

Differential Calculus revision

[82 marks]

Let $f(x) = x - 8$, $g(x) = x^4 - 3$ and $h(x) = f(g(x))$.

1a. Find $h(x)$.

[2 marks]

1b. Let C be a point on the graph of h . The tangent to the graph of h at C is parallel to the graph of f . [5 marks]

Find the x -coordinate of C .

Let θ be an **obtuse** angle such that $\sin \theta = \frac{3}{5}$.

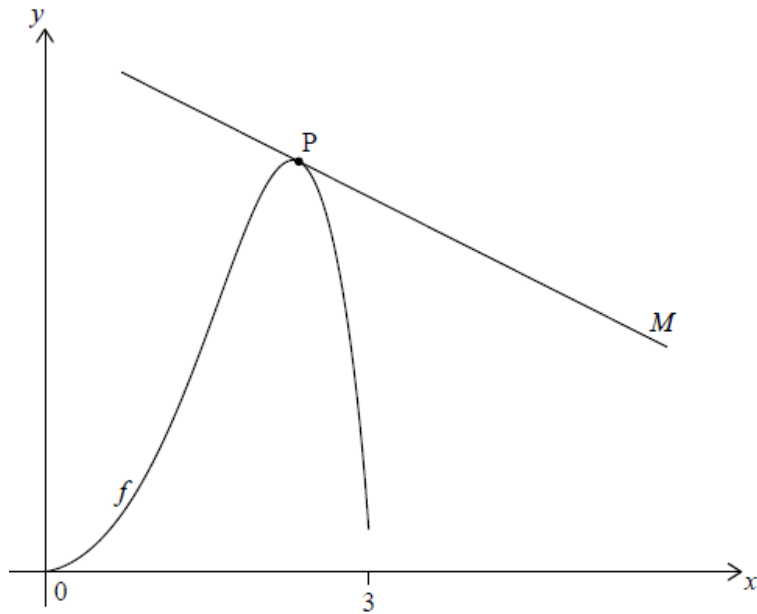
2a. Find the value of $\tan \theta$.

[4 marks]

2b. Line L passes through the origin and has a gradient of $\tan \theta$. Find the equation of L . [2 marks]

Let $f(x) = e^x \sin x - \frac{3x}{4}$.

- 2c. The following diagram shows the graph of f for $0 \leq x \leq 3$. Line M is a tangent to the graph of f at point P. [4 marks]



Given that M is parallel to L , find the x -coordinate of P.

3. Find the coordinates of the points on the curve $y^3 + 3xy^2 - x^3 = 27$ at which $\frac{dy}{dx} = 0$. [9 marks]

The curve C is given by the equation $y = x \tan\left(\frac{\pi xy}{4}\right)$.

- 4a. At the point $(1, 1)$, show that $\frac{dy}{dx} = \frac{2+\pi}{2-\pi}$. [5 marks]

- 4b. Hence find the equation of the normal to C at the point $(1, 1)$. [2 marks]

Consider the function $f(x) = x^2 e^{3x}$, $x \in \mathbb{R}$.

5. The graph of f has a horizontal tangent line at $x = 0$ and at $x = a$. Find a . [2 marks]

Consider the function $f(x) = \frac{1}{3}x^3 + \frac{3}{4}x^2 - x - 1$.

6a. Write down the y -intercept of the graph of $y = f(x)$. [1 mark]

6b. Sketch the graph of $y = f(x)$ for $-3 \leq x \leq 3$ and $-4 \leq y \leq 12$. [4 marks]

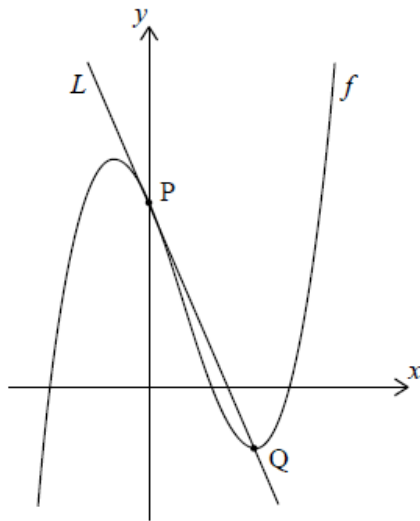
The function has one local maximum at $x = p$ and one local minimum at $x = q$.

6c. Determine the range of $f(x)$ for $p \leq x \leq q$. [3 marks]

7. Let l be the tangent to the curve $y = xe^{2x}$ at the point $(1, e^2)$. [4 marks]
Find the coordinates of the point where l meets the x -axis.

8. Using L'Hôpital's rule, find $\lim_{x \rightarrow 0} \left(\frac{\tan 3x - 3 \tan x}{\sin 3x - 3 \sin x} \right)$. [9 marks]

Let $f(x) = x^3 - 2x^2 + ax + 6$. Part of the graph of f is shown in the following diagram.



The graph of f crosses the y -axis at the point P . The line L is tangent to the graph of f at P .

9a. Find $f'(x)$. [2 marks]

9b. Hence, find the equation of L in terms of a . [4 marks]

9c. The graph of f has a local minimum at the point Q. The line L passes through Q. [8 marks]

Find the value of a .

10. Differentiate from first principles the function $f(x) = 3x^3 - x$. [5 marks]

11. Consider $f(x)$, $g(x)$ and $h(x)$, for $x \in \mathbb{R}$ where $h(x) = (f \circ g)(x)$. [7 marks]

Given that $g(3) = 7$, $g'(3) = 4$ and $f'(7) = -5$, find the gradient of the normal to the curve of h at $x = 3$.