8.1 ARITHMETIC SEQUENCES & SERIES

8.1.1 ARITHMETIC SEQUENCES

A **sequence** is a set of quantities arranged in a definite order.

1, 2, 3, 4, 5, 6,.............. –1, 2, –4, 8, –16....1, 1, 2, 3, 5, 8, 13....

are all examples of sequences. When the terms of a sequence are added, we obtain a series. Sequences and series are used to solve a variety of practical problems in, for example, business.

There are two major types of sequence, **arithmetic** and **geometric**. This section will consider arithmetic sequences (also known as arithmetic progressions, or simply A.P). The characteristic of such a sequence is that there is a common difference between successive terms. For example:

1, 3, 5, 7, 9, 11, . . . (the odd numbers) has a first term of 1 and a common difference of 2.
18, 15, 12, 9, 6, . . . has a first term of 18 and a common difference of –3 (sequence is decreasing).

The terms of a sequence are generally labelled $u_1, u_2, u_3, u_4, \ldots u_n$. The '$n$th term' of a sequence is labelled u_n. In the case of an arithmetic sequence which starts with a and has a **common difference** of d, the **nth term** can be found using the formula:

$$\boxed{u_n = a + (n-1)d} \text{ where } d = u_2 - u_1 = u_3 - u_2 = \ldots$$

EXAMPLE 8.1

For the sequence 7, 11, 15, 19, . . . , find the 20th term.

Solution

In this case $a = 7$ and $d = 4$ because the sequence starts with a 7 and each term is 4 bigger than the one before it, i.e., $d = 11 - 7 = 4$. Therefore the nth term is given by

$$\boxed{u_n = 7 + (n-1) \times 4}$$

That is, $u_n = 4n + 3$

$\therefore u_{20} = 4 \times 20 + 3 = 83$ [$n = 20$ corresponds to the 20th term]

EXAMPLE 8.2

An arithmetic sequence has a first term of 120 and a 10th term of 57. Find the 15th term.

Solution

The data is: $a = 120$ and when $n = 10$, $u_{10} = 57$ [i.e., 10th term is 57].

This gives, $u_{10} = 120 + (10-1)d = 57 \Leftrightarrow 120 + 9d = 57$

$\therefore d = -7$

Using $u_n = a + (n-1)d$, we then have $u_n = 120 + (n-1) \times (-7) = 127 - 7n$.

Therefore, when $n = 15$, $u_{15} = 127 - 7 \times 15 = 22$.

EXAMPLE 8.3

An arithmetic sequence has a 7th term of 16.5 and a 12th term of 24. Find the 24th term.

Solution

In this instance we know neither the first term nor the common difference and so we will need to set up equations to be solved simultaneously.

The data is: $u_7 = a + 6d = 16.5 \quad -(1)$

$u_{12} = a + 11d = 24 \quad -(2)$

We first solve for 'd': $(2) - (1)$: $5d = 7.5 \Leftrightarrow d = 1.5$

Substituting into (1): $a + 6 \times 1.5 = 16.5 \Leftrightarrow a = 7.5$

To find the 24th term we use the general term $u_n = a + (n-1)d$ with $n = 24$:

$$u_{24} = 7.5 + (24-1) \times 1.5 = 42$$

EXAMPLE 8.4

A car whose original value was \$25600 decreases in value by \$90 per month. How long will it take before the car's value falls below \$15000?

Solution

The values can be seen as a sequence: \$25600, \$25510, \$25420 etc.
In this case $a = 25600$ and $d = 25510 - 25600 = -90$ so that:

$$\begin{aligned} u_n &= 25600 + (n-1) \times (-90) \\ &= 25690 - 90n \\ \therefore 15000 &= 25690 - 90n \\ \Leftrightarrow 90n &= 25690 - 15000 \\ \Leftrightarrow n &= 118.777 \end{aligned}$$

The car will be worth less than \$15000 after 119 months

Using a Graphics Calculator

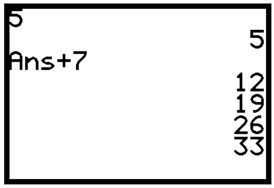
Most graphic calculators have an automatic memory facility (usually called **Ans**) that stores the result of the last calculation as well as an ability to remember the actual calculation. This can be very useful in listing a sequence.

EXAMPLE 8.5

List the arithmetic sequence 5, 12, 19, 26,......

Solution

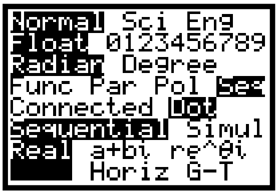
The sequence has a first term of 5. Enter this and press **ENTER** or **EXE**.
The common difference of the sequence is 7 so enter + 7.
The display will show Ans + 7 which means 'add 7 to the previous answer.
From here, every time you press **ENTER** (or **EXE**), you will repeat the calculation, generating successive terms of the sequence.



However, the TI–83 is much more sophisticated than this. It is possible to set up a sequence rule on the TI–83. To do this we use the **MODE** key to switch to **Seq** mode and this changes the Equation editor screen from **Y=** to a sequence version (instead of the usual function form).

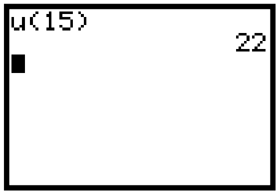
There are three sequence forms; $u(n)$, $v(n)$ and $w(n)$, which can be accessed on the home screen using the 2nd function key with 7, 8 and 9 respectively. Once these equations are defined we can plot their sequence graph.

We now consider Example 8.2, where we obtained the sequence $u_n = 127 - 7n$ and wished to determine the 15th term.



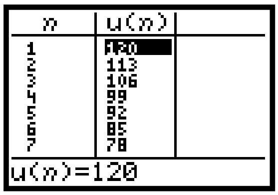
Setting into sequence mode

Define sequence equation

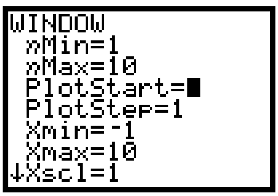


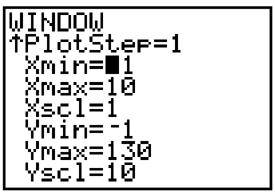
Use 2nd key '7' to call up u.

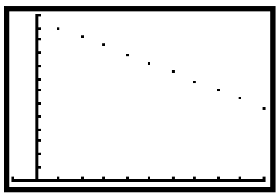
We can also use other features of the TI–83. For example, set up the sequence in a table format:



We can plot the sequence:







The TI–83 has many features that can be used with sequences. Become familiar with all of them.

EXERCISES 8.1.1

1. (a) Show that the following sequences are arithmetic.
(b) Find the common difference.
(c) Define the rule that gives the nth term of the sequence.

i. $\{2, 6, 10, 14, \ldots\}$ ii. $\{20, 17, 14, 11, \ldots\}$
iii. $\{1, -4, -9, \ldots\}$ iv. $\{0.5, 1.0, 1.5, 2.0, \ldots\}$
v. $\{y+1, y+3, y+5, \ldots\}$ vi. $\{x+2, x, x-2, \ldots\}$

2. Find the 10th term of the sequence whose first four terms are $8, 4, 0, -4$.

3. Find the value of x and y in the arithmetic sequence $\{5, x, 13, y, \ldots\}$.

4. An arithmetic sequence has 12 as its first term and a common difference of -5. Find its 12th term.

5. An arithmetic sequence has -20 as its first term and a common difference of 3. Find its 10th term.

6. The 14th term of an arithmetic sequence is 100. If the first term is 9, find the common difference.

7. The 10th term of an arithmetic sequence is -40. If the first term is 5, find the common difference.

8. If $n+5$, $2n+1$ and $4n-3$ are three consecutive terms of an arithmetic sequences, find n.

9. The first three terms of an arithmetic sequence are $1, 6, 11$.
(a) Find the 9th term.
(b) Which term will equal 151?

10. Find x and y given that $4-\sqrt{3}, x, y$ and $2\sqrt{3}-2$ are the first four terms of an arithmetic sequence.

11. For each of the following sequences
(a) $u_n = -5+2n, n \geq 1$. (b) $u_n = 3+4(n+1), n \geq 1$.
determine
i. its common difference
ii. its first term

12. The third and fifth terms of an A.P are $x+y$ and $x-y$ respectively. Find the 12th term.

13. The sum of the fifth term and twice the third of an arithmetic sequence equals the twelth term. If the seventh term is 25 find an expression for the general term, u_n.

14. For a given arithmetic sequence, $u_n = m$ and $u_m = n$. Find
(a) the common difference.
(b) u_{n+m}.

8.1.2 ARITHMETIC SERIES

If the **terms of a sequence are added**, the result is known as a **series**.

The	sequence:	1, 2, 3, 4, 5, 6, . . .
gives the	series:	1 + 2 + 3 + 4 + 5 + 6 + . . .
and the	sequence:	–1, –2, –4, –8, –16 . . .
gives the	series:	(–1) + (–2) + (–4) + (–8) + (–16) + . . .
		[or – 1– 2 – 4 – 8 – 16 – . . .

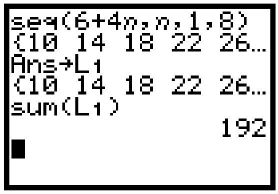
The sum of the terms of a series is referred to as S_n , the **sum of *n* terms of a series**.

For an arithmetic series, we have

$$S_n = u_1 + u_2 + u_3 + \ldots + u_n$$
$$= a + (a + d) + (a + 2d) + \ldots\ldots\ldots\ldots + a + (n - 1)d$$

For example, if we have a sequence defined by $u_n = 6 + 4n, n \geq 1$, then the sum of the first 8 terms is given by

$$S_8 = u_1 + u_2 + u_3 + \ldots + u_8$$
$$= 10 + 14 + 18 + \ldots + 38$$
$$= 192$$



Again, the screen display of the TI–83 shows how readily we can obtain the sum. Once the sequence has been stored as a **List**, use the **sum(** operation to obtain the answer.

There will be many cases in which we can add the terms of a series in this way. If, however, there are a large number of terms to add, a formula is more appropriate.

There is a story that, when the mathematician Gauss was a child, his teacher was having problems with him because he always finished all his work long before the other students. In an attempt to keep Gauss occupied for a period, the teacher asked him to add all the whole numbers from 1 to 100.

'5050' Gauss replied immediately.

It is probable that Gauss used a method similar to this:

1	2	3	4	5	6	......,	96	97	98	99	100
100	99	98	97	96	95	......,	5	4	3	2	1
101	101	101	101	101	101	......,	101	101	101	101	101

Adding each of the pairings gives 100 totals of 101 each. This gives a total of 10100. This is the sum of two sets of the numbers 1 + 2 + 3 + ... + 98 + 99 +100 and so dividing the full answer by 2 gives the answer 5050, as the young Gauss said, 5050.

It is then possible to apply the same approach to such a sequence, bearing in mind that the sequence of numbers must be arithmetic.

Applying this process to the general arithmetic series we have:

a	$a+d$	$a+2d$	. . .	$a+(n-3)d$	$a+(n-2)d$	$a+(n-1)d$
$a+(n-1)d$	$a+(n-2)d$	$a+(n-3)d$	. . .	$a+2d$	$a+d$	a

Each of the pairings comes to the same total.

Here are some examples:

1st pairing: $a+[a+(n-1)d] = 2a+(n-1)d$

2nd pairing: $(a+d)+[a+(n-2)d] = 2a+(n-1)d$

3rd pairing: $(a+2d)+[a+(n-3)d] = 2a+(n-1)d$

$\vdots$

There are n such pairings so $S_n + S_n = n\times[2a+(n-1)d]$

That is, $2S_n = n[2a+(n-1)d]$

Giving the formula, for the sum of n terms of a sequence

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

This formula can now be used to sum large arithmetic series:

EXAMPLE 8.6

Find the sum of 20 terms of the series –2 + 1 + 4 + 7 + 10 +. . .

Solution

We have the following information: $a = u_1 = -2$ and $d = u_2 - u_1 = 1-(-2) = 3$.

Then, the sum to n terms is given by $S_n = \frac{n}{2}[2a+(n-1)d]$

So that the sum to 20 terms is given by

$$\begin{aligned} S_{20} &= \frac{20}{2}[2\times(-2)+(20-1)\times 3] \\ &= 10[-4+19\times 3] \\ &= 530 \end{aligned}$$

EXAMPLE 8.7

Find the sum of 35 terms of the series $-\frac{3}{8}-\frac{1}{8}+\frac{1}{8}+\frac{3}{8}+\frac{5}{8}+\ldots\ldots$

Solution

We have the following information: $a = u_1 = -\frac{3}{8}$ and $d = u_2 - u_1 = -\frac{1}{8}-\left(-\frac{3}{8}\right) = \frac{1}{4}$.

Then, with $n = 35$ we have $S_{35} = \frac{35}{2}\left[2\times-\frac{3}{8}+(35-1)\frac{1}{4}\right] = 17.5\left[-\frac{3}{4}+34\times\frac{1}{4}\right]$

$$= 135\frac{5}{8}$$

EXAMPLE 8.8 An arithmetic series has a third term of 0. The sum of the first 15 terms is –300. What is the first term and the sum of the first ten terms?

Solution

From the given information we have: $u_3 = a + 2d = 0$ – (1)

&: $S_{15} = \frac{15}{2}[2a + 14d] = -300$

i.e., $15a + 105d = -300$

$\therefore a + 7d = -20$ – (2)

The pair of equations can now be solved simultaneously:

(2) – (1): $5d = -20 \Leftrightarrow d = -4$

Substituting result into (1) we have : $a + 2 \times -4 = 0 \Leftrightarrow a = 8$

This establishes that the series is 8 + 4 + 0 + (–4) + (–8) +. . .

So the first term is 8 and the sum of the first ten terms is $S_{10} = \frac{10}{2}[16 + 9 \times -4] = -100$.

Using the TI–83 we have, using the general term $u_n = 8 + (n-1) \times -4 = 12 - 4n$

```
seq(12-4n,n,1,10
)
{8 4 0 -4 -8 -1…
Ans→L1
{8 4 0 -4 -8 -1…
sum(L1)
                -100
```

EXAMPLE 8.9 A new business is selling home computers. They predict that they will sell 20 computers in their first month, 23 in the second month, 26 in the third and so on, in arithmetic sequence. How many months will pass before the company expects to sell their thousandth computer.

Solution

The series is: 20 + 23 + 26 + ...

The question implies that the company is looking at the **total** number of computers sold, so we are looking at a series, not a sequence.

The question asks how many terms (months) will be needed before the total sales reach more than 1000. From the given information we have: $a = 20$, $d = 23 - 20 = 3$.

Therefore, we have the sum to n terms given by $S_n = \frac{n}{2}[2 \times 20 + (n-1) \times 3]$

$= \frac{n}{2}[3n + 37]$

Next, we determine when $S_n = 1000$: $\frac{n}{2}[3n + 37] = 1000 \Leftrightarrow 3n^2 + 37n = 2000$

$\Leftrightarrow 3n^2 + 37n - 2000 = 0$

We solve for n use either of the following methods:

Method 1: Quadratic formula

$$n = \frac{-37 \pm \sqrt{37^2 - 4 \times 3 \times -2000}}{2 \times 3}$$

$= 20.37$ or (-32.7)

Method 2: Graphics Calculator **Solve** function

```
solve(3X²+37X-20
00,X,20)
         20.37941486
```

Method 3: Table of values

Notice that we have entered the expression for S_n as the sequence rule for $u(n)$. In fact, the series itself is made up of terms in a sequence of so-called **partial sums**, often called a **sum sequence**. That is, we have that $\{S_1, S_2, S_3, \ldots\} = \{15, 33, 54, \ldots\}$ forms a sequence.

The answer then, is that the company will sell its thousandth computer during the 20th month.

EXERCISES 8.1.2

1. Find the sum of the first ten terms in the arithmetic sequences
(a) $\{1, 4, 7, 10, \ldots\}$ (b) $\{3, 9, 15, 21, \ldots\}$ (c) $\{10, 4, -2, \ldots\}$.

2. For the given arithmetic sequences, find the sum, S_n, to the requested number of terms.
(a) $\{4, 3, 2, \ldots\}$ for $n = 12$
(b) $\{4, 10, 16, \ldots\}$ for $n = 15$
(c) $\{2.9, 3.6, 4.3, \ldots\}$ for $n = 11$

3. Find the sum of the following sequences:
(a) $\{5, 4, 3, \ldots, -15\}$
(b) $\{3, 9, 15, \ldots, 75\}$
(c) $\{3, 5, 7, \ldots, 29\}$

4. The weekly sales of washing machines from a retail store that has just opened in a new housing complex increases by 2 machines per week. In the first week of January 1995, 24 machines were sold.
(a) How many are sold in the last week of December 1995?
(b) How many machines did the retailer sell in 1995?
(c) When was the 500th machine sold?

5. The fourth term of an arithmetic sequence is 5 while the sum of the first 6 terms is 10. Find the sum of the first nineteen terms.

6. Find the sum of the first 10 terms for the sequences defined by
(a) $u_n = -2 + 8n$ (b) $u_n = 1 - 4n$

7. The sum of the first eight terms of the sequence $\{\ln x, \ln x^2y, \ln x^3y^2, \ldots\}$ is given by $4(a\ln x + b\ln y)$. Find a and b.

8.1.3 SIGMA NOTATION

There is a second notation to denote the sum of terms. This other notation makes use the Greek letter $\sum\ldots$ as the symbol to inform us that we are carrying out a summation.
In short, $\sum\ldots$ stands for '**The sum of . . .**'.

This means that the expression $\boxed{\sum_{i=1}^{n} u_i = u_1 + u_2 + u_3 + \ldots + u_{n-1} + u_n}$.

For example, if $u_i = 2 + 5(i-1)$, i.e., an A.P with first term $a = 2$ and common difference $d = 5$, the expression $S_n = \sum_{i=1}^{n}[2 + 5(i-1)]$ would respesent the sum of the first n terms of the sequence. So, the sum of the first 3 terms would be given by

$$\begin{aligned} S_3 = \sum_{i=1}^{3}[2+5(i-1)] &= \underbrace{[2+5(1-1)]}_{i=1} + \underbrace{[2+5(2-1)]}_{i=2} + \underbrace{[2+5(3-1)]}_{i=3} \\ &= 2 + 7 + 12 \\ &= 21 \end{aligned}$$

Properties of $\sum$

1. $\sum$ is distributive. That is, $\boxed{\sum_{i=1}^{n}[u_i + v_i] = \sum_{i=1}^{n} u_i + \sum_{i=1}^{n} v_i}$.

2. $\boxed{\sum_{i=1}^{n} ku_i = k\sum_{i=1}^{n} u_i}$, for some constant value k.

3. $\boxed{\sum_{i=1}^{n} k = kn}$, i.e., adding a constant term, k, n times is the same as multiplying k by n.

EXAMPLE 8.10

Given that $u_i = 5 + 2i$ and that $v_i = 2 - 5i$ find

(a) $\sum_{i=1}^{5} u_i$ (b) $\sum_{i=1}^{5}[2u_i - v_i]$ (c) $\sum_{i=1}^{1000}[5u_i + 2v_i]$

Solution

(a) $\sum_{i=1}^{5} u_i = u_1 + u_2 + u_3 + u_4 + u_5 = [5+2] + [5+4] + [5+6] + [5+8] + [5+10]$

$= 7 + 9 + 11 + 13 + 15$
$= 55$

(b) $\sum_{i=1}^{5}[2u_i - v_i] = \sum_{i=1}^{5}(2u_i) + \sum_{i=1}^{5}(-v_i) = 2\sum_{i=1}^{5} u_i - \sum_{i=1}^{5} v_i$

Now, $2\sum_{i=1}^{5} u_i = 2 \times 55 = 110$

and $\sum_{i=1}^{5} v_i = \sum_{i=1}^{5}(2-5i) = \sum_{i=1}^{5}(2) - 5\sum_{i=1}^{5} i = 2 \times 5 - 5[1+2+3+4+5]$ [Using properties]

$= -65$

Therefore, $\sum_{i=1}^{5}[2u_i - v_i] = 110 - (-65) = 175$

(c) $\sum_{i=1}^{1000}[5u_i + 2v_i] = \sum_{i=1}^{1000}[5(5+2i) + 2(2-5i)]$

$= \sum_{i=1}^{1000}[25 + 10i + 4 - 10i]$

$= \sum_{i=1}^{1000} 29$

$= 29\,000$ [i.e., 29×1000]

In this example we have tried to show that there are a number of ways to obtain a sum. It is not always necessary to enumerate every term and then add them. Often, an expression can first be simplified.

EXERCISES 8.1.3 – MISCELLANEOUS QUESTIONS

1. Find the twentieth term in the sequence 9, 15, 21, 27, 33, . . .

2. Fill the gaps in this arithmetic sequence: –3, _, _, _, _, _, 12.

3. An arithmetic sequence has a tenth term of 17 and a fourteenth term of 30. Find the common difference.

4. If $u_{59} = \frac{1}{10}$ and $u_{100} = -1\frac{19}{20}$ for an arithmetic sequence, find the first term and the common difference.

5. Find the sum of the first one hundred odd numbers.

6. An arithmetic series has twenty terms. The first term is –50 and the last term is 83, find the sum of the series.

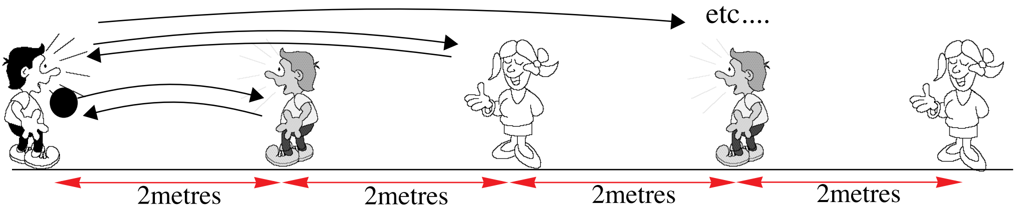
7. Thirty numbers are in arithmetic sequence. The sum of the numbers is 270 and the last number is 38. What is the first number?

8. How many terms of the arithmetic sequence: 2, 2.3, 2.6, 2.9, . . . must be taken before the terms exceed 100?

9. Brian and Melissa save \$50 in the first week of a savings program, \$55 in the second week, \$60 in the third and so on, in arithmetic progression. How much will they save in ten weeks? How long will they have to continue saving if their target is to save \$5000?

10. A printing firm offers to print business cards on the following terms:
\$45 for design and typesetting and then \$0.02 per card.
(i) What is the cost of 500 cards from this printer?
(ii) How many cards can a customer with \$100 afford to order?

11. A children's game consists of the players standing in a line with a gap of 2 metres between each. The child at the left hand end of the line has a ball which s/he throws to the next child in the line, a distance of 2 metres. The ball is then thrown back to the first child who then throws the ball to the third child in the line, a distance of 4 metres. The ball is then returned to the first child, and so on until all the children have touched the ball at least once.



(a) If a total of five children play and they make the least number of throws so that only the leftmost child touches the ball more than once:
  i. What is the largest single throw?
  ii. What is the total distance travelled by the ball?
(b) If seven children play, what is the total distance travelled by the ball?
(c) If n children play, derive a formula for the total distance travelled by the ball.
(d) Find the least number of children who need to play the game before the total distance travelled by the ball exceeds 100 metres.
(e) The children can all throw the ball 50 metres at most
  i. What is the largest number of children that can play the game?
  ii. What is the total distance travelled by the ball?

12. Find each sum,

(a) $\sum_{k=1}^{100} k$ (b) $\sum_{k=1}^{100} (2k+1)$ (c) $\sum_{k=1}^{51} (3k+5)$

13. If $u_i = -3+4i$ and $v_i = 12-3i$ find

(a) $\sum_{i=1}^{10} (u_i+v_i)$ (b) $\sum_{i=1}^{10} (3u_i+4v_i)$ (c) $\sum_{i=1}^{10} u_i v_i$

14. (a) Show that for an arithmetic sequence, $u_n = S_n - S_{n-1}$, where u_n is the nth term and S_n is the sum of the first n terms.

(b) Find the general term, u_n, of the A.P given that $\sum_{i=1}^{n} u_i = \frac{n}{2}(3n-1)$.

5. (a) $I = \frac{a}{n^k}$ **6.** (a) 0.10 (b) $\lambda = \lambda_0 \times 10^{-kx}$ (c) 16.82% (d) $k = -\frac{1}{x}\log\left(\frac{\lambda}{\lambda_0}\right)$

EXERCISE 8.1.1

1. i. (b) 4 (c) $t_n = 4n - 2$ ii. (b) –3 (c) $t_n = -3n + 23$ iii. (b) –5 (c) $t_n = -5n + 6$ iv. (b) 0.5 (c) $t_n = 0.5n$ v. (b) 2 (c) $t_n = y + 2n - 1$ vi. (b) –2 (c) $t_n = x - 2n + 4$ **2.** –28 **3.** 9,17 **4.** –43 **5.** 7 **6.** 7 **7.** –5 **8.** 0 **9.** (a) 41 (b) 31st **10.** 2, $\sqrt{3}$ **11.** (a) i. 2 ii. –3 (b) i. 4 ii. 11 **12.** $x - 8y$ **13.** $t_n = 5 + \frac{10}{3}(n-1)$ **14.** (a) –1 (b) 0

EXERCISE 8.1.2

1. (a) 145 (b) 300 (c) –170 **2.** (a) –18 (b) 690 (c) 70.4 **3.** (a) –105 (b) 507 (c) 224 **4.** (a) 126 (b) 3900 (c) 14th week **5.** 855 **6.** (a) 420 (b) –210 **7.** $a = 9, b = 7$

EXERCISE 8.1.3

1. 123 **2.** –3, –0.5, 2, 4.5, 7, 9.5, 12 **3.** 3.25 **4.** $a = 3$ $d = -0.05$ **5.** 10 000 **6.** 330 **7.** –20 **8.** 328 **9.** \$725, 37wks **10.** i. \$55 ii. 2750 **11.** (a) (i) 8m (ii) 40m (b) 84m (c) Dist = $2n^2 - 2n = 2n(n-1)$ (d) 8 (e) 26 players, 1300m **12.** (a) 5050 (b) 10200 (c) 4233 **13.** (a) 145 (b) 390 (c) –1845 **14.** (b) $3n - 2$

EXERCISE 8.2.1

1. (a) $r = 2, u_5 = 48, u_n = 3 \times 2^{n-1}$ (b) $r = \frac{1}{3}, u_5 = \frac{1}{27}, u_n = 3 \times \left(\frac{1}{3}\right)^{n-1}$ (c) $r = \frac{1}{5}, u_5 = \frac{2}{625}, u_n = 2 \times \left(\frac{1}{5}\right)^{n-1}$ (d) $r = -4, u_5 = -256, u_n = -1 \times (-4)^{n-1}$ (e) $r = \frac{1}{b}, u_5 = \frac{a}{b^3}, u_n = ab \times \left(\frac{1}{b}\right)^{n-1}$ (f) $r = \frac{b}{a}, u_5 = \frac{b^4}{a^2}, u_n = a^2 \times \left(\frac{b}{a}\right)^{n-1}$ **2.** (a) ±12 (b) $\frac{\pm\sqrt{5}}{2}$ **3.** (a) ±96 (b) 15th **4.** (a) $u_n = 10 \times \left(\frac{5}{6}\right)^{n-1}$ (b) $\frac{15625}{3888} \cong 4.02$ (c) $n = 5$ (4 times) **5.** $-2, \frac{4}{3}$ **6.** (a) i. \$4096 ii. \$2097.15 (b) 6.2 yrs **7.** $\left(u_n = \frac{1000}{169} \times \left(\frac{12}{5}\right)^{n-1}\right), \frac{1990656}{4225} \cong 471.16$ **8.** 2.5,5,10 or 10,5,2.5 **9.** 53757 **10.** 108 952 **11.** (a) \$56 156 (b) \$299 284

EXERCISE 8.2.2

1. (a) 3 (b) $\frac{1}{3}$ (c) –1 (d) $-\frac{1}{3}$ (e) 1.25 (f) $-\frac{2}{3}$ **2.** (a) 216513 (b) 1.6384×10^{-10} (c) $\frac{256}{729}$ (d) $\frac{729}{2401}$ (e) $-\frac{81}{1024}$ **3.** (a) 11; 354292 (b) 7; 473 (c) 8; 90.90909 (d) 8; 172.778 (e) 5; 2.256 (f) 13; 111.1111111111 **4.** (a) $\frac{127}{128}$ (b) $\frac{63}{8}$ (c) $\frac{130}{81}$ (d) 60 (e) $\frac{63}{64}$ **5.** 4; 118096 **6.** \$2109.50 **7.** 9.28cm **8.** (a) $V_n = V_0 \times 0.7^n$ (b) 7 **9.** 54 **10.** 53.5gms; 50 weeks. **11.** 7 **12.** 9 **13.** –0.5, –0.7797 **14.** $r = 5$, 1.8×10^{10} **15.** \$8407.35 **16.** 1.8×10^{19} or about 200 billion tonnes.

EXERCISE 8.2.3

1. Term 9 AP = 180, GP = 256. Sum to 11 terms AP = 1650, GP = 2047. **2.** 18. **3.** 12 **4.** 7, 12 **5.** 8 weeks (Ken \$220 & Bo-Youn \$255) **6.** (a) week 8 (b) week 12 **7.** (a) 1.618 (b) 121379 [~121400, depends on rounding errors]

EXERCISE 8.2.4

1. (i) $\frac{81}{2}$ (ii) $\frac{10}{13}$ (iii) 5000 (iv) $\frac{30}{11}$ **2.** $23\frac{23}{99}$ **3.** 6667 fish. [Nb: $t_{43} < 1$. If we use $n = 43$ then ans is 6660 fish]; 20 000 fish. Overfishing means that fewer fish are caught in the long run. [*An alternate estimate for the total catch is* 1665 *fish.*] **4.** 27 **5.** 48,12,3 or 16,12,9 **6.** (a) $\frac{11}{30}$ (b) $\frac{37}{99}$ (c) $\frac{191}{90}$ **7.** 128 cm **8.** $\frac{121}{9}$ **9.** $2 + \frac{4}{3}\sqrt{3}$ **10.** $\frac{1-(-t)^n}{1+t}$ $\frac{1}{1+t}$ **11.** $\frac{1-(-t^2)^n}{1+t^2}$ $\frac{1}{1+t^2}$

EXERCISE 8.2.5

1. 3, –0.2 **2.** $\frac{2560}{93}$ **3.** $\frac{10}{3}$ **4.** (a) $\frac{43}{18}$ (b) $\frac{458}{99}$ (c) $\frac{413}{990}$ **5.** 9900 **6.** 3275 **7.** 3 **8.** $t_n = 6n - 14$ **9.** 6 **10.** $-\frac{1}{6}$ **11.** i. 12 ii. 26 **12.** 9, 12 **13.** ±2 **14.** (5, 5, 5), (5, –10, 20) **15.** (a) 2, 7 (b) 2, 5, 8 (c) $3n - 1$ **16.** (a) 5 (b) 2 m

EXERCISE 8.3

1. \$2773.08 **2.** \$4377.63 **3.** \$1781.94 **4.** \$12216 **5.** \$35816.95 **6.** \$40349.37 **7.** \$64006.80 **8.** \$276971.93, \$281325.41 **9.** \$63762.25 **10.** \$98.62, \$9467.14, interest \$4467.14. Flat interest = \$6000 **11.** \$134.41, \$3790.44, 0.602% /month (or 7.22% p.a)

EXERCISE 9.1

1.	a cm	b cm	c cm	A	B	C
1	3.8	4.1	1.6	67°	90°	23°
2	81.5	98.3	55.0	56°	90°	34°
3	32.7	47.1	33.9	44°	90°	46°
4	1.61	30.7	30.7	3°	90°	87°
5	2.3	2.74	1.49	57°	90°	33°
6	48.5	77	59.8	39°	90°	51°
7	44.4	81.6	68.4	33°	90°	57°
8	2.93	13.0	12.7	13°	90°	77°
9	74.4	94.4	58.1	52°	90°	38°
10	71.8	96.5	64.6	48°	90°	42°
11	23.3	34.1	24.9	43°	90°	47°
12	43.1	43.2	2.3	87°	90°	3°
13	71.5	80.2	36.4	63°	90°	27°
14	33.5	34.1	6.5	79°	90°	11°
15	6.1	7.2	3.82	58°	90°	32°
16	29.1	30	7.3	76°	90°	14°
17	29.0	29.1	2.0	86°	90°	4°
18	34.5	88.2	81.2	23°	90°	67°
19	24.0	29.7	17.5	54°	90°	36°
20	41.2	46.2	21.0	63°	90°	27°
21	59.6	72.9	41.8	55°	90°	35°
22	5.43	6.8	4.09	53°	90°	37°