Differential Calculus revision

[82 marks]

Let $f\left(x\right) = x - 8$, $g\left(x\right) = x^4 - 3$ and $h\left(x\right) = f\left(g\left(x\right)\right)$.

1a. Find $h\left(x\right)$. *[2 marks]*

1b. Let C be a point on the graph of h. The tangent to the graph of h at C is parallel to the graph of f. *[5 marks]*

Find the x-coordinate of C.

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Let θ be an **obtuse** angle such that $\sin\theta = \frac{3}{5}$.

2a. Find the value of $\tan\theta$. *[4 marks]*

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2b. Line L passes through the origin and has a gradient of $\tan\theta$. Find the equation of L. *[2 marks]*

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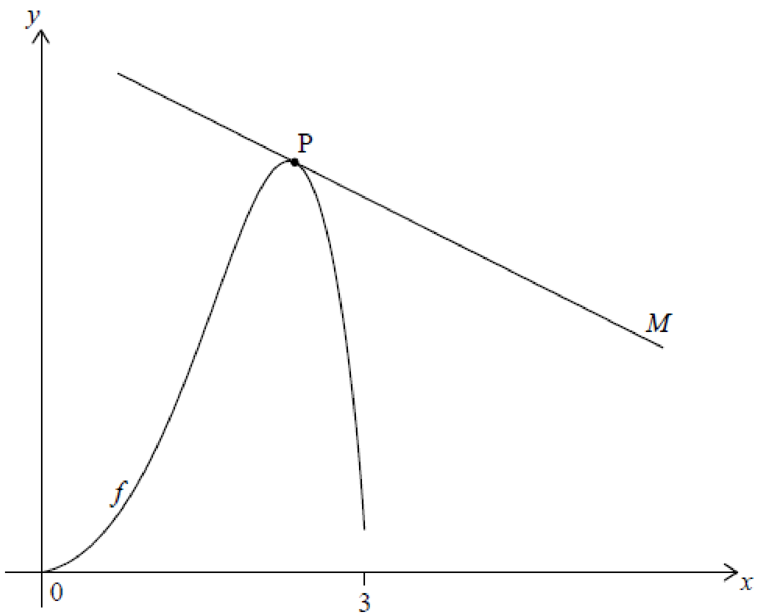
Let $f(x) = e^x \sin x - \frac{3x}{4}$.

2c. The following diagram shows the graph of f for $0 \le x \le 3$. Line M is a tangent to the graph of f at point P. *[4 marks]*



Given that M is parallel to L, find the x-coordinate of P.

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3. Find the coordinates of the points on the curve $y^3 + 3xy^2 - x^3 = 27$ at which $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. *[9 marks]*

The curve C is given by the equation $y = x\tan\left(\frac{\pi xy}{4}\right)$.

4a. At the point (1, 1) , show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2+\pi}{2-\pi}$. *[5 marks]*

4b. Hence find the equation of the normal to C at the point (1, 1). *[2 marks]*

Consider the function $f(x) = x^2 e^{3x}$, $x \in \mathbb{R}$.

5. The graph of f has a horizontal tangent line at $x = 0$ and at $x = a$. Find a. *[2 marks]*

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Consider the function $f(x) = \frac{1}{3}x^3 + \frac{3}{4}x^2 - x - 1$.

6a. Write down the y-intercept of the graph of $y = f(x)$. *[1 mark]*

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6b. Sketch the graph of $y = f(x)$ for $-3 \leq x \leq 3$ and $-4 \leq y \leq 12$. *[4 marks]*

The function has one local maximum at $x = p$ and one local minimum at $x = q$.

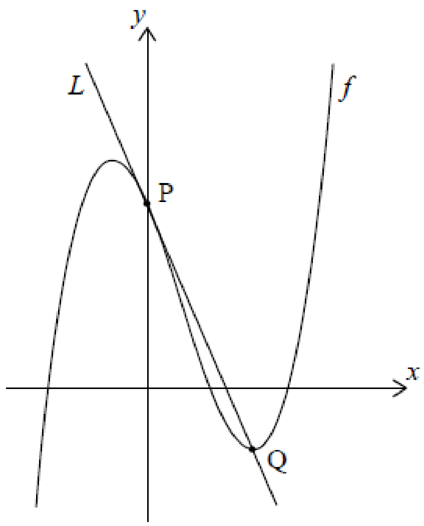
6c. Determine the range of $f(x)$ for $p \leq x \leq q$. *[3 marks]*

7. Let l be the tangent to the curve $y = xe^{2x}$ at the point $(1, e^2)$. *[4 marks]*

Find the coordinates of the point where l meets the x-axis.

8. Using L'Hôpital's rule, find $\lim_{x \to 0} \left(\frac{\tan 3x - 3\tan x}{\sin 3x - 3\sin x} \right)$. *[9 marks]*

Let $f(x) = x^3 - 2x^2 + ax + 6$. Part of the graph of f is shown in the following diagram.



The graph of f crosses the y-axis at the point P. The line *L* is tangent to the graph of f at P.

9a. Find $f'(x)$. *[2 marks]*

9b. Hence, find the equation of *L* in terms of a. *[4 marks]*

9c. The graph of f has a local minimum at the point Q. The line *L* passes through Q. *[8 marks]*

Find the value of a.

10. Differentiate from first principles the function $f\left(x\right) = 3x^3 - x$. *[5 marks]*

11. Consider $f(x)$, $g(x)$ and $h(x)$, for $x \in \mathbb{R}$ where $h(x) = (f \circ g)(x)$. *[7 marks]*

Given that $g(3) = 7$, $g'(3) = 4$ and $f'(7) = -5$, find the gradient of the normal to the curve of h at $x = 3$.