3 + \log_c 5$ (A1)

$= p + q$ A1 N2

(ii) $\log_c 25 = 2 \log_c 5$ (A1)

$= 2q$ A1 N2

(b) **METHOD 1**

$d^{\frac{1}{2}} = 6$ M1

$d = 36$ A1 N1

METHOD 2

For changing base M1

eg $\frac{\log_{10} 6}{\log_{10} d} = \frac{1}{2}$, $2\log_{10} 6 = \log_{10} d$

$d = 36$ A1 N1

[6]

5. Let ln $a = p$, ln $b = q$. Write the following expressions in terms of p and q.

(a) $\ln a^3 b$

(b) $\ln\left(\frac{\sqrt{a}}{b}\right)$

(Total 6 marks)

(a) $\ln a^3 b = 3\ln a + \ln b$ (A1)(A1)

$\ln a^3 b = 3p + q$ A1 N3

(b) $\ln \frac{\sqrt{a}}{b} = \frac{1}{2}\ln a - \ln b$ (A1)(A1)

$\ln \frac{\sqrt{a}}{b} = \frac{1}{2} p - q$ A1 N3

[6]

6. Find the **exact** solution of the equation $9^{2x} = 27^{(1-x)}$.

(Total 6 marks)

METHOD 1

$9 = 3^2,\ 27 = 3^3$ (A1)(A1)

expressing as a power of 3, $(3^2)^{2x} = (3^3)^{1-x}$ (M1)

$3^{4x} = 3^{3-3x}$ (A1)

$4x = 3 - 3x$ (A1)

$7x = 3$

$\Rightarrow x = \frac{3}{7}$ (A1) (C6)

METHOD 2

$2x\log 9 = (1-x)\log 27$ (M1)(A1)(A1)

$\frac{2x}{1-x} = \frac{\log 27}{\log 9}\left(=\frac{3}{2}\right)$ (A1)

$4x = 3 - 3x$ (A1)

$7x = 3$

$\Rightarrow x = \frac{3}{7}$ (A1) (C6)

Notes: *Candidates may use a graphical method.*
Award (M1)(A1)(A1) for a sketch, (A1) for showing the point of intersection, (A1) for 0.4285…., and (A1) for $\frac{3}{7}$.

[6]

7. (a) Given that $\log_3 x - \log_3 (x - 5) = \log_3 A$, express A in terms of x.

(b) Hence or otherwise, solve the equation $\log_3 x - \log_3 (x - 5) = 1$.

(Total 6 marks)

(a) $\log_3 x - \log_3 (x-5) = \log_3\left(\frac{x}{x-5}\right)$ (A1)

$A = \frac{x}{x-5}$ (A1) (C2)

Note: *If candidates have an incorrect or no answer to part (a) award (A1)(A0) if* $\log\left(\frac{x}{x-5}\right)$ *seen in part (b).*

(b) **EITHER**

$\log_3\left(\frac{x}{x-5}\right) = 1$

$\frac{x}{x-5} = 3^1 (= 3)$ (M1)(A1)(A1)

$x = 3x - 15$

$-2x = -15$

$x = \frac{15}{2}$ (A1) (C4)

OR

$\frac{\log_{10}\left(\frac{x}{x-5}\right)}{\log_{10} 3} = 1$ (M1)(A1)

$\log_{10}\left(\frac{x}{x-5}\right) = \log_{10} 3$ (A1)

$x = 7.5$ (A1) (C4)

[6]

8. Let $p = \log_{10} x$, $q = \log_{10} y$ and $r = \log_{10} z$.

Write the expression $\log_{10}\left(\frac{x}{y^2\sqrt{z}}\right)$ in terms of p, q and r.

(Total 6 marks)

$\log_{10}\left(\frac{x}{y^2\sqrt{z}}\right) = \log_{10} x - \log_{10} y^2 - \log_{10}\sqrt{z}$ (A1)(A1)(A1)

$\log_{10} y^2 = 2\log_{10} y$ (A1)

$\log_{10}\sqrt{z} = \frac{1}{2}\log z$ (A1)

$\log_{10}\left(\frac{x}{y^2\sqrt{z}}\right) = \log_{10} x - 2\log y - \frac{1}{2}\log z$

$= p - 2q - \frac{1}{2}r$ (A1) (C2)(C2)(C2)

9. Let $a = \log x$, $b = \log y$, and $c = \log z$.

Write $\log\left(\frac{x^2\sqrt{y}}{z^3}\right)$ in terms of a, b and c.

(Total 6 marks)

$\log x^2 = 2\log x$ (A1)

$\log\sqrt{y} = \frac{1}{2}\log y$ (A1)

$\log z^3 = 3\log z$ (A1)

$2\log x + \frac{1}{2}\log y - 3\log z$ (A1)(A1)

$2a + \frac{1}{2}b - 3c$ (A1) (C6)

10. Given that $\log_5 x = y$, express each of the following in terms of y.

(a) $\log_5 x^2$

(b) $\log_5\left(\frac{1}{x}\right)$

(c) $\log_{25} x$

(Total 6 marks)

(a) $\log_5 x^2 = 2\log_5 x$ (M1)
$= 2y$ (A1) (C2)

(b) $\log_5 \frac{1}{x} = -\log_5 x$ (M1)
$= -y$ (A1) (C2)

(c) $\log_{25} x = \frac{\log_5 x}{\log_5 25}$ (M1)
$= \frac{1}{2}y$ (A1) (C2)

[6]

11. Solve the equation $\log_9 81 + \log_9 \frac{1}{9} + \log_9 3 = \log_9 x$.

(Total 4 marks)

$\log 81 + \log_9\left(\frac{1}{9}\right) + \log_9 3 = \log_9\left[81\left(\frac{1}{9}\right)3\right]$ (M2)

$= \log_9 27$ (A1)

$\Rightarrow x = 27$ (A1) (C4)

[4]

12. Solve the equation $\log_{27} x = 1 - \log_{27}(x - 0.4)$.

(Total 6 marks)

$\log_{27}(x(x - 0.4)) = 1$ (M1)(A1)

$x^2 - 0.4x = 27$ (M1)

$x = 5.4$ or $x = -5$ (G2)

$x = 5.4$ (A1) (C6)

Note: *Award (C5) for giving both roots.*

[6]

13. Let $\log_{10}P = x$, $\log_{10}Q = y$ and $\log_{10}R = z$. Express $\log_{10}\left(\frac{P}{QR^3}\right)^2$ in terms of x, y and z.

(Total 4 marks)

$\log_{10}\left(\frac{P}{QR^3}\right)^2 = 2\log_{10}\left(\frac{P}{QR^3}\right)$ (M1)

$2\log_{10}\left(\frac{P}{QR^3}\right) = 2(\log_{10}P - \log_{10}(QR^3))$ (M1)

$= 2(\log_{10}P - \log_{10}Q - 3\log_{10}R)$ (M1)

$= 2(x - y - 3z)$

$= 2x - 2y - 6z$ or $2(x - y - 3z)$ (A1)

[4]

14. If $\log_a 2 = x$ and $\log_a 5 = y$, find in terms of x and y, expressions for

(a) $\log_2 5$;

(b) $\log_a 20$.

(Total 4 marks)

(a) $\log_2 5 = \frac{\log_a 5}{\log_a 2}$ (M1)

$= \frac{y}{x}$ (A1) (C2)

(b) $\log_a 20 = \log_a 4 + \log_a 5$ or $\log_a 2 + \log_a 10$ (M1)

$= 2\log_a 2 + \log_a 5$

$= 2x + y$ (A1) (C2)

[4]

15. Solve the equation $9^{x-1} = \left(\frac{1}{3}\right)^{2x}$.

(Total 4 marks)

$9^{x-1} = \left(\frac{1}{3}\right)^{2x}$

$3^{2x-2} = 3^{-2x}$ (M1) (A1)

$2x - 2 = -2x$ (A1)

$x = \frac{1}{2}$ (A1) (C4)

[4]

16. Solve the equation $4^{3x-1} = 1.5625 \times 10^{-2}$.

(Total 4 marks)

$4^{3x-1} = 1.5625 \times 10^{-2}$

$(3x - 1)\log_{10} 4 = \log_{10} 1.5625 - 2$ (M1)

$\Rightarrow 3x - 1 = \frac{\log_{10} 1.5625 - 2}{\log_{10} 4}$ (A1)

$\Rightarrow 3x - 1 = -3$ (A1)

$\Rightarrow x = -\frac{2}{3}$ (A1) (C4)

[4]