Kinematics *[147 marks]*

A particle moves in a straight line such that its velocity, $v\ \mathrm{ms}^{-1}$, at time t seconds is given by

$v = 4t^2 - 6t + 9 - 2\sin(4t), 0 \le t \le 1.$

The particle's acceleration is zero at $t = T$.

1a. Find the value of T. *[2 marks]*

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1b. Let s_1 be the distance travelled by the particle from $t = 0$ to $t = T$ and let s_2 be the distance travelled by the particle from $t = T$ to $t = 1$. *[3 marks]*

Show that $s_2 > s_1$.

A particle moves along a straight line so that its velocity, $v \text{ m s}^{-1}$, after t seconds is given by $v(t) = e^{\sin t} + 4 \sin t$ for $0 \le t \le 6$.

2a. Find the value of t when the particle is at rest. *[2 marks]*

2b. Find the acceleration of the particle when it changes direction. *[3 marks]*

2c. Find the total distance travelled by the particle. *[2 marks]*

A particle moves in a straight line such that its velocity, $v\ \mathrm{m\,s^{-1}}$, at time t seconds is given by $v = \frac{(t^2+1)\cos t}{4},\ 0 \le t \le 3$.

3a. Determine when the particle changes its direction of motion. *[2 marks]*

3b. Find the times when the particle's acceleration is $-1.9\ \mathrm{m\,s^{-2}}$. *[3 marks]*

3c. Find the particle’s acceleration when its speed is at its greatest. *[2 marks]*

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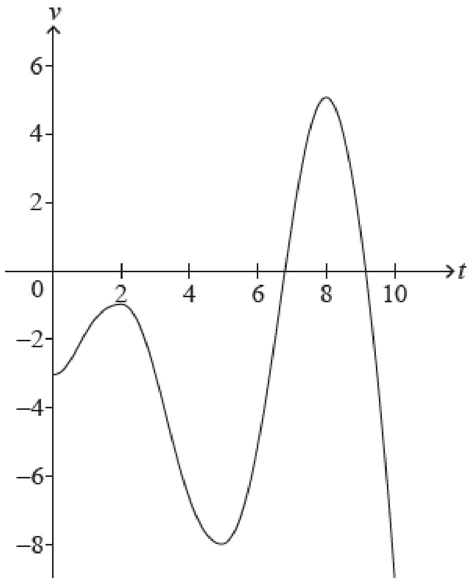
A particle moves in a straight line. The velocity, $v\ \mathrm{ms}^{-1}$, of the particle at time t seconds is given by $v(t) = t\sin t - 3$, for $0 \le t \le 10$.

The following diagram shows the graph of v.



4a. Find the smallest value of t for which the particle is at rest. *[2 marks]*

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4b. Find the total distance travelled by the particle. *[2 marks]*

4c. Find the acceleration of the particle when $t = 7$. *[2 marks]*

A particle P moves in a straight line such that after time t seconds, its velocity, v in $\mathrm{m\,s^{-1}}$, is given by $v = \mathrm{e}^{-3t} \sin 6\, t$, where $0 < t < \frac{\pi}{2}$.

5a. Find the times when P comes to instantaneous rest. *[2 marks]*

At time t, P has displacement $s(t)$; at time $t = 0$, $s(0) = 0$.

5b. Find an expression for s in terms of t. *[7 marks]*

5c. Find the maximum displacement of P, in metres, from its initial position. *[2 marks]*

5d. Find the total distance travelled by P in the first 1.5 seconds of its motion. *[2 marks]*

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At successive times when the acceleration of P is $0\ \mathrm{m\,s^{-2}}$, the velocities of P form a geometric sequence. The acceleration of P is zero at times t_1, t_2, t_3 where $t_1 < t_2 < t_3$ and the respective velocities are v_1, v_2, v_3.

5e. Show that, at these times, $\tan 6t = 2$. *[2 marks]*

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5f. Hence show that $\frac{v_2}{v_1} = \frac{v_3}{v_2} = -e^{-\frac{\pi}{2}}$. *[5 marks]*

A rocket is travelling in a straight line, with an initial velocity of $140\,\mathrm{m\,s^{-1}}$. It accelerates to a new velocity of $500\,\mathrm{m\,s^{-1}}$ in two stages.

During the first stage its acceleration, $a\,\mathrm{m\,s^{-2}}$, after t seconds is given by $a(t) = 240\sin(2t)$, where $0 \leqslant t \leqslant k$.

6a. Find an expression for the velocity, $v\,\mathrm{m\,s^{-1}}$, of the rocket during the first stage. *[4 marks]*

The first stage continues for k seconds until the velocity of the rocket reaches 375 m s^{-1}.

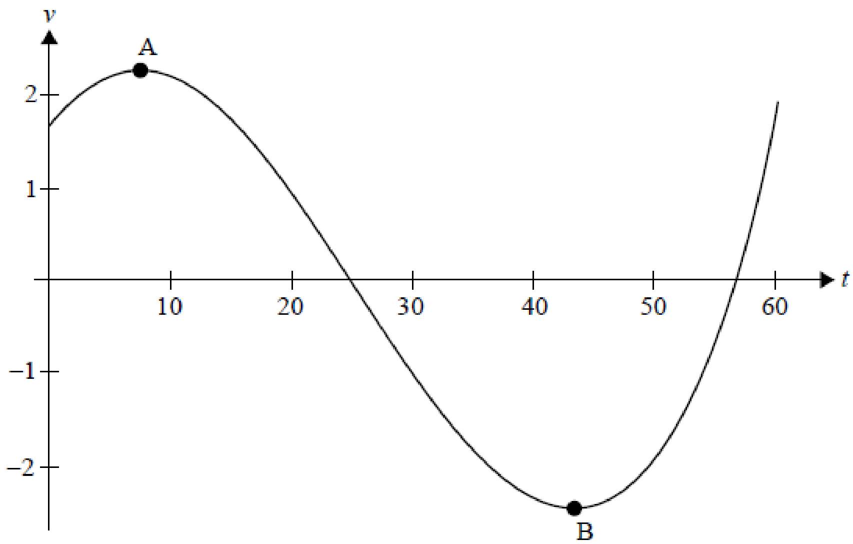
6b. Find the distance that the rocket travels during the first stage. *[4 marks]*

6c. During the second stage, the rocket accelerates at a constant rate. The distance which the rocket travels during the second stage is the same as the distance it travels during the first stage. *[6 marks]*

Find the total time taken for the two stages.

A body moves in a straight line such that its velocity, $v\,\mathrm{ms}^{-1}$, after t seconds is given by $v = 2\sin\left(\frac{t}{10} + \frac{\pi}{5}\right)\csc\left(\frac{t}{30} + \frac{\pi}{4}\right)$ for $0 \leqslant t \leqslant 60$.

The following diagram shows the graph of v against t. Point A is a local maximum and point B is a local minimum.



7a. Determine the coordinates of point A and the coordinates of point B. *[4 marks]*

7b. Hence, write down the maximum speed of the body. *[1 mark]*

The body first comes to rest at time $t = t_1$. Find

7c. the value of t_1. *[2 marks]*

7d. the distance travelled between $t = 0$ and $t = t_1$. *[2 marks]*

7e. the acceleration when $t = t_1$. *[2 marks]*

7f. Find the distance travelled in the first 30 seconds. *[3 marks]*

8. A particle moves along a horizontal line such that at time t seconds, $t \geq 0$, its acceleration a is given by $a = 2t - 1$. When $t = 6$, its displacement s from a fixed origin O is 18.25 m. When $t = 15$, its displacement from O is 922.75 m. Find an expression for s in terms of t. *[6 marks]*

A particle moves along a straight line so that its velocity, v m s^{-1}, after t seconds is given by $v\left(t\right) = 1.4^t - 2.7$, for $0 \le t \le 5$.

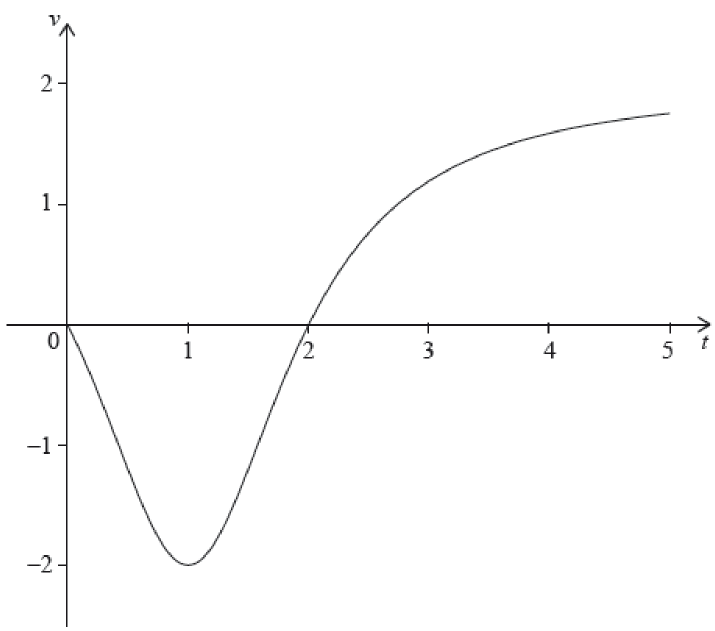
9a. Find when the particle is at rest. *[2 marks]*

9b. Find the acceleration of the particle when $t = 2$. *[2 marks]*

9c. Find the total distance travelled by the particle. *[3 marks]*

10. **Note: In this question, distance is in metres and time is in seconds.** *[6 marks]*

A particle moves along a horizontal line starting at a fixed point A. The velocity v of the particle, at time t, is given by $v(t) = \frac{2t^2-4t}{t^2-2t+2}$, for $0 \leqslant t \leqslant 5$. The following diagram shows the graph of v



There are t-intercepts at $(0,\ 0)$ and $(2,\ 0)$.

Find the maximum distance of the particle from A during the time $0 \leqslant t \leqslant 5$ and justify your answer.

A particle P moves along the x-axis. The velocity of P is $v \mathrm{~m~s}^{-1}$ at time t seconds, where $v(t) = 4 + 4t - 3t^2$ for $0 \leq t \leq 3$. When $t = 0,\ P$ is at the origin O.

11a. Find the value of t when P reaches its maximum velocity. *[2 marks]*

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11b. Show that the distance of P from O at this time is $\frac{88}{27}$ metres. *[5 marks]*

11c. Sketch a graph of v against t, clearly showing any points of intersection with the axes. *[4 marks]*

11d. Find the total distance travelled by P. *[5 marks]*

The acceleration, $a \text{ ms}^{-2}$, of a particle moving in a horizontal line at time t seconds, $t \geq 0$, is given by $a = -(1 + v)$ where $v \text{ ms}^{-1}$ is the particle's velocity and $v > -1$.

At $t = 0$, the particle is at a fixed origin O and has initial velocity $v_0 \text{ ms}^{-1}$.

12a. By solving an appropriate differential equation, show that the particle's velocity at time t is given by $v(t) = (1 + v_0)\mathrm{e}^{-t} - 1$. *[6 marks]*

Initially at O, the particle moves in the positive direction until it reaches its maximum displacement from O. The particle then returns to O.

Let s metres represent the particle's displacement from O and $s_{\max}$ its maximum displacement from O.

12b. Show that the time T taken for the particle to reach $s_{\max}$ satisfies the equation $\mathrm{e}^T = 1 + v_0$. *[2 marks]*

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12c. By solving an appropriate differential equation and using the result from part (b) (i), find an expression for $s_{\max}$ in terms of v_0. *[5 marks]*

Let $v(T-k)$ represent the particle's velocity k seconds before it reaches $s_{\max}$, where

$$v(T-k) = (1+v_0)e^{-(T-k)} - 1.$$

12d. By using the result to part (b) (i), show that $v(T-k) = e^k - 1$. *[2 marks]*

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Similarly, let $v(T+k)$ represent the particle's velocity k seconds after it reaches $s_{\max}$.

12e. Deduce a similar expression for $v(T+k)$ in terms of k. *[2 marks]*

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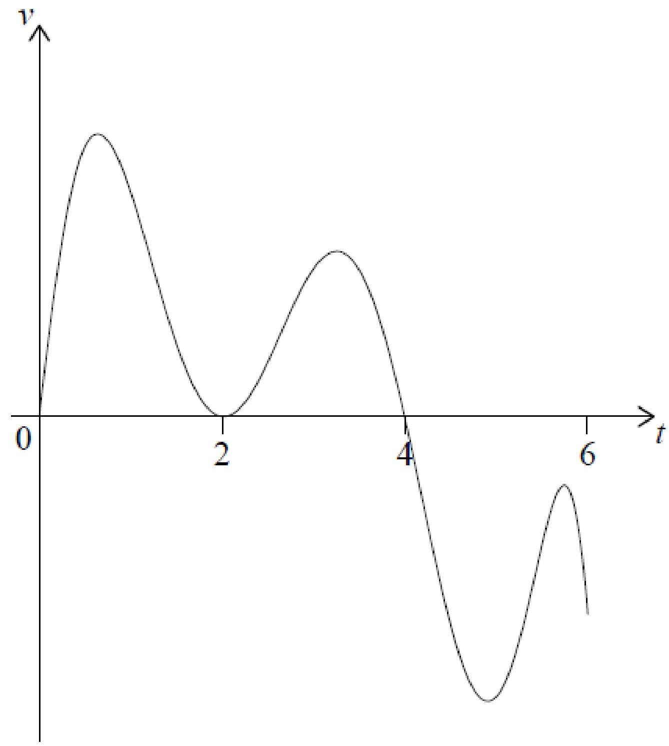
12f. Hence, show that $v(T-k)+v(T+k)\geq 0$. *[3 marks]*

A particle P starts from point O and moves along a straight line. The graph of its velocity, v ms^{-1} after t seconds, for $0 \le t \le 6$, is shown in the following diagram.



The graph of v has t-intercepts when t = 0, 2 and 4.

The function $s(t)$ represents the displacement of P from O after t seconds.

It is known that P travels a distance of 15 metres in the first 2 seconds. It is also known that $s(2) = s(5)$ and $\int_2^4 v \, \mathrm{d}t = 9$.

13a. Find the value of $s(4) - s(2)$. *[2 marks]*

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13b. Find the total distance travelled in the first 5 seconds. *[5 marks]*

14. A particle moves in a straight line such that at time t seconds $(t \geqslant 0)$, its velocity v, in ms^{-1}, is given by $v = 10t\mathrm{e}^{-2t}$. Find the exact distance travelled by the particle in the first half-second. *[5 marks]*

A particle moves along a straight line. Its displacement, s metres, at time t seconds is given by $s = t + \cos 2t, t \geqslant 0$. The first two times when the particle is at rest are denoted by t_1 and t_2, where $t_1 < t_2$.

15a. Find t_1 and t_2. *[5 marks]*

15b. Find the displacement of the particle when $t = t_1$ *[2 marks]*