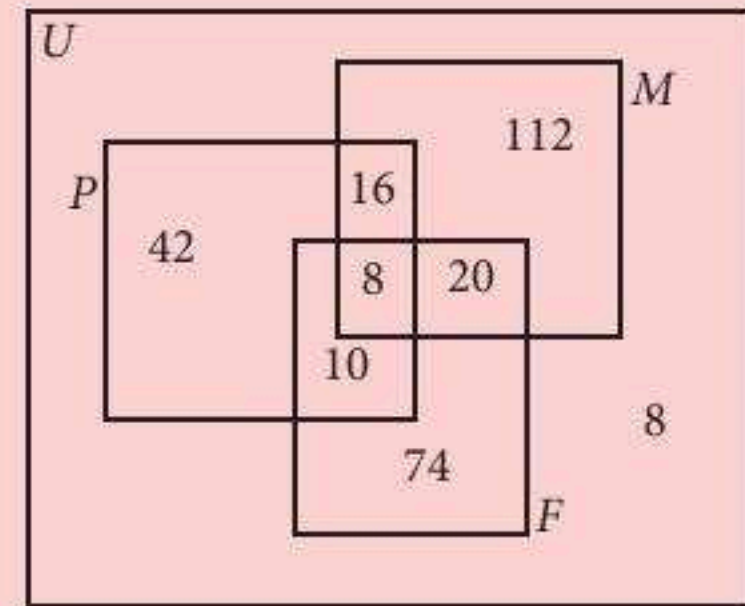1. In a study of least favourite subjects for students in a high school, students mentioned physics (P), mathematics (M) and French (F). The results of the study are shown in the Venn diagram.

   (a) Write down the total number of students in this school.

   (b) A student is chosen at random. What is the probability that the least favourite subject is:

      (i) mathematics only

      (ii) physics or French

      (iii) all three subjects

      (iv) physics or mathematics, but not French.

   (c) What is the probability that a student chosen at random does not include French as one of the least favourites?

2. Beartown has three local newspapers: *The Art Journal*, *The Beartown News*, and *The Currier*.

   A survey shows that

   32% of the town's population read *The Art Journal*,
   46% read *The Beartown News*,
   54% read *The Currier*,
   3% read *The Art Journal* and *The Beartown News* only,
   8% read *The Art Journal* and *The Currier* only,
   12% read *The Beartown News* and *The Currier* only, and
   5% of the population reads all three newspapers.

   (a) Draw a Venn diagram to represent this information. Label A the set that represents *The Art Journal* readers, B the set that represents *The Beartown News* readers, and C the set that represents *The Currier* readers.

   (b) Find the percentage of the population that does not read any of the three newspapers.

   (c) Find the percentage of the population that reads exactly one newspaper.

   (d) Find the percentage of the population that reads *The Art Journal* or *The Beartown News* but not *The Currier*.

A local radio station states that 83% of the population reads either *The Beartown News* or *The Currier*.

(e) Use your Venn diagram to decide whether the statement is true. Justify your answer.

The population of Beartown is 120 000. The local radio station claimed that 34 000 of the town's citizens read at least two of the local newspapers.

(f) Find the percentage error in this claim.

3. Two independent events A and B are given such that

   $P(A) = k$, $P(B) = k + 0.3$ and $P(A \cap B) = 0.18$

   (a) Find k

   (b) Find $P(A \cup B)$

4. Many airport authorities test prospective employees for drug use. This procedure has plenty of opponents who claim that it creates difficulties for some people and that it prevents some others from getting these jobs even if they were not drug users. The claim depends on the fact that these tests are not 100% accurate. To test this claim, assume that a test is 98% accurate in that it identifies a person as a user or non-user 98% of the time. Each job applicant takes this test twice.
   The tests are done at separate times and are designed to be independent of each other. What is the probability that:

   (a) a non-user fails both tests

   (b) a drug user is detected (i.e., they fail at least one test)

   (c) a drug user passes both tests?

5. In a group of 200 students taking the IB examination, 120 take Spanish, 60 take French and 10 take both.

   (a) If a student is selected at random, what is the probability that the student:

       (i) takes either French or Spanish

       (ii) takes either French or Spanish but not both

       (iii) does not take any French or Spanish?

   (b) Given that a student takes the Spanish exam, what is the chance that they take French?

6. In a factory producing disk drives for computers, there are three machines that work independently to produce one of the components. In any production process, machines are not 100% fault free. The production after one batch from each machine is listed in the table.

	Defective	Non-defective
Machine I	6	120
Machine II	4	80
Machine III	10	150

(a) A component is chosen at random from the batches. Find the probability that the chosen component is:

(i) from machine I

(ii) a defective component from machine II

(iii) non-defective or from machine I

(iv) from machine I given that it is defective.

(b) Is the quality of the component dependent on the machine used?

7. At a school, the students are organising a lottery to raise money for their community. The tickets consist of coloured envelopes with a small note inside. The note says: 'You won!' or 'No prize.' The envelopes have several colours. They have 70 red envelopes that contain two prizes, and the rest (130 tickets) contain four other prizes.

(a) You want to help this class and you buy a ticket hoping that it does not have a prize. You pick your ticket at random by closing your eyes. What is the probability that your ticket does not have a prize?

(b) You are surprised – you picked a red envelope. What is the probability that you did not win a prize?

8. Two events A and B have the conditions:

$$P(A|B) = 0.30, \quad P(B|A) = 0.60, \quad P(A \cap B) = 0.18$$

(a) Find $P(B)$.

(b) Are A and B independent? Why?

9. In several ski resorts in Switzerland, the local sports authorities use senior high school students as 'ski instructors' to help deal with the surge in demand during vacations. To become an instructor, you have to pass a test and have to be a senior at your school. Here are the results of a survey of 120 students who are training to become instructors. In this group, there are 70 boys and 50 girls. 74 students took the test, 32 boys and 16 girls passed the test, and the rest, including 12 girls, failed. 10 of the students, including 6 girls, were too young to take the ski test.

(a) Copy and complete the Table 8.5.

	Boys	Girls
Passed	32	16
Failed		12
Training		
Too young		

Table 8.5 Table for question 9

(b) Find the probability that:

   (i)   a student chosen at random has taken the test

   (ii) a girl chosen at random has taken the test

   (iii) a randomly chosen boy and randomly chosen girl have both
            passed the ski test.

10. Events X and Y have the conditions $P(X) = 0.6$, $P(Y) = 0.8$ and
    $P(X \cup Y) = 1$

    Find:

    (a) $P(X \cap Y)$

    (b) $P(X' \cup Y')$

11. In a survey, 100 managers were asked 'Do you prefer to watch the news
    or play sport?' Of the 46 men in the survey, 33 said they would choose
   sport, while 29 women also made this choice.

	Men	Women	Total
News			
Sport	33	29	
Total	46		100

    Find the probability that:

    (a) a manager selected at random prefers to watch the news

    (b) a manager prefers to watch the news, given that the manager is a
            man.

12. Two unbiased, six-sided dice are rolled, and the total score is noted.

    (a) Copy and complete the tree diagram by entering probabilities and
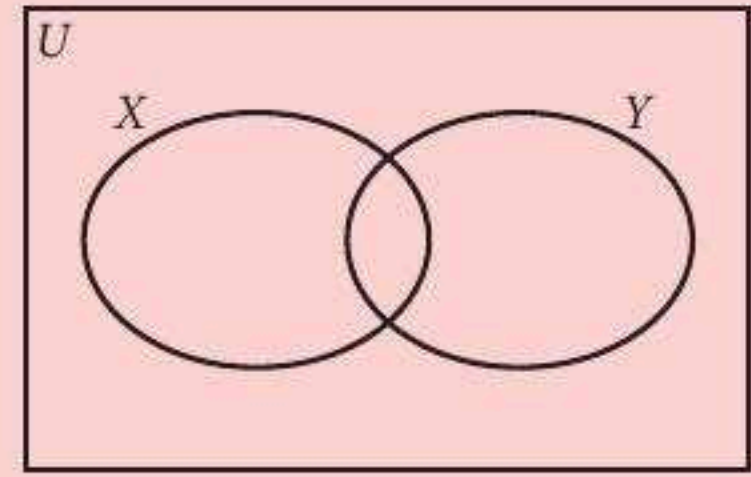   listing outcomes.
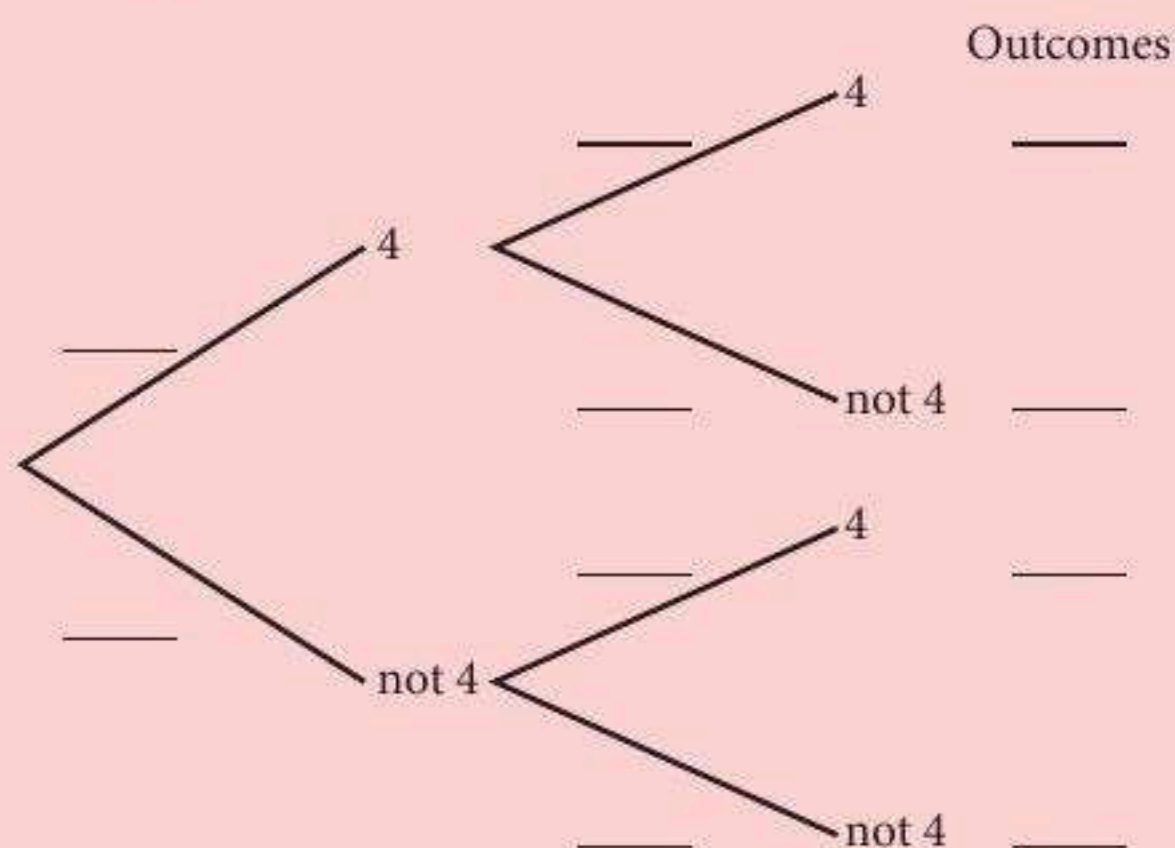
    (b) Find the probability of getting one or more fours.

13. The Venn diagram shows a sample space U and events X and Y.

$n(U) = 36, n(X) = 11, n(Y) = 6$ and $n(X \cup Y)' = 21$

  (a) Copy the diagram and shade the region $(X \cup Y)'$

  (b) Find:

    (i) $n(X \cap Y)$ (ii) $P(X \cap Y)$

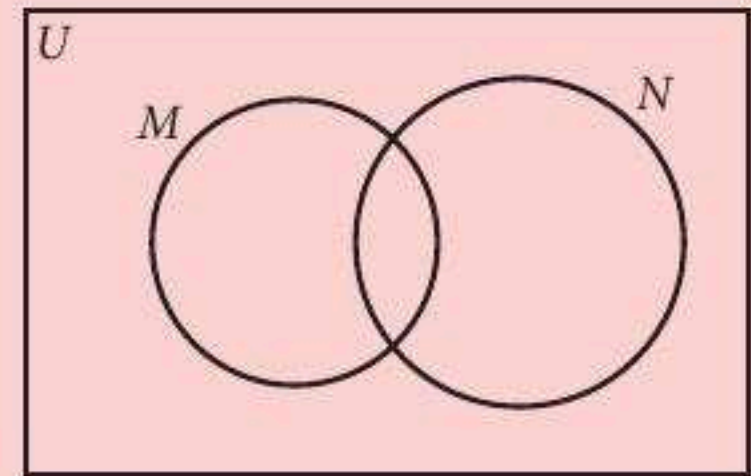 (c) Are events X and Y mutually exclusive? Explain why or why not.

14. The Venn diagram shows the universal set U and the subsets M and N.

  (a) Copy the diagram and shade the area in the diagram which represents the set $M \cap N'$

$n(U) = 100, n(M) = 30, n(N) = 50,$
$n(M \cup N) = 65$

  (b) Find $n(N \cap M')$

  (c) An element is selected at random from U. What is the probability that this element is in $N \cap M'$?

15. Two fair dice are thrown and the number showing on each is noted. Find the probability that:

  (a) the sum of the numbers is less than or equal to 7

  (b) at least one dice shows a 3

  (c) at least one dice shows a 3, given that the sum is less than 8.

16. For events A and B, the probabilities are $P(A) = \dfrac{3}{11}$, $P(B) = \dfrac{4}{11}$

  Calculate the value of $P(A \cap B)$ if:

  (a) $P(A \cup B) = \dfrac{6}{11}$

  (b) events A and B are independent.

17. Consider events A, B such that $P(A) \neq 0$, $P(A) \neq 1$, $P(B) \neq 0$, and $P(B) \neq 1$.

  For each of the below, state whether A and B are mutually exclusive, independent, or neither.

  (a) $P(A|B) = P(A)$

  (b) $P(A \cap B) = 0$

  (c) $P(A \cap B) = P(A)$

18. Sophia is a student at an IB school.

The probability that she will be woken by her alarm clock is $\frac{7}{8}$

If she is woken by her alarm clock the probability that she will be late for school is $\frac{1}{4}$

If she is not woken by her alarm clock the probability that she will be late for school is $\frac{3}{5}$

Let W be the event 'Sophia is woken by her alarm clock'.

Let L be the event 'Sophia is late for school'.

(a) Copy and complete the tree diagram.

(b) Calculate the probability that Sophia will be late for school.

(c) Given that Sophia is late for school what is the probability that she was woken by her alarm clock?
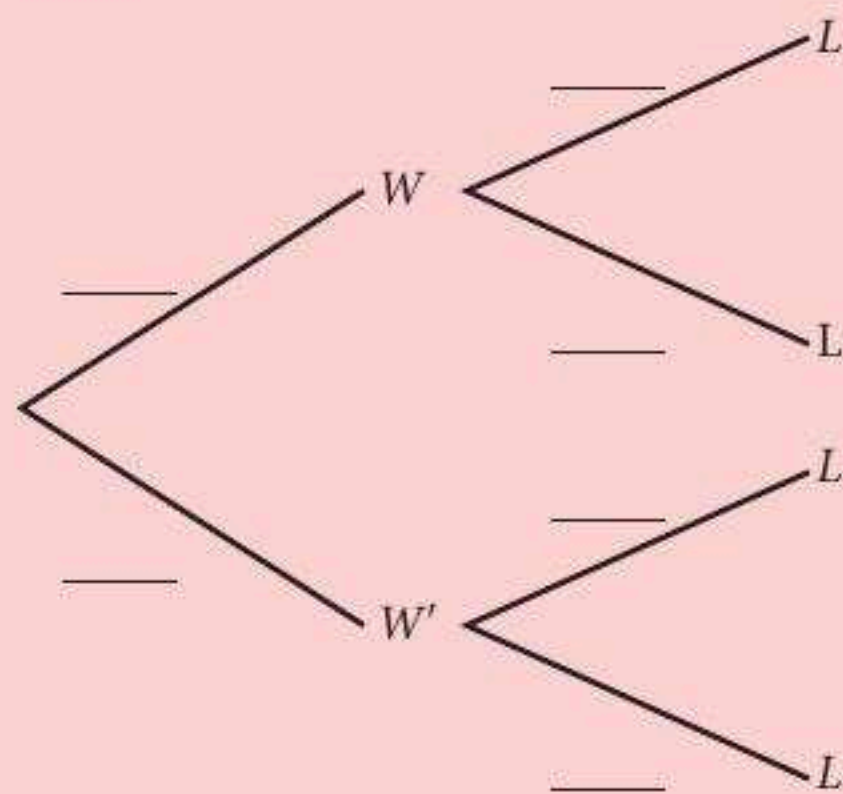
19. The diagram shows a circle divided into three sectors A, B and C. The angles at the centre of the circle are 90°, 120° and 150°. Sectors A and B are shaded as shown.

The arrow is spun. It cannot land on the lines between the sectors. Let A, B, C and S be the events defined by

　　　A: Arrow lands in sector A

　　　B: Arrow lands in sector B

　　　C: Arrow lands in sector C
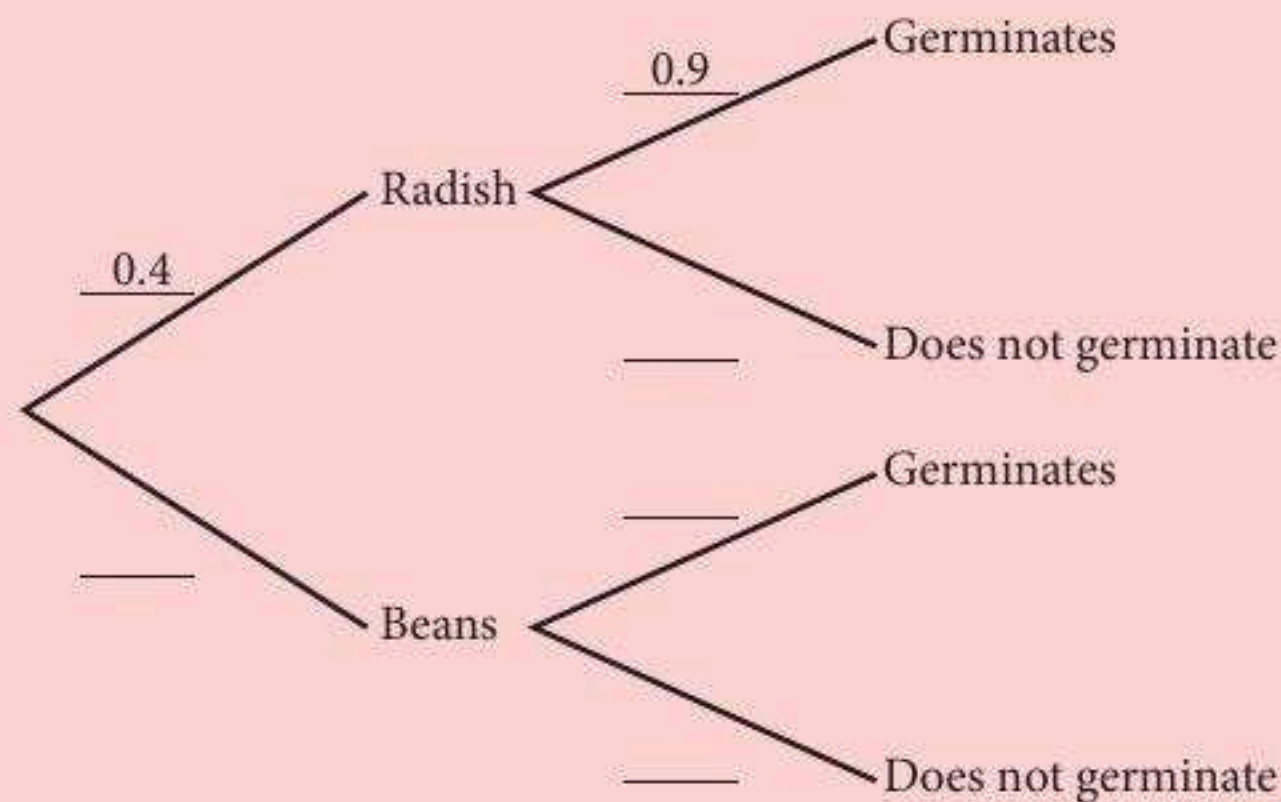
　　　S: Arrow lands in a shaded region.

Find:

(a) P(B)　　(b) P(S)　　(c) P($A|S$)
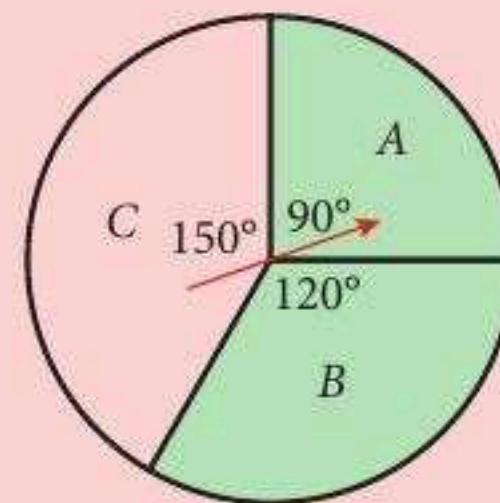
20. A packet of seeds contains 40% radish seeds and 60% bean seeds. The probability that a radish seed germinates is 0.9, and that a bean seed germinates is 0.8. A seed is chosen at random from the packet.

(a) Copy and complete the probability tree diagram.

(b) (i) Calculate the probability that the chosen seed is radish and germinates.

(ii) Calculate the probability that the chosen seed germinates.

(iii) Given that the seed germinates, calculate the probability that it is radish.

21. Two unbiased six-sided dice of different colours are rolled.

Find:

(a) P(the same number appears on both dice)

(b) P(the sum of the numbers is 10)

(c) P(the sum of the numbers is 10 or the same number appears on both dice).

22. The table shows the subjects studied by 210 students at a college.

	Year 1	Year 2	Totals
History	50	35	85
Science	15	30	45
Art	45	35	80
Totals	110	100	210

(a) A student from the college is selected at random.

Let A be the event the student studies art.

Let B be the event the student is in Year 2.

(i) Find P(A)

(ii) Find the probability that the student is a Year 2 art student.

(iii) Are the events A and B independent? Justify your answer.

(b) Given that a history student is selected at random, calculate the probability that the student is in Year 1.

(c) Two students are selected at random from the college. Calculate the probability that one student is in Year 1, and the other in Year 2.

16.

P(A)	P(B)	Conditions for events A and B	P(A ∩ B)	P(A ∪ B)	P(A|B)
0.3	0.4	Mutually exclusive	0.00	0.70	0.00
0.3	0.4	Independent	0.12	0.58	0.30
0.1	0.5	Mutually exclusive	0.00	0.60	0.00
0.2	0.5	Independent	0.10	0.60	0.20

17 (a) 0.30   (b) yes

18. (a) 65%   (b) 35%   (c) 52%

19. (a) 0.56   (b) 0.10

20. (a) $\dfrac{1}{216}$ (b) $\dfrac{91}{216}$ (c) $\dfrac{25}{72}$

21. (a) 0.3   (b) 0.441   (c) 0.657

22. (a) $\dfrac{23}{144}$ (b) $\dfrac{11}{144}$ (c) $\dfrac{15}{144}$ (d) $\dfrac{5}{12}$

23. (a)
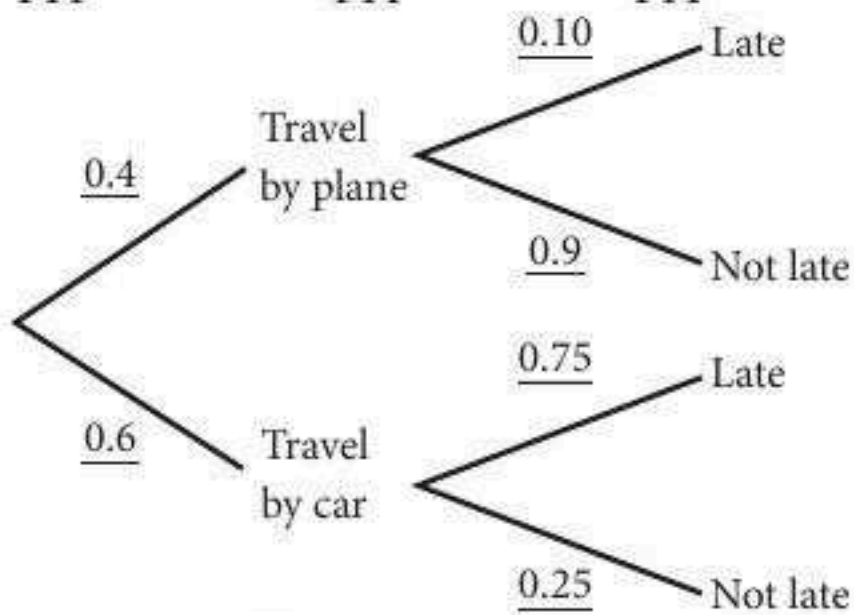
(b) 0.51   (c) 0.706

24. (a) 0.8805   (b) 0.0471

25. (a) 0.325   (b) 0.427   (c) 0.553

Chapter 8 Practice questions

1. (a) 290
    (b) (i) 0.386
       (ii) 0.434
       (iii) 0.028
       (iv) 0.586
    (c) 0.614

2. (a)

(b) 1   (c) 71   (d) 45   (e) True
(f) 1.19%

3. (a) 0.30   (b) 0.72

4. (a) 0.0004   (b) 0.9996   (c) 0.0004

5. (a) (i) 0.85
       (ii) 0.80
       (iii) 0.15
    (b) 0.083

6. (a) (i) 0.3405
       (ii) 0.0108
       (iii) 0.9622
       (iv) 0.30
    (b) Yes.

7. (a) 0.97   (b) 0.971

8. (a) 0.60   (b) Yes, P(B\|A)=P(B)=0.60

9. (a)

	Boys	Girls
Passed the ski test	32	16
Failed the ski test	12	12
Training, but did not take the test yet	22	16
Too young to take the test	4	6

(b) (i) 0.6
      (ii) 0.56
      (iii) 0.1463

10. (a) 0.4   (b) 0.6

11. (a) 0.38   (b) 0.283

12. (a)   (b) $\dfrac{11}{36}$

13. (a)
    (b) (i) 2
       (ii) $\dfrac{1}{18}$
    (c) No, $n(X \cap Y) \neq 0$

14. (a)   (b) 35   (c) 0.35

15. (a) $\dfrac{7}{12}$ (b) $\dfrac{11}{36}$ (c) $\dfrac{1}{3}$

16. (a) $\dfrac{1}{11}$ (b) $\dfrac{12}{121}$

17. (a) Ind.
    (b) Mutually excl.
    (c) Neither

18. (a)   (b) $\dfrac{47}{160}$ (c) $\dfrac{35}{47}$

19. (a) $\dfrac{1}{3}$ (b) $\dfrac{7}{12}$ (c) $\dfrac{3}{7}$

20. (a)
    (b) (i) 0.36
       (ii) 0.84
       (iii) 0.429

21. (a) $\dfrac{1}{6}$ (b) $\dfrac{1}{12}$ (c) $\dfrac{2}{9}$

22. (a) (i) $\dfrac{8}{21}$
       (ii) $\dfrac{1}{6}$
       (iii) No $P(A \cap B) \neq P(A)P(B)$
    (b) $\dfrac{10}{17}$
    (c) $\dfrac{220}{441}$

Chapter 9

Exercise 9.1

1. Achilles can get arbitrarily close to the tortoise. Select any distance between the tortoise and Achilles and after some number of intervals Achilles must be that close. Therefore, the distance between the tortoise and Achilles approaches zero.

2. (a) 4   (b) −5   (c) 6   (d) 3
    (e) 0.354

3. (a) (i) 31 cm s⁻¹   (ii) 92.5 cm s⁻¹   (iii) 153 cm s⁻¹
    (b) Around 92 cm s⁻¹ by drawing a line by eye and estimating two points on the line, then calculating slope, e.g. (5, 325) and (1.5, 0) yields 92.9 cm s⁻¹.