

k	$P(x \leq k)$
0	
1	
2	
3	
4	
5	
6	

Table 12.22 Table for question 3 (a)

2. A poll of 20 adults is taken in a large city. The purpose is to determine whether they support banning smoking in restaurants. It is known that approximately 60% of the population supports the decision. Let x represent the number of respondents in favour of the decision.
- What is the probability that 5 respondents support the decision?
 - What is the probability that none of the 20 supports the decision?
 - What is the probability that at least 1 respondent supports the decision?
 - What is the probability that at least two respondents support the decision?
 - Find the mean and standard deviation of the distribution.
3. Consider the binomial random variable with $n = 6$ and $p = 0.3$.
- Copy Table 12.22 and fill in the probabilities.
 - Copy and complete the following table. Some cells have been filled in to guide you.

Number of successes x	List the values of x	Write the probability statement	Explain it, if needed	Find the required probability
At most 3				
At least 3				
More than 3	4, 5, 6	$P(x > 3)$	$1 - P(x \leq 3)$	0.07047
Fewer than 3				
Between 3 and 5 (inclusive)				
Exactly 3				

4. Repeat question 3 with $n = 7$ and $p = 0.4$
5. A box contains 8 balls: 5 are green and 3 are white, red and yellow. Three balls are chosen at random without replacement, and the number of green balls y is recorded.
- Explain why y is not a binomial random variable.
 - Explain why, when we repeat the experiment with replacement, then y is a binomial random variable.
 - Give the values of n and p and display the probability distribution in tabular form.
 - What is the probability that at most 2 green balls are drawn?
 - What is the expected number of green balls drawn?
 - What is the variance of the number of balls drawn?
 - What is the probability that some green balls will be drawn?

6. On a multiple-choice test, there are 10 questions, each with 5 possible answers, one of which is correct. Nick is unaware of the content of the material so he guesses on all questions. Find:
- (a) the probability that Nick does not answer any question correctly
 - (b) the probability that Nick answers at most half of the questions correctly
 - (c) the probability that Nick answers at least one question correctly.
 - (d) How many questions should Nick expect to answer correctly?
7. Houses in a large city are equipped with alarm systems to protect them from burglary. A company claims their system to be 98% reliable. That is it will trigger an alarm in 98% of the cases. In a certain neighbourhood, 10 houses equipped with this system experience an attempted burglary. Find the probability that:
- (a) all the alarms work properly
 - (b) at least half of the houses trigger an alarm
 - (c) at most 8 alarms will work properly.
8. Harry Potter books are purchased by readers of all ages. 40% of Harry Potter books were purchased by readers 30 years of age or older. 15 readers are chosen at random. Find the probability that:
- (a) at least 10 of them are 30 years or older
 - (b) 10 of them are 30 or older
 - (c) at most 10 of them are younger than 30.
9. A factory makes computer hard disks. Over a long period, 1.5% of them are found to be defective. A random sample of 50 hard disks is tested.
- (a) Write down the expected number of defective hard disks in the sample.
 - (b) Find the probability that three hard disks are defective.
 - (c) Find the probability that more than one hard disk is defective.
10. Car colour preferences change over time and according to the area the customer lives in and the car model he/she is interested in. In a certain city a large dealer of one brand of cars noticed that 10% of the cars sold are metallic grey. Twenty of his customers are selected at random, and their car orders are checked for colour.
- (a) Find the probability that:
 - (i) at least 5 cars are metallic grey
 - (ii) at most 6 cars are metallic grey
 - (iii) more than 5 are metallic grey
 - (iv) between 4 and 6 are metallic grey
 - (v) more than 15 are not metallic grey.

- (b) In a sample of 100 customer records, find:
- (i) the expected number of metallic grey car orders
 - (ii) the standard deviation of metallic grey car orders.

According to the empirical rule, 95% of the orders of metallic grey orders are between a and b .

- (c) Find a and b .

11. Owners of dogs in many countries buy health insurance for their dogs. 3% of all dogs have health insurance. In a random sample of 100 dogs in a large city find:
- (a) the expected number of dogs with health insurance
 - (b) the probability that five of the dogs have health insurance.
 - (c) the probability that more than ten dogs have health insurance.
12. A balanced coin is flipped five times. Let x be the number of heads observed.
- (a) Using a table, construct the probability distribution of x .
 - (b) What is the probability that no heads are observed?
 - (c) What is the probability that all flips are heads?
 - (d) What is the probability that at least one head is observed?
 - (e) What is the probability that at least one tail is observed?
 - (f) Another coin is unbalanced so that it shows 2 heads in every 10 flips. Repeat questions (a)–(e) for this coin.
13. When John throws a stone at a target, the probability that he hits the target is 0.4.
- He throws a stone 6 times.
- (a) Find the probability that he hits the target exactly 4 times.
 - (b) Find the probability that he hits the target for the first time on his third throw.
14. On a television channel the news is shown at the same time each day. The probability that Alice watches the news on a given day is 0.4. Calculate the probability that on five consecutive days, she watches the news on at most three days.
15. A satellite relies on solar cells for its power and will operate provided that at least one of the cells is working. Cells fail independently of each other, and the probability that an individual cell fails within one year is 0.8
- (a) For a satellite with ten solar cells, find the probability that all ten cells fail within one year.
 - (b) For a satellite with ten solar cells, find the probability that the satellite is still operating at the end of one year.
 - (c) For a satellite with n solar cells, write down the probability that the satellite is still operating at the end of one year. Hence, find the smallest number of solar cells required so that the probability of the satellite still operating at the end of one year is at least 0.95

- 18.** A machine produces bearings with diameters that are normally distributed with mean 3.0005 cm and standard deviation 0.0010 cm. Specifications require the bearing diameters to lie in the interval 3.000 ± 0.0020 cm. Those outside the interval are considered scrap and must be disposed of. What fraction of the production will be scrap?
- 19.** A soft-drink machine can be regulated so that it discharges an average 216 cc per bottle. The amount of fill is normally distributed with standard deviation 9 cc. Give the maximum size of the bottles so that they will overflow only 1% of the time.
- 20.** A soft-drink machine can be regulated so that it discharges an average 216 cc per bottle. The amount of fill is normally distributed with standard deviation 9 cc. The amount of drink discharged 95% of the time lies between a and b above and below the mean. Find a and b .
- 21.** The speeds of cars on a main highway are approximately normal with mean 111.89 km h^{-1} and standard deviation 17.9 km h^{-1} . The speed limit on this highway is set to be 140 km h^{-1} . Cars travelling slower than 90 km h^{-1} are considered a hazard because they are too slow for the rest of cars.
- (a) What percent of cars travel within the acceptable limits?
- (b) Find the proportion of cars that travel at speeds exceeding 110 km h^{-1} .

Chapter 12 practice questions

- 1.** Residents of a small town have savings which are normally distributed with a mean of \$3000 and a standard deviation of \$500.
- (a) What percentage of townspeople have savings greater than \$3200?
- (b) Two townspeople are chosen at random. What is the probability that both of them have savings between \$2300 and \$3300?
- (c) The percentage of townspeople with savings less than d dollars is 74.22%. Find the value of d .
- 2.** The mass, W , of bags of rice follows a normal distribution with mean 1000 g and standard deviation 4 g.
- (a) Find the probability that a bag of rice chosen at random has a mass between 990 g and 1004 g.
- 95% of the bags of rice have a mass less than k grams.
- (b) Find the value of k .
- For a bag of rice chosen at random, $P(1000 - a < W < 1000 + a) = 0.9$
- (c) Find the value of a .
- 3.** A fair coin is flipped eight times. Calculate the probability of obtaining:
- (a) exactly 4 heads (b) exactly 3 heads (c) 3, 4 or 5 heads

4. The lifespan of a particular species of insect is normally distributed with a mean of 57 hours and a standard deviation of 4.4 hours.
 - (a) Find the probability that the lifespan of an insect of this species is:
 - (i) more than 55 hours
 - (ii) between 55 and 60 hours.
 - (b) 90% of the insects die after t hours. Find the value of t .
5. Intelligence quotient (IQ) in a certain population is normally distributed with a mean of 100 and a standard deviation of 15.
 - (a) What percentage of the population has an IQ between 90 and 125?
 - (b) If two people are chosen at random from the population, what is the probability that both have an IQ greater than 125?
6. The mass of packets of a breakfast cereal is normally distributed with a mean of 750 g and standard deviation of 25 g.
 - (a) Find the probability that a packet chosen at random has mass:
 - (i) less than 740 g
 - (ii) at least 780 g
 - (iii) between 740 g and 780 g
 - (b) Two packets are chosen at random. What is the probability that both packets have a mass which is less than 740 g?
 - (c) The mass of 70% of the packets is more than x grams. Find the value of x .
7. In a village in Wales, the height of adults is normally distributed with a mean of 187.5 cm and a standard deviation of 9.5 cm.
 - (a) What percentage of adults in the village have a height greater than 197 cm?
 - (b) A standard doorway in the village is designed so that 99% of adults have a space of at least 17 cm over their heads when going through a doorway. Find the height of a standard doorway in the village. Give your answer to the nearest cm.
8. It is claimed that the masses of a population of lions are normally distributed with a mean mass of 310 kg and a standard deviation of 30 kg.
 - (a) Calculate the probability that a lion selected at random will have a mass of 350 kg or more.
 - (b) The probability that the mass of a lion lies between a and b is 0.95, where a and b are symmetric about the mean. Find the value of a and of b .
9. Reaction times of human beings are normally distributed with a mean of 0.76 seconds and a standard deviation of 0.06 seconds.
 - (a) Calculate the probability that the reaction time of a person chosen at random is:
 - (i) greater than 0.70 seconds
 - (ii) between 0.70 and 0.79 seconds.

Three percent (3%) of the population have a reaction time less than c seconds.

(b) (i) Represent this information on a diagram, indicating the area representing 3%.

(ii) Find c

10. A factory makes calculators. Over a long period, 2% of them are found to be faulty. A random sample of 100 calculators is tested.
- (a) Write down the expected number of faulty calculators in the sample.
- (b) Find the probability that three calculators are faulty.
- (c) Find the probability that more than one calculator is faulty.
11. Ball bearings are used in engines in large quantities. A car manufacturer buys these bearings from a factory. They agree on the following terms. The car company chooses a sample of 50 ball bearings from the shipment. If they find more than 2 defective bearings, the shipment is rejected. It is a fact that the factory produces 4% defective bearings.
- (a) What is the probability that the sample is clear of defects?
- (b) What is the probability that the shipment is accepted?
- (c) What is the expected number of defective bearings in the sample of 50?

12. The table shows the probability distribution of a random variable X .

X	0	1	2	3
$P(x)$	$2k$	$2k^2$	$k^2 + k$	$2k^2 + k$

(a) Calculate the value of k .

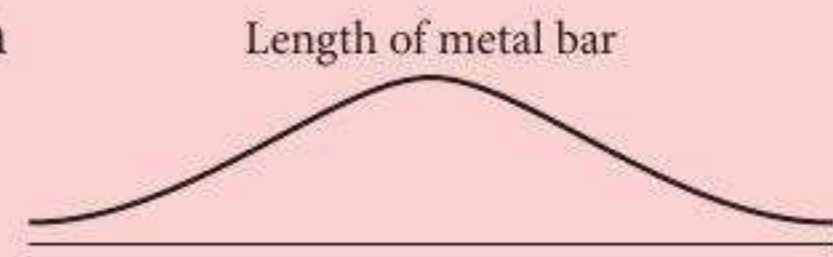
(b) Find $E(X)$.

13. It is estimated that 2.3% of the cherry tomatoes produced on a certain farm are considered to be small and cannot be sold for commercial purposes. The farmers have to separate such fruits and use them for domestic consumption instead.
- (a) 12 tomatoes are randomly selected from the produce. Calculate the probability that:
- (i) three are not fit to be sold (ii) at least 4 are not fit to be sold.
- (b) It is known that the sizes of such tomatoes are normally distributed with a mean of 3 cm and a standard deviation of 0.5 cm. Tomatoes that are categorised as large have to be larger than 2.5 cm. What proportion of the produce is large?

14. A factory makes metal bars. Their lengths are assumed to be normally distributed with a mean of 180 cm and a standard deviation of 5 cm.
- (a) On the provided diagram, shade the region representing the probability that a metal bar, chosen at random, will have a length less than 175 cm.

A metal bar is chosen at random.

- (b) (i) The probability that the length of the metal bar is less than 175 cm is equal to the probability that the length is greater than h cm. Write down the value of h .

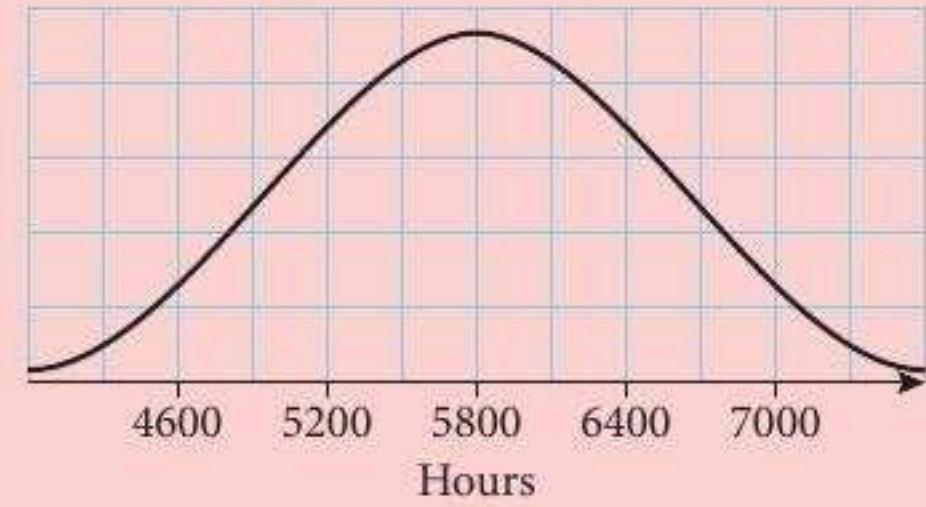


- (ii) Find the probability that the length of the metal bar is greater than one standard deviation above the mean.

15. The lifetime, L , of light bulbs made by a company follows a normal distribution.

L is measured in hours.

The normal distribution curve of L is shown in the diagram.



- (a) Write down the mean lifetime of the light bulbs.

The standard deviation of the lifetime of the light bulbs is 850 hours.

- (b) Find the probability that $5000 \leq L \leq 6000$, for a randomly chosen light bulb.

The company states that 90% of the light bulbs have a lifetime of at least k hours.

- (c) Find the value of k . Give your answer correct to the nearest hundred.

16. A speed camera on Peterson Road records the speed of each passing vehicle. The speeds are found to be normally distributed with a mean of 67 km h^{-1} and a standard deviation of 3.4 km h^{-1} .

- (a) Sketch a diagram of this normal distribution and shade the region representing the probability that the speed of a vehicle is between 60 and 70 km h^{-1} .

A vehicle on Peterson Road is chosen at random.

- (b) Find the probability that the speed of this vehicle is:

- (i) more than 60 km h^{-1}
 (ii) less than 70 km h^{-1}
 (iii) between 60 and 70 km h^{-1} .

It is found that 19% of the vehicles are exceeding the speed limit of $s \text{ km h}^{-1}$.

- (c) Find the value of s , correct to the nearest integer.

There is a fine of US\$65 for exceeding the speed limit on Peterson Road. On a particular day the total value of fines issued was US\$14,820.

- (d) (i) Calculate the number of fines that were issued on this day.
 (ii) Estimate the total number of vehicles that passed the speed camera on Peterson Road on this day.