Differential Calculus Intro *[93 marks]*

1. Find the equation of the tangent to the curve $y = \mathrm{e}^{2x} - 3x$ at the point where $x = 0$. *[5 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$(x = 0 \Rightarrow) y = 1$ ***(A1)***

appreciate the need to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ ***(M1)***

$\left(\frac{\mathrm{d}y}{\mathrm{d}x} =\right) 2\mathrm{e}^{2x} - 3$ ***A1***

$(x = 0 \Rightarrow) \frac{\mathrm{d}y}{\mathrm{d}x} = -1$ ***A1***

$\frac{y-1}{x-0} = -1 \quad (y = 1 - x)$ ***A1***

[5 marks]

2. Consider the curve with equation $y = \left(2x - 1\right)\mathrm{e}^{kx}$, where $x \in \mathbb{R}$ and $k \in \mathbb{Q}$. *[5 marks]*

The tangent to the curve at the point where $x = 1$ is parallel to the line $y = 5\mathrm{e}^{k}x$.

Find the value of k.

Markscheme

evidence of using product rule ***(M1)***

$\frac{\mathrm{d}y}{\mathrm{d}x} = (2x-1) \times \left(k\mathrm{e}^{kx}\right) + 2 \times \mathrm{e}^{kx} \ \left(= \mathrm{e}^{kx}(2kx - k + 2)\right)$ ***A1***

correct working for one of (seen anywhere) ***A1***

$\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 1 \Rightarrow k\mathrm{e}^{k} + 2\mathrm{e}^{k}$

OR

slope of tangent is $5\mathrm{e}^{k}$

their $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 1$ equals the *slope* of $y = 5\mathrm{e}^{k}x \ \left(= 5\mathrm{e}^{k}\right)$ (seen anywhere) ***(M1)***

$k\mathrm{e}^{k} + 2\mathrm{e}^{k} = 5\mathrm{e}^{k}$

$k = 3$ ***A1***

[5 marks]

The function f is defined for all $x \in \mathbb{R}$. The line with equation $y = 6x - 1$ is the tangent to the graph of f at $x = 4$.

3a. Write down the value of $f'(4)$. *[1 mark]*

Markscheme

$f'(4) = 6$ ***A1***

[1 mark]

3b. Find $f(4)$. *[1 mark]*

Markscheme

$f(4) = 6 \times 4 - 1 = 23$ ***A1***

[1 mark]

The function g is defined for all $x \in \mathbb{R}$ where $g(x) = x^2 - 3x$ and $h(x) = f(g(x))$.

3c. Find $h(4)$. *[2 marks]*

Markscheme

$h(4) = f(g(4))$ ***(M1)***

$h(4) = f\left(4^2 - 3 \times 4\right) = f(4)$

$h(4) = 23$ ***A1***

[2 marks]

3d. Hence find the equation of the tangent to the graph of h at $x = 4$. *[3 marks]*

Markscheme

attempt to use chain rule to find h' ***(M1)***

$f'(g(x)) \times g'(x)$ OR $\left(x^2 - 3x\right)' \times f'\left(x^2 - 3x\right)$

$h'(4) = (2 \times 4 - 3) f'\left(4^2 - 3 \times 4\right)$ ***A1***

$= 30$

$y - 23 = 30(x - 4)$ OR $y = 30x - 97$ ***A1***

[3 marks]

The curve C has equation $\mathrm{e}^{2y} = x^3 + y$.

4a. Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{2\mathrm{e}^{2y}-1}$. *[3 marks]*

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

attempts implicit differentiation on both sides of the equation **M1**

$2\mathrm{e}^{2y}\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + \frac{\mathrm{d}y}{\mathrm{d}x}$ **A1**

$\left(2\mathrm{e}^{2y} - 1\right)\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2$ **A1**

so $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{2\mathrm{e}^{2y}-1}$ **AG**

[3 marks]

4b. The tangent to C at the point P is parallel to the y-axis. *[4 marks]*

Find the x-coordinate of P.

Markscheme

attempts to solve $2\mathrm{e}^{2y} - 1 = 0$ for y **(M1)**

$y = -0.346\ldots\left(= \frac{1}{2}\ln\frac{1}{2}\right)$ **A1**

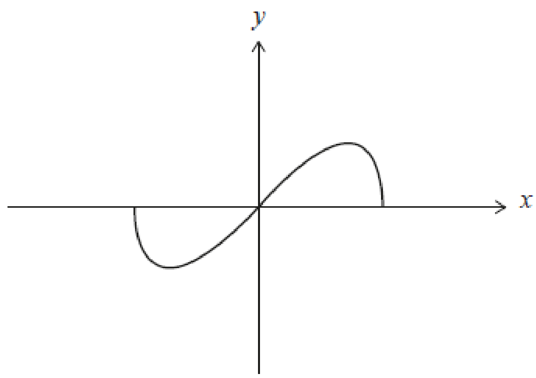
attempts to solve $\mathrm{e}^{2y} = x^3 + y$ for x given their value of y **(M1)**

$x = 0.946\left(= \left(\frac{1}{2}\left(1 - \ln\frac{1}{2}\right)\right)^{\frac{1}{3}}\right)$ **A1**

[4 marks]

A function f is defined by $f(x) = x\sqrt{1-x^2}$ where $-1 \le x \le 1$.

The graph of $y = f(x)$ is shown below.



5a. Show that f is an odd function. *[2 marks]*

Markscheme

attempts to replace x with $-x$ ***M1***

$f(-x) = -x\sqrt{1-(-x)^2}$

$= -x\sqrt{1-(-x)^2}(= -f(x))$ ***A1***

Note: Award ***M1A1*** for an attempt to calculate both $f(-x)$ and $-f(-x)$ independently, showing that they are equal.
Note: Award ***M1A0*** for a graphical approach including evidence that **either** the graph is invariant after rotation by 180° about the origin **or** the graph is invariant after a reflection in the y-axis and then in the x-axis (or vice versa).

so f is an odd function ***AG***

[2 marks]

5b. The range of f is $a \le y \le b$, where $a,\ b \in \mathbb{R}$. *[6 marks]*

Find the value of a and the value of b.

Markscheme

attempts both product rule and chain rule differentiation to find $f'(x)$ ***M1***

$f'(x) = x \times \frac{1}{2} \times (-2x) \times \left(1-x^2\right)^{-\frac{1}{2}} + \left(1-x^2\right)^{\frac{1}{2}} \times 1 \left(= \sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}}\right)$ ***A1***

$= \frac{1-2x^2}{\sqrt{1-x^2}}$

sets their $f'(x) = 0$ ***M1***

$\Rightarrow x = \pm\frac{1}{\sqrt{2}}$ ***A1***

attempts to find at least one of $f\left(\pm\frac{1}{\sqrt{2}}\right)$ ***(M1)***

Note: Award **M1** for an attempt to evaluate $f(x)$ at least at one of their $f'(x) = 0$ roots.

$a = -\frac{1}{2}$ and $b = \frac{1}{2}$ ***A1***

Note: Award ***A1*** for $-\frac{1}{2} \leq y \leq \frac{1}{2}$.

[6 marks]

Consider the curve C given by $y = x - xy\ln(xy)$ where $x > 0,\ y > 0$.

6a. Show that $\frac{\mathrm{d}y}{\mathrm{d}x} + \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)(1 + \ln(xy)) = 1$. *[3 marks]*

Markscheme

METHOD 1

attempts to differentiate implicitly including at least one application of the product rule ***(M1)***

$u = xy,\ v = \ln(xy),\ \frac{\mathrm{d}u}{\mathrm{d}x} = x\frac{\mathrm{d}y}{\mathrm{d}x} + y,\ \frac{\mathrm{d}v}{\mathrm{d}x} = \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)\frac{1}{xy}$

$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - \left[\frac{xy}{xy}\left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right) + \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)\ln(xy)\right]$ ***A1***

Note: Award ***(M1)A1*** for implicitly differentiating $y = x(1 - y\ln(xy))$ and obtaining $\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - \left[\frac{xy}{xy}\left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right) + x\frac{\mathrm{d}y}{\mathrm{d}x}\ln(xy) + y\ln(xy)\right]$.

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - \left[\left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right) + \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)\ln(xy)\right]$$

$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)(1 + \ln(xy))$ ***A1***

$\frac{\mathrm{d}y}{\mathrm{d}x} + \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)(1 + \ln(xy)) = 1$ ***AG***

METHOD 2

$$y = x - xy\ln x - xy\ln y$$

attempts to differentiate implicitly including at least one application of the product rule ***(M1)***

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - \left(\frac{xy}{x} + \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)\ln x\right) - \left(\frac{xy}{y}\frac{\mathrm{d}y}{\mathrm{d}x} + \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)\ln y\right)$$

A1

or equivalent to the above, for example

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - \left(x\ln x\frac{\mathrm{d}y}{\mathrm{d}x} + (1 + \ln x)y\right) - \left(y\ln y + x\left(\ln y\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{\mathrm{d}y}{\mathrm{d}x}\right)\right)$$

$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - x\frac{\mathrm{d}y}{\mathrm{d}x}(\ln x + \ln y + 1) - y(\ln x + \ln y + 1)$ ***A1***

or equivalent to the above, for example

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - x\frac{\mathrm{d}y}{\mathrm{d}x}(\ln(xy) + 1) - y(\ln(xy) + 1)$$

$\frac{\mathrm{d}y}{\mathrm{d}x} + \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)(1 + \ln(xy)) = 1$ ***AG***

METHOD 3

attempt to differentiate implicitly including at least one application of the product rule ***M1***

$$u = x\ln(xy),\ v = y,\ \frac{\mathrm{d}u}{\mathrm{d}x} = \ln(xy) + \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)\frac{x}{xy},\ \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - \left(x\frac{\mathrm{d}y}{\mathrm{d}x}\ln(xy) + y\ln(xy) + \frac{xy}{xy}\left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)\right)$ ***A1***

$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - x\frac{\mathrm{d}y}{\mathrm{d}x}(\ln(xy) + 1) - y(\ln(xy) + 1)$ ***A1***

$\frac{\mathrm{d}y}{\mathrm{d}x} + \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)(1 + \ln(xy)) = 1$ ***AG***

METHOD 4

lets $w = xy$ and attempts to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ where $y = x - w\ln w$ ***M1***

$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - \left(\frac{\mathrm{d}w}{\mathrm{d}x} + \frac{\mathrm{d}w}{\mathrm{d}x}\ln w\right) \left(= 1 - \frac{\mathrm{d}w}{\mathrm{d}x}(1 + \ln w)\right)$ ***A1***

$\frac{\mathrm{d}w}{\mathrm{d}x} = x\frac{\mathrm{d}y}{\mathrm{d}x} + y$ ***A1***

$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y + \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)\ln(xy)\right) \ \left(= 1 - \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)(1 + \ln(xy))\right)$

$\frac{\mathrm{d}y}{\mathrm{d}x} + \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)(1 + \ln(xy)) = 1$ ***AG***

[3 marks]

6b. Hence find the equation of the tangent to C at the point where $x = 1$. *[5 marks]*

Markscheme

METHOD 1

substitutes $x=1$ into $y=x-xy\ln(xy)$ ***(M1)***

$y=1-y\ln y \Rightarrow y=1$ ***A1***

substitutes $x=1$ and their non-zero value of y into
$\frac{\mathrm{d}y}{\mathrm{d}x}+\left(x\frac{\mathrm{d}y}{\mathrm{d}x}+y\right)(1+\ln(xy))=1$ ***(M1)***

$2\frac{\mathrm{d}y}{\mathrm{d}x}=0\ \left(\frac{\mathrm{d}y}{\mathrm{d}x}=0\right)$ ***A1***

equation of the tangent is $y=1$ ***A1***

METHOD 2

substitutes $x=1$ into $\frac{\mathrm{d}y}{\mathrm{d}x}+\left(x\frac{\mathrm{d}y}{\mathrm{d}x}+y\right)(1+\ln(xy))=1$ ***(M1)***

$\frac{\mathrm{d}y}{\mathrm{d}x}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}+y\right)(1+\ln(y))=1$

EITHER

correctly substitutes $\ln y=\frac{1-y}{y}$ into $\frac{\mathrm{d}y}{\mathrm{d}x}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}+y\right)(1+\ln(xy))=1$
A1

$\frac{\mathrm{d}y}{\mathrm{d}x}\left(1+\frac{1}{y}\right)=0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=0\ (y=1)$ ***A1***

OR

correctly substitutes $y+y\ln y=1$ into $\frac{\mathrm{d}y}{\mathrm{d}x}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}+y\right)(1+\ln(xy))=1$
A1

$\frac{\mathrm{d}y}{\mathrm{d}x}(2+\ln y)=0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=0\ (y=1)$ ***A1***

THEN

substitutes $x=1$ into $y=x-xy\ln(xy)$ ***(M1)***

$y=1-y\ln y \Rightarrow y=1$

equation of the tangent is $y=1$ ***A1***

[5 marks]

Consider the functions $f(x)= -(x-h)^2 + 2k$ and $g(x)= \mathrm{e}^{x-2} + k$ where $h,\ k \in \mathbb{R}$.

7a. Find $f'(x)$. *[1 mark]*

Markscheme

$f'(x)= -2(x-h)$ ***A1***

[1 mark]

The graphs of f and g have a common tangent at $x = 3$.

7b. Show that $h = \frac{\mathrm{e}+6}{2}$. *[3 marks]*

Markscheme

$g'(x)= \mathrm{e}^{x-2}$ OR $g'(3)= \mathrm{e}^{3-2}$ (may be seen anywhere) ***A1***

Note: The derivative of g must be explicitly seen, either in terms of x or 3.

recognizing $f'(3)= g'(3)$ ***(M1)***

$-2(3-h)= \mathrm{e}^{3-2}(= e)$

$-6 + 2h = \mathrm{e}$ OR $3 - h = -\frac{\mathrm{e}}{2}$ ***A1***

Note: The final ***A1*** is dependent on one of the previous marks being awarded.

$h = \frac{\mathrm{e}+6}{2}$ ***AG***

[3 marks]

7c. Hence, show that $k = \mathrm{e} + \frac{\mathrm{e}^2}{4}$. *[3 marks]*

Markscheme

$f(3) = g(3)$ ***(M1)***

$-(3-h)^2 + 2k = e^{3-2} + k$

correct equation in k

EITHER

$-\left(3 - \frac{e+6}{2}\right)^2 + 2k = e^{3-2} + k$ ***A1***

$k = e + \left(\frac{6-e-6}{2}\right)^2 \left(= e + \left(\frac{-e}{2}\right)^2\right)$ ***A1***

OR

$k = e + \left(3 - \frac{e+6}{2}\right)^2$ ***A1***

$k = e + 9 - 3e - 18 + \frac{e^2+12e+36}{4}$ ***A1***

THEN

$k = e + \frac{e^2}{4}$ ***AG***

[3 marks]

Let $y = \frac{\ln x}{x^4}$ for $x > 0$.

8a. Show that $\frac{dy}{dx} = \frac{1-4\ln x}{x^5}$. *[3 marks]*

Markscheme

attempt to use quotient or product rule ***(M1)***

$\frac{dy}{dx} = \frac{x^4\left(\frac{1}{x}\right) - (\ln x)(4x^3)}{(x^4)^2}$ OR $(\ln x)(-4x^{-5}) + (x^{-4})\left(\frac{1}{x}\right)$ ***A1***

correct working ***A1***

$= \frac{x^3(1-4\ln x)}{x^8}$ OR cancelling x^3 OR $\frac{-4\ln x}{x^5} + \frac{1}{x^5}$

$= \frac{1-4\ln x}{x^5}$ ***AG***

[3 marks]

Consider the function defined by $f(x)\frac{\ln x}{x^4}$ for $x > 0$ and its graph $y = f(x)$.

8b. The graph of f has a horizontal tangent at point P. Find the coordinates of P. *[5 marks]*

Markscheme

$f'(x) = \frac{\mathrm{d}y}{\mathrm{d}x} = 0$ ***(M1)***

$\frac{1-4\ln x}{x^5} = 0$

$\ln x = \frac{1}{4}$ ***(A1)***

$x = \mathrm{e}^{\frac{1}{4}}$ ***A1***

substitution of their x to find y ***(M1)***

$y = \frac{\ln \mathrm{e}^{\frac{1}{4}}}{\left(\mathrm{e}^{\frac{1}{4}}\right)^4}$

$= \frac{1}{4\mathrm{e}} \left(= \frac{1}{4}\mathrm{e}^{-1}\right)$ ***A1***

$\mathrm{P}\left(\mathrm{e}^{\frac{1}{4}}, \frac{1}{4\mathrm{e}}\right)$

[5 marks]

8c. Given that $f''(x) = \frac{20\ln x - 9}{x^6}$, show that P is a local maximum point. *[3 marks]*

Markscheme

$f\,''\left(\mathrm{e}^{\frac{1}{4}}\right)=\frac{20\ln\mathrm{e}^{\frac{1}{4}}-9}{\left(\mathrm{e}^{\frac{1}{4}}\right)^{6}}$ ***(M1)***

$=\frac{5-9}{\mathrm{e}^{1.5}}\left(=-\frac{4}{\mathrm{e}^{1.5}}\right)$ ***A1***

which is negative ***R1***

hence P is a local maximum ***AG***

Note: The ***R1*** is dependent on the previous ***A1*** being awarded.

[3 marks]

8d. Solve $f(x)>0$ for $x>0$. *[2 marks]*

Markscheme

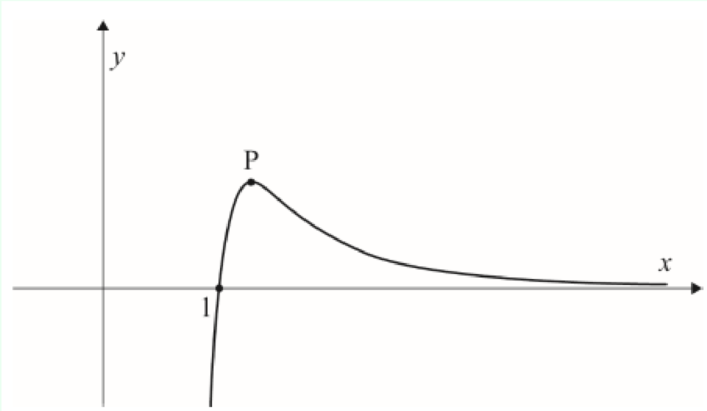
$\ln x>0$ ***(A1)***

$x>1$ ***A1***

[2 marks]

8e. Sketch the graph of f, showing clearly the value of the x-intercept and the approximate position of point P. *[3 marks]*

Markscheme



A1A1A1

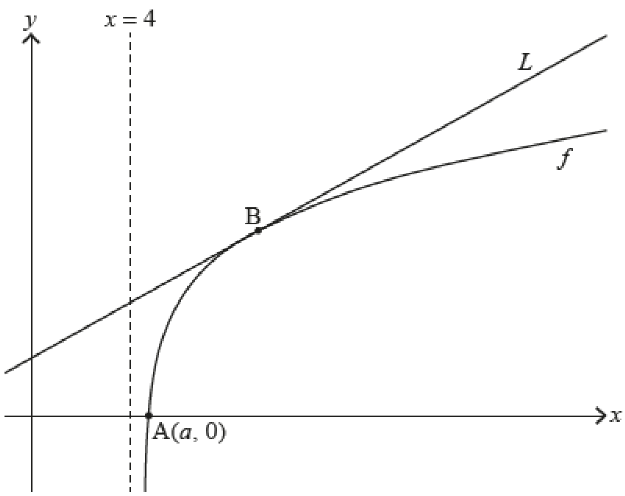
Note: Award ***A1*** for one x-intercept only, located at 1

A1 for local maximum, P, in approximately correct position
A1 for curve approaching x-axis as $x \to \infty$ (including change in concavity).

[3 marks]

Consider the function f defined by $f(x) = \ln(x^2 - 16)$ for $x > 4$.

The following diagram shows part of the graph of f which crosses the x-axis at point A, with coordinates $(a, 0)$. The line L is the tangent to the graph of f at the point B.



9a. Find the exact value of a. *[3 marks]*

Markscheme

$\ln\left(x^2-16\right)=0$ ***(M1)***

$e^0=x^2-16(=1)$

$x^2=17$ OR $x=\pm\sqrt{17}$ ***(A1)***

$a=\sqrt{17}$ ***A1***

[3 marks]

9b. Given that the gradient of L is $\frac{1}{3}$, find the x-coordinate of B. *[6 marks]*

Markscheme

attempt to differentiate (must include $2x$ and/or $\frac{1}{x^2-16}$) ***(M1)***

$f'(x)=\frac{2x}{x^2-16}$ ***A1***

setting their derivative $=\frac{1}{3}$ ***M1***

$\frac{2x}{x^2-16}=\frac{1}{3}$

$x^2-16=6x$ OR $x^2-6x-16=0$ (or equivalent) ***A1***

valid attempt to solve their quadratic ***(M1)***

$x=8$ ***A1***

Note: Award ***A0*** if the candidate's final answer includes additional solutions (such as $x=-2, 8$).

[6 marks]

Consider the graph of the function $f(x)=x^2-\frac{k}{x}$.

10a. Write down $f'(x)$. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure. It appeared in a paper that permitted the use of a calculator, and so might not be suitable for all forms of practice.

$2x + \frac{k}{x^2}$ ***(A1)(A1)(A1)*** ***(C3)***

Note: Award ***(A1)*** for $2x$, ***(A1)*** for $+k$, and ***(A1)*** for x^{-2} or $\frac{1}{x^2}$.
Award at most ***(A1)(A1)(A0)*** if additional terms are seen.

[3 marks]

The equation of the tangent to the graph of $y = f(x)$ at $x = -2$ is $2y = 4 - 5x$.

10b. Write down the gradient of this tangent. *[1 mark]*

Markscheme

-2.5 $\left(\frac{-5}{2}\right)$ ***(A1)*** ***(C1)***

[1 mark]

10c. Find the value of k. *[2 marks]*

Markscheme

$-2.5 = 2\times(-2)+\frac{k}{(-2)^2}$ ***(M1)***

Note: Award ***(M1)*** for equating their gradient from part (b) to their substituted derivative from part (a).

$(k=)\ 6$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from parts (a) and (b).

[2 marks]

Consider the curve C defined by $y^2 = \sin(xy), y \neq 0$.

11a. Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y\cos(xy)}{2y - x\cos(xy)}$. *[5 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

attempt at implicit differentiation ***M1***

$2y\frac{\mathrm{d}y}{\mathrm{d}x} = \cos(xy)\left[x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right]$ ***A1M1A1***

Note: Award ***A1*** for LHS, ***M1*** for attempt at chain rule, ***A1*** for RHS.

$2y\frac{\mathrm{d}y}{\mathrm{d}x} = x\frac{\mathrm{d}y}{\mathrm{d}x}\cos(xy) + y\cos(xy)$

$2y\frac{\mathrm{d}y}{\mathrm{d}x} - x\frac{\mathrm{d}y}{\mathrm{d}x}\cos(xy) = y\cos(xy)$

$\frac{\mathrm{d}y}{\mathrm{d}x}(2y - x\cos(xy)) = y\cos(xy)$ ***M1***

Note: Award ***M1*** for collecting derivatives and factorising.

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y\cos(xy)}{2y - x\cos(xy)}$ ***AG***

[5 marks]

11b. Prove that, when $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, $y = \pm 1$. *[5 marks]*

Markscheme

setting $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

$y \cos(xy) = 0$ ***(M1)***

$(y \neq 0) \Rightarrow \cos(xy) = 0$ ***A1***

$\Rightarrow \sin(xy)\left(= \pm\sqrt{1-\cos^2(xy)} = \pm\sqrt{1-0}\right) = \pm 1$ ***OR***

$xy = (2n+1)\frac{\pi}{2}\ (n \in \mathbb{Z})$ ***OR*** $xy = \frac{\pi}{2},\ \frac{3\pi}{2}, \ldots$ ***A1***

Note: If they offer values for xy, award ***A1*** for at least two correct values in two different ‘quadrants’ and no incorrect values.

$y^2 (= \sin(xy)) > 0$ ***R1***

$\Rightarrow y^2 = 1$ ***A1***

$\Rightarrow y = \pm 1$ ***AG***

[5 marks]

11c. Hence find the coordinates of all points on C, for $0 < x < 4\pi$, where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. *[5 marks]*

Markscheme

$y = \pm 1 \Rightarrow 1 = \sin(\pm x) \Rightarrow \sin x = \pm 1$ OR
$y = \pm 1 \Rightarrow 0 = \cos(\pm x) \Rightarrow \cos x = 0$ ***(M1)***

$(\sin x = 1 \Rightarrow)\left(\frac{\pi}{2},\ 1\right), \left(\frac{5\pi}{2},\ 1\right)$ ***A1A1***

$(\sin x = -1 \Rightarrow)\left(\frac{3\pi}{2},\ -1\right), \left(\frac{7\pi}{2},\ -1\right)$ ***A1A1***

Note: Allow ‘coordinates’ expressed as $x = \frac{\pi}{2},\ y = 1$ for example.

Note: Each of the ***A*** marks may be awarded independently and are not dependent on ***(M1)*** being awarded.

Note: Mark only the candidate’s first two attempts for each case of $\sin x$.

[5 marks]