

Arithmetics sequences [42 marks]

The n^{th} term of an arithmetic sequence is given by $u_n = 15 - 3n$.

1a. State the value of the first term, u_1 .

[1 mark]

Markscheme

$$u_1 = 12 \quad \mathbf{A1}$$

[1 mark]

1b. Given that the n^{th} term of this sequence is -33 , find the value of n .

[2 marks]

Markscheme

$$15 - 3n = -33 \quad (\mathbf{A1})$$

$$n = 16 \quad \mathbf{A1}$$

[2 marks]

1c. Find the common difference, d .

[2 marks]

Markscheme

valid approach to find d $(\mathbf{M1})$

$$u_2 - u_1 = 9 - 12 \quad \text{OR} \quad \text{recognize gradient is } -3 \quad \text{OR} \quad \text{attempts to solve}$$
$$-33 = 12 + 15d$$

$$d = -3 \quad \mathbf{A1}$$

[2 marks]

2. Consider an arithmetic sequence where $u_8 = S_8 = 8$. Find the value of the first term, u_1 , and the value of the common difference, d . [5 marks]

Markscheme

METHOD 1 (finding u_1 first, from S_8)

$$4(u_1 + 8) = 8 \text{ (A1)}$$

$$u_1 = -6 \text{ A1}$$

$$u_1 + 7d = 8 \text{ OR } 4(2u_1 + 7d) = 8 \text{ (may be seen with their value of } u_1) \text{ (A1)}$$

attempt to substitute their u_1 (M1)

$$d = 2 \text{ A1}$$

METHOD 2 (solving simultaneously)

$$u_1 + 7d = 8 \text{ (A1)}$$

$$4(u_1 + 8) = 8 \text{ OR } 4(2u_1 + 7d) = 8 \text{ OR } u_1 = -3d \text{ (A1)}$$

attempt to solve linear or simultaneous equations (M1)

$$u_1 = -6, d = 2 \text{ A1A1}$$

[5 marks]

An arithmetic sequence has first term 60 and common difference -2.5 .

- 3a. Given that the k th term of the sequence is zero, find the value of k . [2 marks]

Markscheme

attempt to use $u_1 + (n - 1)d = 0$ (M1)

$$60 - 2.5(k - 1) = 0$$

$$k = 25 \text{ A1}$$

[2 marks]

- 3b. Let S_n denote the sum of the first n terms of the sequence. [3 marks]
Find the maximum value of S_n .

Markscheme

METHOD 1

attempting to express S_n in terms of n **(M1)**

use of a graph or a table to attempt to find the maximum sum **(M1)**
 $= 750$ **A1**

METHOD 2

EITHER

recognizing maximum occurs at $n = 25$ **(M1)**

$$S_{25} = \frac{25}{2}(60 + 0), S_{25} = \frac{25}{2}(2 \times 60 + 24 \times -2.5) \text{ **(A1)**}$$

OR

attempting to calculate S_{24} **(M1)**

$$S_{24} = \frac{24}{2}(2 \times 60 + 23 \times -2.5) \text{ **(A1)**}$$

THEN

$= 750$ **A1**

[3 marks]

In an arithmetic sequence, $u_2 = 5$ and $u_3 = 11$.

4a. Find the common difference.

[2 marks]

Markscheme

valid approach **(M1)**

eg $11 - 5, 11 = 5 + d$

$d = 6$ **A1 N2**

[2 marks]

4b. Find the first term.

[2 marks]

Markscheme

valid approach **(M1)**

eg $u_2 - d, 5 - 6, u_1 + (3 - 1)(6) = 11$

$u_1 = -1$ **A1 N2**

[2 marks]

4c. Find the sum of the first 20 terms.

[2 marks]

Markscheme

correct substitution into sum formula

eg $\frac{20}{2}(2(-1) + 19(6)), \frac{20}{2}(-1 + 113)$ **(A1)**

$S_{20} = 1120$ **A1 N2**

[2 marks]

Sergei is training to be a weightlifter. Each day he trains at the local gym by lifting a metal bar that has heavy weights attached. He carries out successive lifts. After each lift, the same amount of weight is **added** to the bar to increase the weight to be lifted.

The weights of each of Sergei's lifts form an arithmetic sequence.

Sergei's friend, Yuri, records the weight of each lift. Unfortunately, last Monday, Yuri misplaced all but two of the recordings of Sergei's lifts.

On that day, Sergei lifted 21 kg on the third lift and 46 kg on the eighth lift.

5a. For that day find how much weight was added after each lift.

[2 marks]

Markscheme

$5d = 46 - 21$ **OR** $u_1 + 2d = 21$ and $u_1 + 7d = 46$ **(M1)**

Note: Award **(M1)** for a correct equation in d or for two correct equations in u_1 and d .

$(d =) 5$ (kg) **(A1) (C2)**

[2 marks]

5b. For that day find the weight of Sergei's first lift.

[2 marks]

Markscheme

$$u_1 + 2 \times 5 = 21 \quad (\mathbf{M1})$$

OR

$$u_1 + 7 \times 5 = 46 \quad (\mathbf{M1})$$

Note: Award **(M1)** for substitution of their d into either of the two equations.

$$(u_1 =) 11 \text{ (kg)} \quad (\mathbf{A1})(\mathbf{ft}) \quad (\mathbf{C2})$$

Note: Follow through from part (a)(i).

[2 marks]

5c. On that day, Sergei made 12 successive lifts. Find the total combined weight of these lifts. [2 marks]

Markscheme

$$\frac{12}{2}(2 \times 11 + (12 - 1) \times 5) \quad (\mathbf{M1})$$

Note: Award **(M1)** for correct substitution into arithmetic series formula.

$$= 462 \text{ (kg)} \quad (\mathbf{A1})(\mathbf{ft}) \quad (\mathbf{C2})$$

Note: Follow through from parts (a) and (b).

[2 marks]

The 3rd term of an arithmetic sequence is 1407 and the 10th term is 1183.

6. Calculate the number of positive terms in the sequence.

[3 marks]

Markscheme

$$1471 + (n - 1)(-32) > 0 \quad (\mathbf{M1})$$

$$\Rightarrow n < \frac{1471}{32} + 1$$

$$n < 46.96... \quad (\mathbf{A1})$$

so 46 positive terms **A1**

[3 marks]

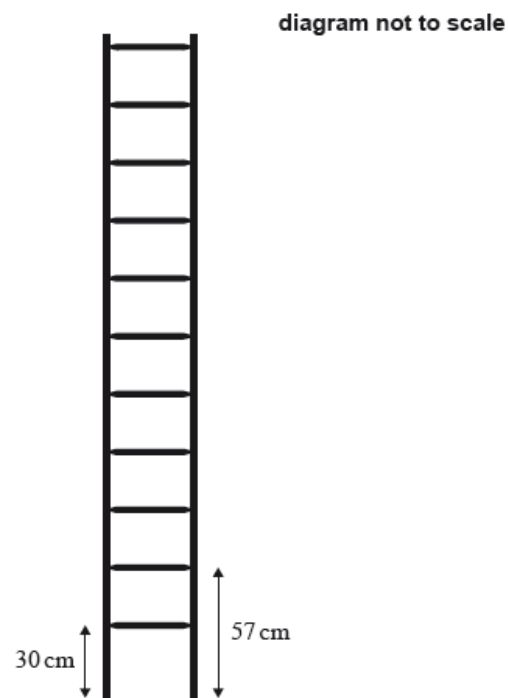
The company Snakezen's Ladders makes ladders of different lengths. All the ladders that the company makes have the same design such that:

the first rung is 30 cm from the base of the ladder,

the second rung is 57 cm from the base of the ladder,

the distance between the first and second rung is equal to the distance between all adjacent rungs on the ladder.

The ladder in the diagram was made by this company and has eleven equally spaced rungs.



7a. Find the distance from the base of this ladder to the top rung.

[3 marks]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$$30 + (11 - 1) \times 27 \quad \text{(M1)(A1)}$$

Note: Award **(M1)** for substituted arithmetic sequence formula, **(A1)** for correct substitutions.

$$= 300 \text{ (cm)} \quad \text{(A1)} \quad \text{(C3)}$$

Note: Units are not required.

[3 marks]

7b. The company also makes a ladder that is 1050 cm long.

[3 marks]

Find the maximum number of rungs in this 1050 cm long ladder.

Markscheme

$$1050 \geq 30 + (n - 1) \times 27 \quad \text{(M1)(A1)(ft)}$$

Note: Award **(M1)** for substituted arithmetic sequence formula ≤ 1050 , accept an equation, **(A1)** for correct substitutions.

$$n = 38 \quad \text{(A1)(ft)} \quad \text{(C3)}$$

Note: Follow through from their 27 in part (a). The answer must be an integer and rounded down.

[3 marks]

Tomás is playing with sticks and he forms the first three diagrams of a pattern. These diagrams are shown below.



Diagram 1



Diagram 2



Diagram 3

Tomás continues forming diagrams following this pattern.

8a. Diagram n is formed with 52 sticks. Find the value of n .

[3 marks]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$$4 + 3(n - 1) = 52 \quad (M1)(A1)$$

Note: Award **(M1)** for substitution into the formula of the n th term of an arithmetic sequence, **(A1)** for correct substitution.

$$n = 17 \quad (A1) \quad (C3)$$

[3 marks]

Tomás forms a total of 24 diagrams.

8b. Find the total number of sticks used by Tomás for all 24 diagrams.

[3 marks]

Markscheme

$$\frac{24}{2}(2 \times 4 + 23 \times 3) \text{ OR } \frac{24}{2}(4 + 73) \quad (M1)(A1)(ft)$$

Notes: Award **(M1)** for substitution into the sum of the first n terms of an arithmetic sequence formula, **(A1)(ft)** for their correct substitution, consistent with part (a).

$$924 \quad (A1)(ft) \quad (C3)$$

Note: Follow through from part (a).

[3 marks]